Vibration Sensitivity Analysis of Nano-mechanical Piezo-Laminated Beams with Consideration of Size Effects

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Abstract: The presented article investigates vibration sensitivity analysis of Nano-mechanical piezo-laminated beams with consideration of size effects. To do this, the vibration governing equation of the stepped Nano-mechanical piezo-laminated beam is firstly derived by implementation of the nonlocal elasticity theory. The nonlocal formulation is considered for both of the beam and the piezoelectric layer and the obtained equation is solved analytically. Moreover, there is a need to recognize the importance and relative effects of the beam parameters on the natural frequencies and resonant amplitudes of the nonlocal beam. Therefore, the Sobol sensitivity analysis is utilized to investigate the relative effects of geometrical and the nonlocal parameters on the natural frequencies and the resonant amplitude of the nanobeam. The obtained results show that the length and the thickness of the piezoelectric layer have prominent effects on the vibration characteristics of the beam. Moreover, it is indicated that nonlocal parameter effect on the resonant amplitudes is more than resonant frequency. Also, the effect of the nonlocal term is more important at higher modes of vibration. Therefore, the nonlocal size effects cannot be ignored in vibration analysis of the nanobeam especially at higher modes.

Keywords: Nano-mechanical Beam, Piezoelectric, Size Effects, Vibration, Sensitivity

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Biographical notes: Mostafa Nazemizadeh received his PhD in Mechanical Engineering from Amirkabir University of Technology, in 2016. His current research interests are robotic nonlinear dynamics and optimal control, micro-to-nano dynamics and nonlinear vibration. **Firooz Bakhtiari-Nejad** received his PhD in Mechanical Engineering all from Kansas State University, in 1975. He is now the adjunct professor in the department of Mechanical Engineering, University of Maryland from 2017 to present. His current research interests are the nonlinear and random vibrations and controls of macro-to-nano systems. **Behrooz Shahriari** received PhD in aerospace engineering from Malek Ashtar University of Technology in 2016. He performed some industrial and academic projects and education in the fields of aerospace structural design and analysis.

1 INTRODUCTION

Nowadays, it is possible to manufacture infinitesimal structures and systems in the dimension of micro-to-Nano due to recent developments in Nano manufacturing technologies [1]. Proceeding along this line, a new miniature structure known as Nano-mechanical beams have emerged into nanotechnology applications such as Nano-resonators, Atomic Force Microscopes (AFM), bio-sensors, etc. [2-4]. The Nano implementations of the Nano-mechanical beams are originated from their novel features like low weight, small size, simple fabrication and high frequency operation [5].

In addition, the Nano-mechanical beams are mostly used in vibration modes of operation. With respect to the extensive implementation of vibrating Nano-mechanical beams, their precise vibration modelling and study are essential to identify their operation. Thus, Nano-researchers have focused on vibration analysis of the Nano-mechanical beams: Jiang et al. [6] investigated vibration modelling and analysis of a Nano-mechanical resonator. They presented a numerical solution to determine the vibration characteristics of the Nano-mechanical beam for Nano-scale deformations. Taheri [7] studied dynamic modelling of an atomic force Nano-mechanical beam in adjacent of a surface considering tip-sample interaction forces. Also they [8] studied dynamic modeling and simulation of a piezoelectric atomic force microscope. However, they modeled the system as a lumped mass which cannot be approved as an accurate model for a continuous beam especially in Nano scale. Souayeh et al. [9] presented a computational model for vibration of a carbon nanotube beam. Employing the Galerkin discretization method, they transformed the partial differential equation to a finite degrees of freedom system and solved it by the harmonic balance method.

As it can be seen from literature, the recent studies are based on classical continuum models of the small size beams. However, the capability of this theory for dynamic modelling of such systems is challenged recently [10]. The key motivation is due to importance of material structures such as lattice spacing in the micro-to-nano scale. As experiments and atomistic simulations are difficult and computationally expensive, the size-dependent elasticity theories have been established for vibration analysis of the Nano-mechanical beams. Jalali et al. [11] studied the free vibration of functionally graded micro-beams under thermal environment based on modified couple stress theory. They employed an approximate method to solve the eigenvalue problem for obtaining the natural frequencies of the beam. Also, in [12] the modified couple stress model of the Nano-mechanical beam was presented to investigate the size effects on the free vibration of the non-uniform beam. Demir et al. [13] studied response of Nano-beams resting on elastic foundations. They employed the finite element method for the analysis of Nano-beams under the Winkler foundation and the uniform load. Also, they considered the small-scale effect along with nonlocal elasticity theory to model the Nano-beam in the Nano-scale. Nazemizadeh and Bakhtiari-Nejad [14] presented the size-dependent free vibration of Nano-beams with piezoelectric-layered actuators. They studied the size effects on the linear vibration of the beam and reported that the non-local and dimensional parameters have significant effects on the free vibration of the beam. However, their study was limited by only changing one a parameter and keeping other parameters constant, simultaneously. Eftekhari et al. [15] investigated optimal vibration control of multi-layer micro-beams actuated by piezoelectric layer based on modified couple stress and surface stress elasticity theories. They analyzed effects of the independent material length scale on the resonant vibration characteristics of the structure. They indicate that the small material length scale plays a key role in the dynamic respond and vibration control of micro-beam integrated with piezoelectric layers.

With respect to the literature review, there is a need to dynamic modelling and vibration analysis of the nanobeam with consideration of longitudinal discontinuities and nonlocal effects of the beam and piezoelectric layers. Furthermore, several input parameters including geometrical and size effect ones can be considered for the Nano-beam and the relative effectiveness of each parameter on the vibration characteristics should be determined. The sensitivity analysis methods are known as a powerful instrument to investigate the effects of changing all parameters simultaneously and detect the relative effects of parameters on vibration behaviour of the beam. Therefore, the presented paper studies the vibration sensitivity analysis of the Nano-mechanical piezo-laminated beam with consideration of size effects. The sensitivity analysis includes the natural frequencies and the vibration amplitudes at the first two modes. At first, the vibration governing equation of the stepped Nano-mechanical piezo-laminated beam is derived employing the nonlocal elasticity theory. The nonlocal formulation is considered for both of the beam and the piezoelectric layer and the obtained equation is solved analytically. Then, the Sobol sensitivity analysis is employed to investigate the effects of different parameters on the vibration characteristics of the beam. Five parameters of the Nano-mechanical piezo-laminated beam are analyzed to investigate the relative effectiveness of each parameter on the natural frequency and the resonant amplitude. The parameters are chosen as the length of the piezoelectric layer, the length of the middle section of the beam (uncovered by piezoelectric layer), the length of the tip section of the beam (with narrower width), the thickness ratio of the piezoelectric layer to the beam and the nonlocal term. The obtained results show that the length and the thickness of the piezoelectric layer have prominent effects on the resonant amplitude of the beam. Also, it is indicated that nonlocal parameter effect on the resonant amplitudes is more than resonant frequency. Moreover, the effect of the nonlocal term increases with increment of the resonant mode of vibration. The rest of the paper is organized as follows: in section 2, the Sobol sensitivity formulation is presented. The governing equation of the piezo-laminated Nano-mechanical beam is developed in section 3. The simulation is presented in section 4 and then the paper is concluded.

2 SOBOL SENSETIVITY METHOD

The sensitivity analysis is known as an important tool for forming and understanding mathematical models in various forms. It affords assured information about the system with regards to the behaviour achieved from the simulation. In general, the sensitivity analysis is applied to wide ranges of applications such as model identification, model simplification and construction. If the purpose is to compare and evaluate the relative effects of several input parameters on a given output, the sensitivity analysis methods are used. Using the sensitivity analysis method, one can study the effect and relative importance of each defined input on the system output. The Sensitivity Analysis (SA) methods are classified into two groups: local SA and global SA. The local SA method estimates or approximates the partial derivatives of model outputs with respect to model inputs at some nominal settings. On the other hand, in contrast to the local SA approaches, the global SA methods evaluate the effect of model outputs in the whole permitted ranges of inputs [16]. Therefore, the global sensitivity analysis approaches have achieved more importance. The main idea in the global methods is the estimation of variance components for inputs or a group of inputs. The Sobol sensitivity analysis is a global and statistical SA approach. The Sobol sensitivity analysis method is recognized as a useful implementation for studying complex and multivariate systems.

In order to express the Sobol SA method, the input parameter range Ω is defined as [17]:

$$\Omega^k = \left\{ X \middle| 0 \le x_i \le 1; i = 1,2,...,k \right\} \tag{1}$$

The output function is stated as:

$$f(x_1,...,x_k) = f_0 + \sum_{i=1}^{k} f_i(x_i) + \sum_{1\le i<j\le k} f_{ij}(x_i,x_j) + ... \\ ... + f_{1,2,...,k}(x_1,...,x_k) \tag{2}$$

Where, f_0 is a constant value and is equal to:

$$f_0 = \int_{\Omega^k} f(x)\,dx \tag{3}$$

Moreover, Sobol proved the other terms of "Eq. (2)" are constant and the flowing relations are expressed:

$$f_i(x_i) = -f_0 + \int_0^1 ... \int_0^1 f(X)\,dX_{-i} \tag{4}$$

$$f_{ij}(x_i,x_j) = -f_0 - f_i(x_i) + \int_0^1 ... \int_0^1 f(X)\,dX_{-ij} \tag{5}$$

Where, dX_{-i} indicates an integral on all variables except the variable x_i and dX_{-ij} expresses the integral on all variables except the variables x_i and x_i . Also, in the Sobol statistical method, variance sensitivity index is expressed as:

$$D = \int_{\Omega^k} f^2(X)\,dX - f_0^2 \tag{6}$$

And the partial variances are as:

$$D_{i_1,...,i_s} = \int_0^1 ... \int_0^1 f^2_{i_1,...,i_s}(x_{i1},...,x_{is})\,dX_{i1}...dX_{is} \tag{7}$$

Where, $1 \le i_1 < ... < i_s \le k$ and the total variance is given as:

$$D = \sum_{i=1}^{k} D_i + \sum_{1\le i<j\le k} D_{ij} + ... + D_{1,2,...,k} \tag{8}$$

Therefore, the sensitivity index is obtained by dividing the variance of each variable into the total variance:

$$S_{1,2,...,k} = \frac{D_{1,2,...,k}}{D} \qquad 1 \le i \tag{9}$$

Where, S_i is the first-order sensitivity index and states the effect of the input x_i on the output.

3 VIBRATION FORMULATION

To sensitivity analysis of the piezo-laminated Nano-mechanical beam, a cantilever is coated by a piezoelectric layer on its top surface based on the nonlocal elasticity theory. The main feature of the nonlocal continuum mechanics is that the nonlocal stress tensor at a reference point depends not only on strain tensor of the same coordinate but also on all other points of the body. Eringen presented the nonlocal stress formulation as an integral and proposed an alternative differential equation as [18]:

$$\left(1-\mu^2\nabla^2\right)\sigma_{ij}=c_{ijkl}\,\varepsilon_{kl} \tag{10}$$

In which, σ_{ij} is the nonlocal stress tensor, ε_{ij} is the strain tensor, ∇^2 is the Laplacian operator, and μ is a scale coefficient that interprets the size-dependent nonlocal effects.

Furthermore, the nonlocal elasticity has been developed for the piezoelectric materials, recently. The nonlocal differential equation [19]:

$$\begin{aligned}\left(1-\mu^2\nabla^2\right)\sigma_{ij}&=c_{ijkl}\,\varepsilon_{kl}-e_{kij}E_k\\ \left(1-\mu^2\nabla^2\right)D_i&=e_{ikl}\,\varepsilon_{kl}-\xi_{kij}E_k\end{aligned} \tag{11}$$

Where, E_k and e_{kij} are the electric field and piezoelectric constants. The above nonlocal constitutive relations can be rewritten in a one-dimensional form as:

$$\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)\sigma_{xx,1}=c_1\,\varepsilon_{xx} \tag{12}$$

$$\begin{aligned}\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)\sigma_{xx,2}&=c_2\,\varepsilon_{xx}-e_2E_z\\ \left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)D_x&=e_2\,\varepsilon_{xx}-\xi_2\,E_z\end{aligned} \tag{13}$$

Where, $\sigma_{xx,1}$ and $\sigma_{xx,2}$ are the nonlocal stress components in the beam and the piezoelectric layer along the x-direction.

Now to utilize the nonlocal constitutive equations, a Nano-mechanical cantilever with a piezoelectric patch covered on its top surface is considered ("Fig. 1"). The total length of the beam is l. The beam has three different sections along the length. The first section is designed until l_1 with width and height of b_1 and h_1, respectively. The piezoelectric patch is bonded along the x-direction in this section and the width and thickness of the piezoelectric layer are b_2 and h_2, respectively. The second section of the beam is from l_1 until l_2 with width and height of b_1 and h_1. Also, the third section of the beam is from l_2 until l with narrower width and height of b_3 and h_3.

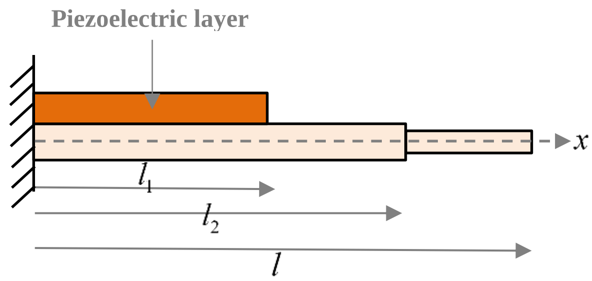


Fig. 1 A Nano-mechanical beam with a piezoelectric layer.

It is considered that lateral displacement of the beam is parallel to z-axis. So, the longitudinal and lateral displacements of an arbitrary point in a beam section are:

$$U(x,y,z,t)=-(z-z_n)\frac{\partial w}{\partial x},W(x,y,z,t)=w(x,t) \tag{14}$$

So, the non-zero strain component ε_{xx} of the beam is:

$$\varepsilon_{xx}=-(z-z_n)\frac{\partial^2 w(x,t)}{\partial x^2} \tag{15}$$

Furthermore, to derive the vibration equation of the Nano-beam, the Hamilton's principle is implemented. Therefore, the kinetic energy T of the piezo-laminated can be summarized as:

$$T=\frac{1}{2}\int_0^l\begin{Bmatrix}(\rho_1A_1+\rho_2A_2)H(0l_1)\\+\rho_1A_1\,H(l_1l_2)+\rho_1A_3\,H(l_2l)\end{Bmatrix}\left(\frac{\partial w}{\partial t}\right)^2dx \tag{16}$$

Where, ρ_1 and ρ_2 are the density of the beam and the piezoelectric layer, respectively. Also, $H(x)$ is the Heaviside function and $H(ij)=H(x-i)-H(x-j)$ is defined.

Moreover, the potential energy of the piezoelectric laminated beam can be written as:

$$\pi=\frac{1}{2}\iiint_{V_1}\sigma_{xx,1}\,\varepsilon_{xx}\,dV_1+\frac{1}{2}\iiint_{V_2}\left(D_zE_z-\sigma_{xx,2}\,\varepsilon_{xx}\right)dV_2 \tag{17}$$

Now by substituting equation (15) into (12) and (13), and then submitting these equations into (17), the potential energy can be rewritten as:

$$\pi = \frac{1}{2}\int_0^l \left(M_{xx} \frac{\partial^2 w(x,t)}{\partial x^2} + \kappa E_z \right) dx \qquad (18)$$

In which the following relations are given:

$$\begin{aligned} M_{xx} = &\iint_{A_1} \sigma_{xx,1}(z-z_n) H(0l_1) dA_1 + \iint_{A_1} \sigma_{xx,1} zH(l_1 l_2) dA_1 \\ &+ \iint_{A_2} \sigma_{xx,2}(z-z_n) H(0l_1) dA_2 + \iint_{A_3} \sigma_{xx,3} zH(l_1 l_2) dA_3 \end{aligned} \qquad (19)$$

$$\kappa(x) = \iint_{A_2} D_z H(0l_1) dA_2 \qquad (20)$$

Furthermore, the Hamilton's principle states:

$$\int_{t_0}^{t} (\delta T - \delta\pi + \delta W_{nc}) dt = 0 \qquad (21)$$

Where, W_{nc} is the work of non-conservative forces. Now, by substituting relations (16) and (18) into (21) and performing some mathematical calculations, the governing vibration equation of the piezo-laminated Nano-mechanical based on the elasticity theory is obtained as:

$$\rho_{eff} A_{eff} \frac{\partial^2 w(x,t)}{\partial t^2} = \frac{\partial^2 M_{eff}}{\partial x^2} \qquad (22)$$

Where, the effective inertia of the system is given as:

$$\begin{aligned} \rho A_{eff} = &\rho_1 A_1 H(0l_1) + \rho_2 A_2 H(0l_1) \\ &+ \rho_1 A_1 H(l_1 l_2) + \rho_1 A_3 H(l_1 l_2) \end{aligned} \qquad (23)$$

Moreover, by integrating equations (12) and (13) on the cross-sectional areas of the Nano-mechanical beam and the piezoelectric layers, the nonlocal formulation will be achieved as:

$$M_{eff} - \mu^2 \frac{\partial^2 M_{eff}}{\partial x^2} = -cI_{eff} \frac{\partial^2 w}{\partial x^2} - \kappa e_2 Q_2 E_z \qquad (24)$$

Where the following equations are defined as:

$$\begin{aligned} cI_{eff} = &cI_1 H(0l_1) + cI_2 H(0l_1) \\ &+ cI_3 H(l_1 l_2) + cI_3 H(l_2 l) \end{aligned} \qquad (25)$$

$$\begin{aligned} &\{I_1, I_2\} = \iint_{\{A_1, A_2\}} (z - z_n)^2 \{dA_1, dA_2\} \\ &\{I_3\} = \iint_{A_3} z^2 \, dA_3 \\ &Q_2 = \iint_{A_2} (z - z_n) \, dA_2 \end{aligned} \qquad (26)$$

Now, by substituting equation (22) into (24), it will be:

$$\begin{aligned} M_{eff} = &-cI_{eff} \frac{\partial^2 w(x,t)}{\partial x^2} + k_p(x) V_p \\ &+ \mu^2 \rho A_{eff} \frac{\partial^2 w(x,t)}{\partial t^2} \end{aligned} \qquad (27)$$

Where, V_p and k_p are the piezoelectric voltage and coefficient of applied voltage, respectively. Therefore, by substituting equation (27) into equations (22), the governing equation of motion of the piezo-laminated Nano-mechanical beam can be summarized as:

$$\left[\begin{aligned} &\left(1 - \mu^2 \frac{\partial^2}{\partial x^2}\right)\left(\rho A_{eff} \frac{\partial^2 w(x,t)}{\partial t^2}\right) \\ &+ \frac{\partial^2}{\partial x^2}\left(cI_{eff} \frac{\partial^2 w(x,t)}{\partial x^2}\right) \end{aligned} \right] = \frac{\partial^2}{\partial x^2}\left[k_p(x) V_p\right] \qquad (28)$$

And the boundary conditions are given as:

$$\begin{aligned} &w(0,t) = 0 \quad and \quad \frac{\partial w(0,t)}{\partial x} = 0 \\ &-cI_{eff} \frac{\partial^2 w(l,t)}{\partial x^2} + \mu^2 \rho A_{eff} \frac{\partial^2 w(l,t)}{\partial t^2} = 0 \\ &\frac{\partial}{\partial x}\left[-cI_{eff} \frac{\partial^2 w(l,t)}{\partial x^2} + \mu^2 \rho A_{eff} \frac{\partial^2 w(l,t)}{\partial t^2} \right] = 0 \end{aligned} \qquad (29)$$

4 FREQUENCY SOLUTION

In section 2, governing equations and boundary conditions of a piezo-laminated Nano-mechanical beam are derived. For sensitivity analysis of the problem, at first the vibration equation is analytically solved. To do this, the Nano-mechanical beam is considered as three longitudinal sections. Therefore, the governing equation (28) can be rewritten as:

$$\begin{cases}\begin{bmatrix}\rho A_{eff,1}\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2 w_1(x,t)}{\partial t^2}\right)\\+cI_{eff,1}\frac{\partial^4 w_1(x,t)}{\partial x^4}\end{bmatrix}=\frac{\partial^2}{\partial x^2}\left(k_pV_p\right) & 0<x<l_1\\ \begin{bmatrix}\rho A_{eff,2}\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2 w_2(x,t)}{\partial t^2}\right)\\+cI_{eff,2}\frac{\partial^4 w_2(x,t)}{\partial x^4}\end{bmatrix}=0 & l_1<x<l_2\\ \begin{bmatrix}\rho A_{eff,3}\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2 w_3(x,t)}{\partial t^2}\right)\\+cI_{eff,3}\frac{\partial^4 w_3(x,t)}{\partial x^4}\end{bmatrix}=0 & l_2<x<l\end{cases} \quad (30)$$

Moreover, the boundary conditions and compatibility conditions can be given as:

$$w_1(0,t)=0,\quad \frac{\partial w_1(0,t)}{\partial x}=0 \quad (31)$$

$$w_1(l_1,t)=w_2(l_1,t),\quad \frac{\partial w_1(l_1,t)}{\partial x}=\frac{\partial w_2(l_1,t)}{\partial x}$$

$$\begin{bmatrix}-cI_{eff,1}\frac{\partial^2 w_1(l_1,t)}{\partial x^2}\\+\mu^2\rho A_{eff,1}\frac{\partial^2 w_1(l_1,t)}{\partial t^2}\end{bmatrix}=\begin{bmatrix}-cI_{eff,2}\frac{\partial^2 w_2(l_1,t)}{\partial x^2}\\+\mu^2\rho A_{eff,2}\frac{\partial^2 w_1(l_1,t)}{\partial t^2}\end{bmatrix} \quad (32)$$

$$\begin{bmatrix}-cI_{eff,1}\frac{\partial^3 w_1(l_1,t)}{\partial x^3}\\+\mu^2\rho A_{eff,1}\frac{\partial^3 w_1(l_1,t)}{\partial x\partial t^2}\end{bmatrix}=\begin{bmatrix}-cI_{eff,2}\frac{\partial^3 w_2(l_1,t)}{\partial x^3}\\+\mu^2\rho A_{eff,2}\frac{\partial^3 w_2(l_1,t)}{\partial x\partial t^2}\end{bmatrix}$$

$$w_2(l_2,t)=w_3(l_2,t),\quad \frac{\partial w_2(l_2,t)}{\partial x}=\frac{\partial w_3(l_2,t)}{\partial x}$$

$$\begin{bmatrix}-cI_{eff,2}\frac{\partial^2 w_2(l_2,t)}{\partial x^2}\\+\mu^2\rho A_{eff,2}\frac{\partial^2 w_2(l_2,t)}{\partial t^2}\end{bmatrix}=\begin{bmatrix}-cI_{eff,3}\frac{\partial^2 w_3(l_2,t)}{\partial x^2}\\+\mu^2\rho A_{eff,3}\frac{\partial^2 w_3(l_2,t)}{\partial t^2}\end{bmatrix} \quad (33)$$

$$\begin{bmatrix}-cI_{eff,2}\frac{\partial^3 w_2(l_2,t)}{\partial x^3}\\+\mu^2\rho A_{eff,2}\frac{\partial^3 w_2(l_2,t)}{\partial x\partial t^2}\end{bmatrix}=\begin{bmatrix}-cI_{eff,3}\frac{\partial^3 w_3(l_2,t)}{\partial x^3}\\+\mu^2\rho A_{eff,3}\frac{\partial^3 w_3(l_2,t)}{\partial x\partial t^2}\end{bmatrix}$$

$$\begin{bmatrix}-cI_{eff,3}\frac{\partial^2 w_3(l,t)}{\partial x^2}\\+\mu^2\rho A_{eff,3}\frac{\partial^2 w_3(l,t)}{\partial t^2}\end{bmatrix}=0$$

$$\begin{bmatrix}-cI_{eff,3}\frac{\partial^3 w_3(l,t)}{\partial x^3}\\+\mu^2\rho A_{eff,3}\frac{\partial^3 w_3(l,t)}{\partial x\partial t^2}\end{bmatrix}=0 \quad (34)$$

Therefore, the harmonic solution for the transverse vibration of the Nano-mechanical beam in each section is in the form of $w_i(x,t)=W_i(x)e^{j\omega t}$ in which ω is the natural frequencies of the beam. So, the harmonic solution substituted into "Eq. (30)" results in:

$$\begin{aligned}W_i(x)=&C_{1,i}\cos(\alpha_i x)+C_{2,i}\sin(\alpha_i x)\\&+C_{3,i}\cosh(\beta_i x)+C_{4,i}\sinh(\beta_i x)\end{aligned} \quad (35)$$

Now, by substituting the boundary conditions into "Eq. (35)", a 12×12 matrix is achieved that the natural frequencies of the system can be obtained from solution of its determinant. In addition, the natural frequency and the shape function of each resonance vibration mode of the Nano-mechanical beam are obtained. Then the response of the system is presumed as $w_i(x,t)=\sum W_i(x)\,q_i(t)$, and substituting the solution into the governing equation (28), the following equation is obtained:

$$M\,\ddot{\vec{q}}+C\,\dot{\vec{q}}+K\,\vec{q}=\vec{Q} \quad (36)$$

In which the indices of matrices are determined as:

$$
\begin{aligned}
m_{ij} &= \int_0^l \left[W_j(x)\left(1-\mu^2 \frac{d^2}{dx^2}\right)\left(\rho A_{eff,i} W_i(x)\right)\right]dx \\
c_{ij} &= \int_0^l \left[W_j(x)\left(1-\mu^2 \frac{d^2}{dx^2}\right)\left(B\, W_i(x)\right)\right]dx \\
k_{ij} &= \int_0^l \left[W_j(x)\, \frac{d^2}{dx^2}\left(cI_{eff,i}\, \frac{d^2 W_i(x)}{dx^2}\right)\right]dx \\
Q_i &= \int_0^l \left[W_j(x)\, \frac{d^2}{dx^2}\left(k_p(x)\, V_p\right)\right]dx
\end{aligned}
\tag{37}
$$

In the next section, the natural frequency and resonant amplitude of the Nano-mechanical beam are obtained for a lot of data proposed by the Sobol's method.

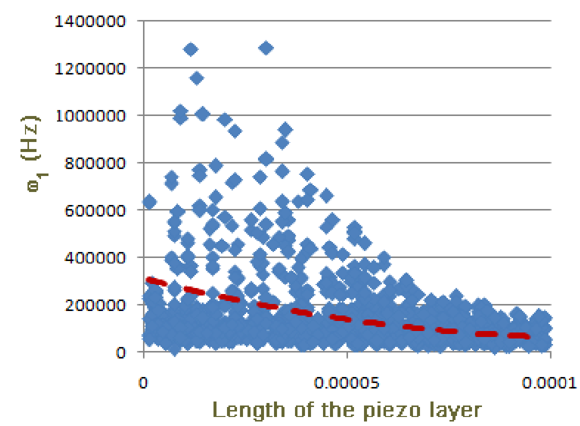
5 SENSITIVITY ANALYSIS RESULTS

In common studies, the effect of various parameters on the vibration characteristics of the systems is investigated while other parameters are fixed. On the other hand, to improve performance of the system, determining of these parameters cannot be by trial and error. In spite, the sensitivity analysis can be an efficient procedure to study and optimize the system behavior which can change all parameters simultaneously and detect the effect of them on the systems. In the sensitivity analysis, the variation of input parameters versus an arbitrary output is determined. Moreover, the relative effect of each parameter on the output is obtained. The sensitivity analysis methods are widely applied to the complex systems with various input parameters such as micro-to-nano systems. Matviykiv et al. [20] studied the sensitivity analysis of MEMS cantilever sensors. They only modeled a uniform cantilever. Also, they did not consider size effects in their analysis which cannot be ignored in micro and Nano scales. Moreover, in [21], the sensitivity analysis of various adhesion and friction formulations on manipulation in Nano scale are presented.
At first to validate the analytical solution of the problem, the non-dimensional first natural frequency of the present word is compared with Ref. [22] for a uniform nonlocal beam ("Table 1").

Table 1 Comparison of the first natural frequency

	1st natural frequency	
Nonlocal term	Present work	Ref. [22]
0.1	1.8794	1.8792

Furthermore, in this section, five parameters of the Nano-mechanical beam are analyzed to investigate the relative effectiveness of each parameter on the vibration characteristics. The parameters are chosen as the length of the piezoelectric layer, the length of the middle section of the beam (uncovered by piezoelectric), the length of the tip section of the beam (with narrower width), the thickness ratio of the piezoelectric layer to the beam and the nonlocal term. Using the Sobol's sensitivity analysis, 3584 random data is generated in which the length parameters are chosen on the interval [200, 10000] nm, thickness ratio is chosen on the interval [0.2, 2] and the nonlocal term on the interval [0, 0.1]. In "Fig. 2", the sensitivity analysis of the fundamental natural frequency (ω_1) versus the input parameters is plotted.



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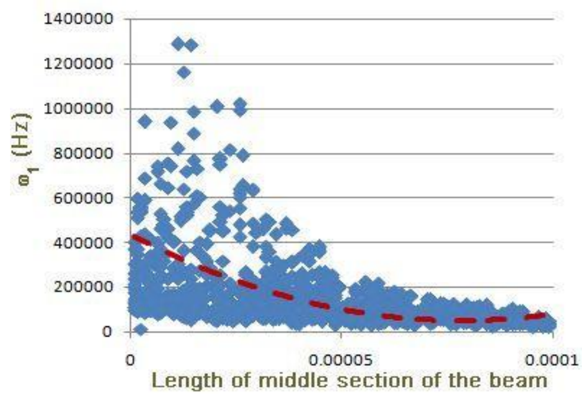

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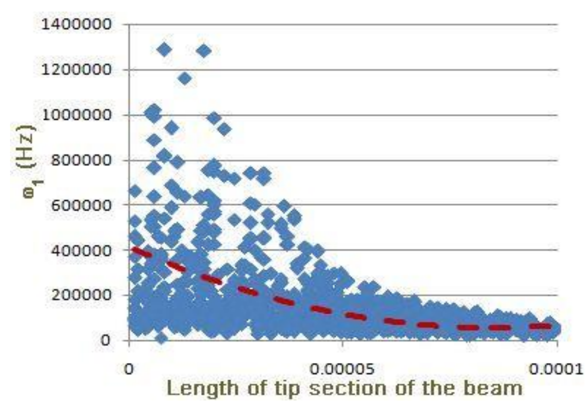

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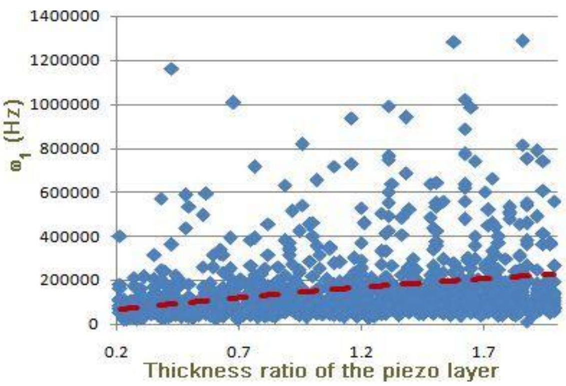

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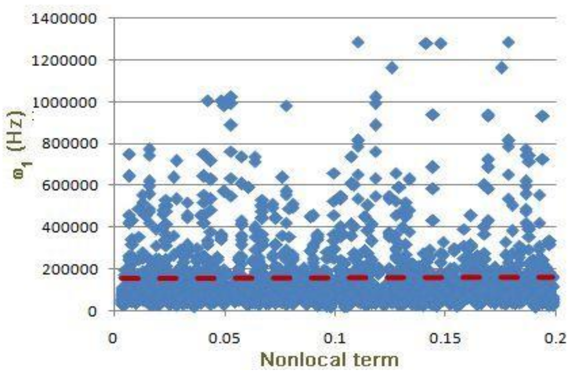
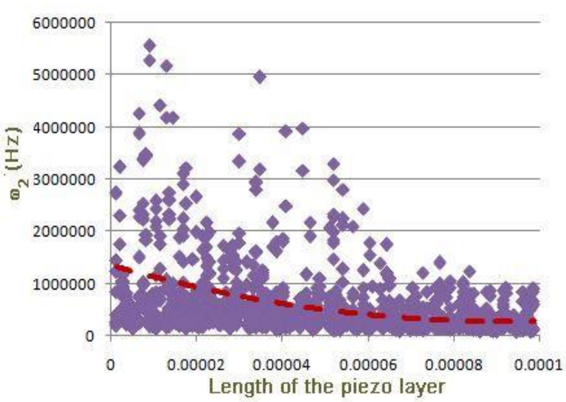


e

Fig. 2 The first frequency versus length of the: (a): first section, (b): middle section, and, the first frequency versus length of the: (c): tip section, (d): piezo thickness ratio, and (e): nonlocal term

As it is seen in “Fig. 2”, increment of each length parameter decreases the fundamental frequency of the beam because the length parameter is in denominator of the frequency equations. However, the effect of the tip section length is more prominent. The first frequency is increased by increment of the piezoelectric thickness ratio. Also, the nonlocal term has a little increasing effect on the first natural frequency.

In “Fig. 3”, the sensitivity analysis of the second natural frequency of the Nano-mechanical beam (ω_2) is presented.



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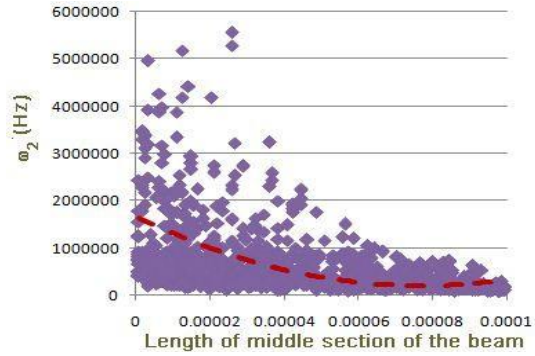

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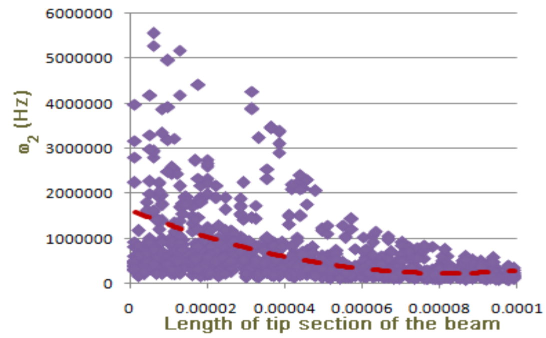

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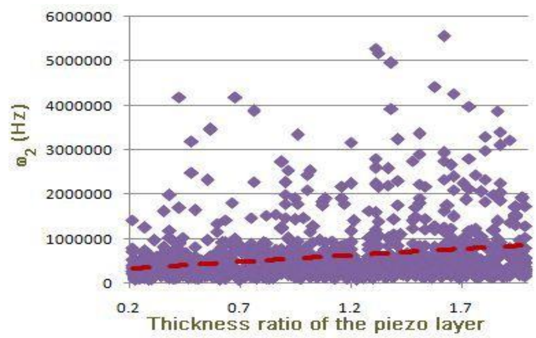

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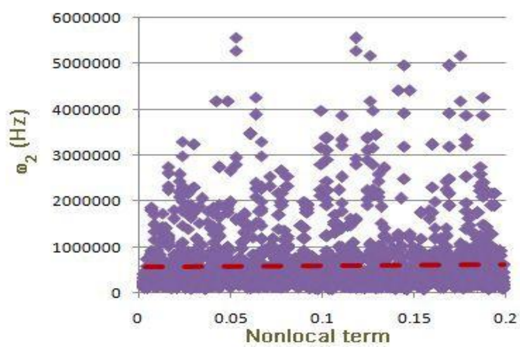
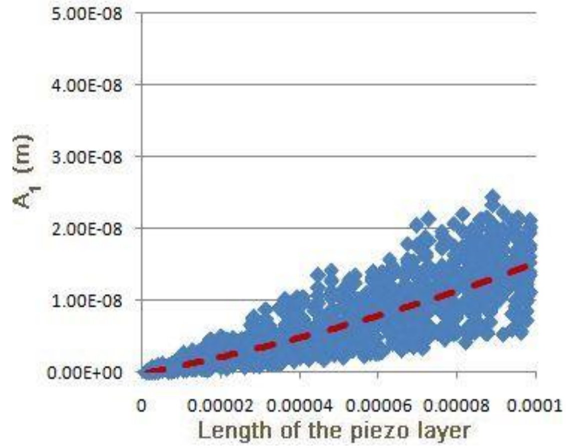


e

Fig. 3 The second frequency versus length of the: (a): first section, (b): middle section, and, the second frequency versus length of the: (c): tip section, (d): piezo thickness ratio, and (e): nonlocal term

According to “Fig. 3”, the second natural frequency is reduced with increasing the length of each section. The thickness ratio increases the frequency. In addition, increasing the nonlocal term leads to decrement of the second frequency. This reduction is related to decreasing of the Nano-mechanical beam stiffness and can be explained as the nonlocal elasticity theory which assumes that atoms are connected by an elastic matrix while the classic theory assumes that atoms are linked rigidly.

In "Fig. 4", the sensitivity analysis of the first resonant amplitude of the beam tip A_1 is presented.



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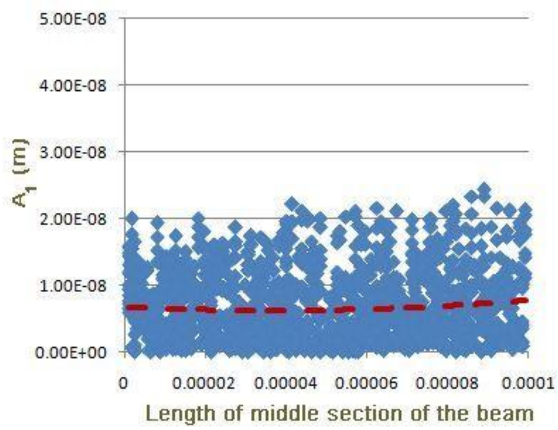

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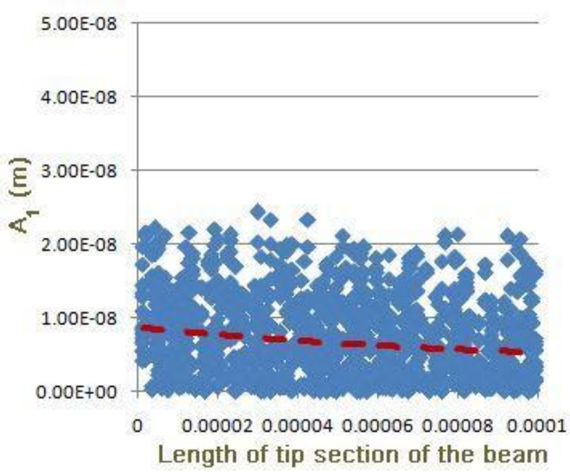

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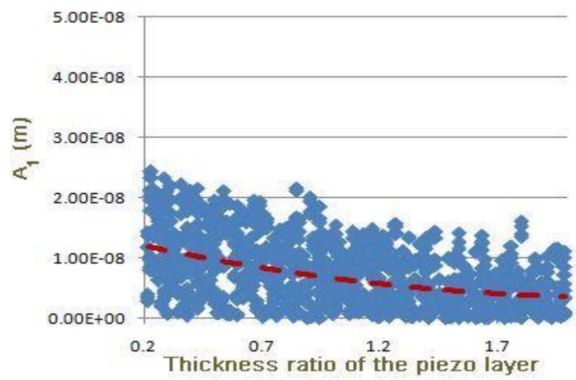

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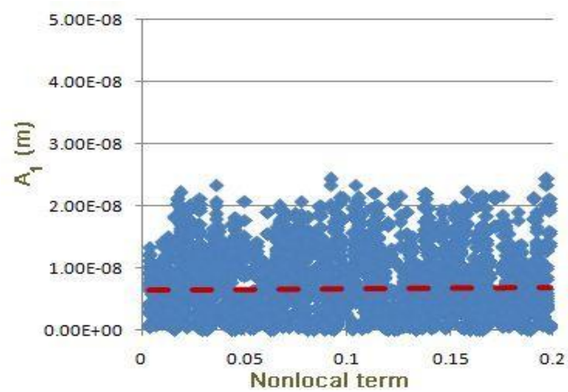

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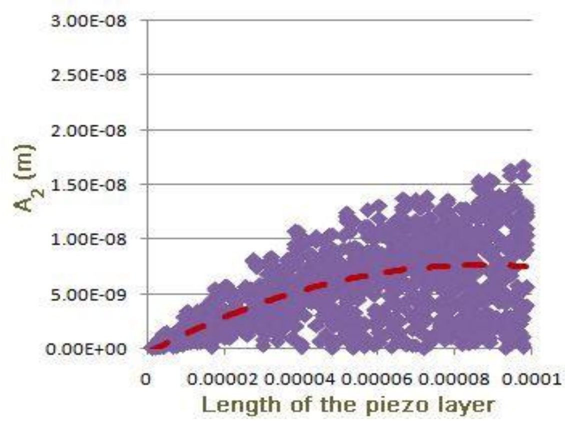
Fig. 4 The first resonant amplitude versus length of the: (a): first section, (b): middle section, (c): tip section, and, the first resonant amplitude versus the: (d): piezo thickness ratio, (e): nonlocal term.

As it is seen in "Fig. 4", the first resonance amplitude is increased by increment of the length of the piezoelectric layer. Also increasing the tip length and the piezo height ratio decreases the first resonance amplitude. Hence, the effects of the middle length and the nonlocal term are small, the effects of the length of the piezo layer and the thickness ratio are prominent, respectively. Also, the sensitivity analysis of the second resonant amplitude of the beam tip A_2 is presented in "Fig. 5".



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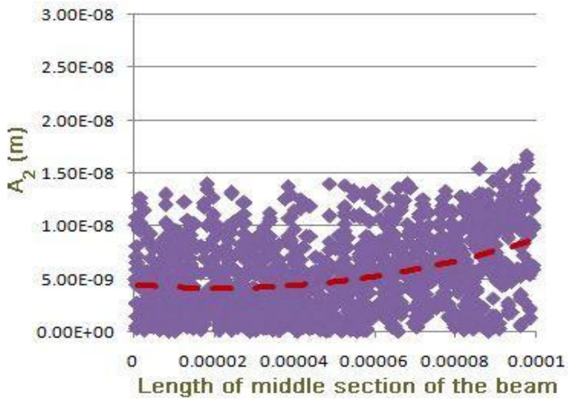

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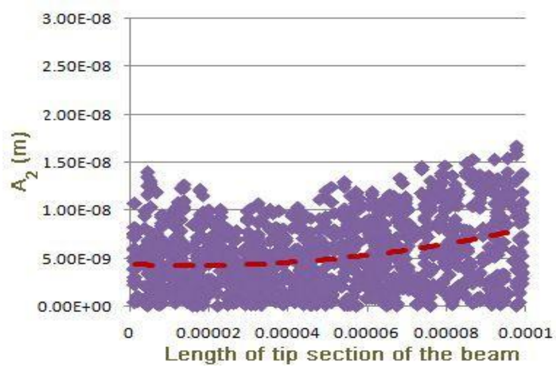

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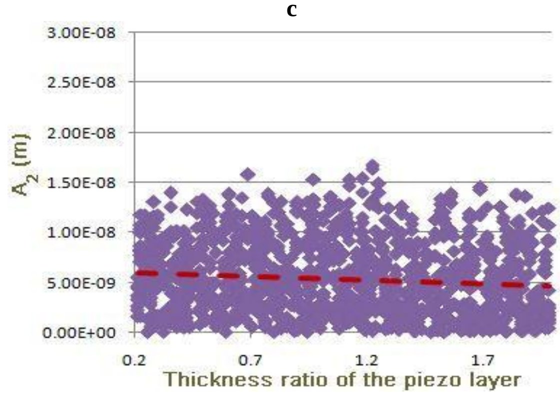

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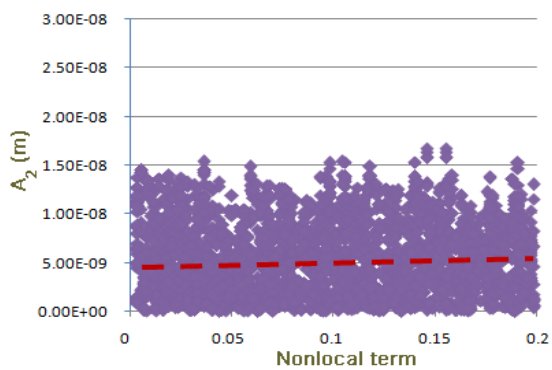

e

Fig. 5 The second resonant amplitude versus length of the: (a): first section, (b): middle section, (c): tip section, and, the second resonant amplitude versus the: (d): piezo thickness ratio, (e): nonlocal term.

As it is seen in “Fig. 5”, the second resonance response is increased with increment of the length of the piezo layer, the middle and the tip sections and the nonlocal term. It is seen that the effect of nonlocal term on the second mode is more obvious. It can be explained by the reality that wavelengths are decreased for higher modes and the stronger interactions between atoms leads to increasing of the nonlocal elasticity effect. Moreover, increasing the thickness ratio decreases the second resonance response. This matter is originated from increasing of the Nano-mechanical beam stiffness. On the other hand, the relative effect of the input parameters on the natural frequency of the Nano-mechanical beam is reported in “Table 2”.

Table 2 Sensitivity analysis of the frequencies

Input parameter	Output parameter	
	ω_1	ω_2
piezo length	15.17	21.73
middle length	32.16	29.25
tip length	44.24	35.11
piezo height	7.41	9.89
nonlocal term	1.02	3.02

According to “Table 2”, the effect of the tip length is prominent and can be a crucial parameter in Nano-mechanical resonators. Moreover, the relative effect of the nonlocal term and piezoelectric height increases by increment of the vibrating modes.

Also, the relative effect of the input parameters on the resonant amplitudes of the Nano-mechanical beam is presented in “Table 3”.

Table 3 Sensitivity analysis of the resonant amplitudes

Input parameter	Output parameter	
	A_1	A_2
piezo length	61.12	41.18
middle length	3.73	17.42
tip length	10.41	22.42
piezo height	23.55	15.41
nonlocal term	2.19	3.57

As it is seen in “Table 3”, the piezoelectric length has the most effect on the resonant response especially in the first mode. Also, the effect of the nonlocal parameter on the resonant amplitudes is more than the natural frequencies.

6 CONCLUSION

In the presented research, the vibration sensitivity analysis of the Nano-mechanical piezo-laminated beam has been investigated with consideration of size effects.

At first, the vibration governing equation of the beam is using the nonlocal elasticity theory. Then, the Sobol sensitivity analysis is utilized to analyze the effects of different parameters on the natural frequencies and the resonant amplitude of the beam. The input parameters are chosen as the length of the piezoelectric layer, the length of the middle section of the beam, the length of the tip section of the beam, the thickness ratio of the piezoelectric layer to the beam and the nonlocal term. Regarding to the sensitivity analysis, the following results can be highlighted:

- The resonant frequencies and amplitudes of the Nano-mechanical beam are sensitive to its geometrical characteristics and size effect. The relative effectiveness of each parameter can be an important attitude for other researches such as nano manufacturing, sensing applications, atomic force microscopy and etc.
- The sensitivity analysis shows that the length and the thickness of the piezoelectric layer have prominent effects on the first resonant amplitude of the Nano-mechanical beam. However, the length of the tip and middle sections has more effect on the first resonant frequency.
- However, the nonlocal term has less effect on the vibration characteristics of the Nano-mechanical beam, its effect on the resonant amplitudes is more than resonant frequency. Moreover, the effect of the nonlocal term increases with increment of the resonant mode of vibration and cannot be neglected.
- Although the length of the piezoelectric is the most effective parameter of the resonant amplitudes, but this relative effect decreases by increment of the mode number. Also, the length of the tip section has prominent effect on the resonant frequencies of the Nano-mechanical beam, but the relative effectiveness decreases with increment of the mode vibration.

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