Investigating the Effect of Piezoelectric layers on Circular Plates under Forced Vibration

A. A. Jandaghian*
Department of Mechanical Engineering,
University of Khajeh Nasir Toosi, Tehran, Iran
E-mail: Aliakbar_Jandaghian@yahoo.com
*Corresponding author

A. A. Jafari
Department of Mechanical Engineering,
University of Khajeh Nasir Toosi, Tehran, Iran
E-mail: ajafari@kntu.ac.ir

Received: 27 November 2011, Revised: 19 July 2011, Accepted: 6 May 2012

Abstract: This paper deals with the harmonic forced vibration of a circular plate surface bonded by two piezoelectric layers, based on the Kirchhoff plate model. The electric potential field in the piezoelectric layer is assumed such that the Maxwell static electricity equation is satisfied. The differential equations of motion are solved analytically for clamped edge boundary condition of the plate. The solutions are expressed by elementary Bessel functions. The results are verified by those obtained from finite-element analysis as well as the findings stated in the literature.

Keywords: Circular Plates, Classical Plate Theory Harmonic Vibration, Piezoelectric

Reference: Jandaghian, A. A. and Jafari, A. A. “Investigating effect of using piezoelectric layers on the forced vibration of circular plates”, Int J of Advanced Design and Manufacturing Technology, Vol. 5/ No.5, 2012, pp. 1-9.

Biographical notes: A. A. Jafari received his PhD in Mechanical Engineering from University of Wollongong, Australia. He is currently Associate Professor at the Department of Mechanical Engineering, K.N.T University of Technology Tehran, Iran. His current research interest includes Modal Analysis. **A. A. Jandaghian** received his BSc in Mechanical Engineering. He has also obtained his master's degrees in Mechanical Engineering at the K.N.T University of Technology Tehran, Iran. He is interested in vibration of plates.

1 INTRODUCTION

The analysis of a coupled piezoelectric structure has recently been keenly researched because piezoelectric materials are more extensively used either as actuators or sensors. Examples include the analytical modeling and behavior of a beam with surface-bonded or embedded piezoelectric sensors and actuators [1-3], use of piezoelectric materials in composite laminates and vibration control [4], [5]. The use of finite-element method in the analysis of piezoelectric coupled structures has been studied [6-10] and implemented in commercial FEA codes [11], [12].

The challenge of developing a basic mechanical model for the piezoelectric coupled structure has been met by many researchers. Crawley and de Luis developed a uniform strain model for a beam with surface bonded and embedded piezoelectric actuator patches accounting for the shear lag effects of the adhesive layer between the piezoelectric actuator and the beam [13]. A model to account for the coupling effect was later proposed based on the Euler beam assumption by Crawley and Anderson [14]. Leibowitz and Vinson derived a model based on Hamilton's principle in which the elastic layers, soft-core layers or piezoelectric layers are included [15]. Ding *et al* obtained the general solutions for the coupled dynamic equations of a transversely isotropic piezoelectric medium [16]. Recently, Sun *et a*l and Zhang *et al* presented their research on the analysis of a sandwich beam and plate structure containing a piezoelectric core [17-19]. The piezoelectric core is positioned such that an electric field in the thickness direction would generate shear deformation within the core. Models for composite structures with piezoelectric materials as sensors and actuators have also been published [20], [21]. Wang and Quek [22] presented their research on the free vibration of a piezoelectric sandwich beam structure, in which the piezoelectric effect on the resonance frequencies of the structure and the distribution of the electric potential are studied and analyzed. Also, they analyzed free vibration of piezoelectric coupled circular plate [23]. For isotropic elastic materials, a good account of axisymmetric static bending of circular plates is recorded in [24]. Similar problems for piezoelectric materials are discussed in [25], [26]. Zhang et al investigated the static and transient bending of a piezoelectric circular plate [27].

However, to the author's best knowledge, no researches dealing with the forced vibration of circular plate integrated with the piezoelectric layer have been reported. Therefore, the present work attempts to solve the problem of providing an exact solution for forced vibration of thin circular plate with two full size surface-bonded piezoelectric layers on the top and bottom of plate. The formulation is based on CPT. a consistent formulation that satisfies the Maxwell static electricity equation is presented so the full coupling effect of the piezoelectric layer based on the dynamic Characteristics of the circular plate can be estimated. The solutions are expressed by elementary Bessel results obtained by the mentioned method are compared against finite element analysis as a measure of validity verification.

2 DISPLACEMENT AND ELECTRIC POTENTIAL FIELD MODELS FOR CIRCULAR PLATE

A key issue in the analysis of a piezoelectric coupled circular plate is in the modeling of the displacement field and the electric potential field. The cylindrical coordinate system is adopted.

2.1. Displacement field based on the Kirchhoff thin plate model

The cross section of a circular plate with a piezoelectric layer mounted on its surface is shown in Figure 1. In most practical applications, the thin plate is applicable, whereby the shear deformation and rotary inertia can be omitted. The applied external load is assumed to be axisymmetric.

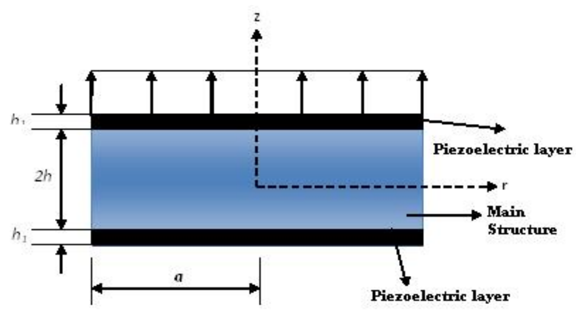

Fig. 1 Cross section of a circular plate with two piezoelectric layers mounted on its surfaces

Due to symmetry, the displacement and strains in the plate are expressed as:

$$u_z = u_z(r,t) = w(r,t) \tag{1}$$

$$u_r = u_r(r,t) = -z\frac{\partial u_z}{\partial r} \tag{2}$$

$$u_\theta = 0 \quad (3)$$

where 'u_z', 'u_r' and 'u_θ' are the displacements in the transverse z-direction, radial r-direction, and tangential θ-direction of the plate, respectively.

The poling direction of the piezoelectric material is assumed to be in the z-direction. The strain 'ε' in the plate and the piezoelectric layer with respect to the radial and tangential directions and the shear component are given by:

$$\varepsilon_r = \frac{\partial u_r}{\partial r} = -z\frac{\partial^2 w}{\partial r^2} \quad (4)$$

$$\varepsilon_\theta = \frac{u_r}{r} = -z\frac{\partial w}{r\partial r} \quad (5)$$

$$\varepsilon_{r\theta} = 0 \quad (6)$$

The stress components in the main plate are expressed as [24]:

$$\sigma_r^1 = \frac{E(\varepsilon_r + \upsilon\varepsilon_\theta)}{1-\upsilon^2} \quad (7)$$

$$\sigma_\theta^1 = \frac{E(\varepsilon_\theta + \upsilon\varepsilon_r)}{1-\upsilon^2} \quad (8)$$

Substituting Eqs. (4) and (5) into Eqs. (7) and (8) result in:

$$\sigma_r^1 = \frac{-zE}{1-\upsilon^2}\left[\frac{d^2w}{dr^2} + \frac{\upsilon}{r}\frac{dw}{dr}\right] \quad (9)$$

$$\sigma_\theta^1 = \frac{-zE}{1-\upsilon^2}\left[\frac{1}{r}\frac{dw}{dr} + \upsilon\frac{d^2w}{dr^2}\right] \quad (10)$$

$$\tau_{r\theta} = 0 \quad (11)$$

The stress components in the piezoelectric layer can be written as:

$$\sigma_r^2 = C_{11}^E\varepsilon_r + C_{12}^E\varepsilon_\theta + C_{13}^E\varepsilon_z - e_{31}E_z \quad (12)$$

$$\sigma_\theta^2 = C_{12}^E\varepsilon_r + C_{11}^E\varepsilon_\theta + C_{13}^E\varepsilon_z - e_{31}E_z \quad (13)$$

$$\sigma_z^2 = C_{13}^E\varepsilon_r + C_{13}^E\varepsilon_\theta + C_{33}^E\varepsilon_z - e_{33}E_z \quad (14)$$

And ε_z for plane stress is obtained from Eq. (14)

$$\varepsilon_z = \frac{e_{33}E_z - C_{13}(\varepsilon_r + \varepsilon_\theta)}{C_{33}} \quad (15)$$

Substituting Eq. (15) into Eqs. (12) and (13) results in:

$$\sigma_r^2 = \overline{C}_{11}^E\varepsilon_r + \overline{C}_{12}^E\varepsilon_\theta - \overline{e}_{31}E_z \quad (16)$$

$$\sigma_\theta^2 = \overline{C}_{12}^E\varepsilon_r + \overline{C}_{11}^E\varepsilon_\theta - \overline{e}_{31}E_z \quad (17)$$

where the superscripts 1 and 2 represent the variables in the main structure and the piezoelectric material, respectively; $\overline{C}_{11}^E$, $\overline{C}_{12}^E$ and $\overline{e}_{31}$ are transformed reduced material constants of piezoelectric medium for the plane stress problem, and are given as follows:

$$\overline{C}_{11}^E = C_{11}^E - \frac{\left(C_{13}^E\right)^2}{C_{33}^E}, \overline{C}_{12}^E = C_{12}^E - \frac{\left(C_{13}^E\right)^2}{C_{33}^E}, \quad \overline{C}_{11}^E = C_{11}^E - \frac{\left(C_{13}^E\right)^2}{C_{33}^E} \quad (18)$$

where, E is the Young's modulus of the main material; C_{11}^E and C_{12}^E are the elastic modulus of the piezoelectric material in the radial and tangential directions, measured at constant electric field; and e_{31} is the piezoelectric constant of the piezoelectric layer.

2.2. Distribution of electric potential in the piezoelectric layer

As the piezoelectric layers are shortly connected, the electric potential is zero throughout the surfaces. There are several different models representing the input electric potential for such a piezoelectric layer. In this paper we decided to adopt the following Wang et al. electric potential function [23].

$$\phi(r,z,t) = \left[1 - \left(\frac{2z - 2h - h_1}{h_1}\right)^2\right]\varphi(r,t) \quad (19)$$

where, z is measured from the mid-plane of the plate in the global z-direction, 'h_1' is the thickness of the piezoelectric layer, and $\varphi(r,t)$ is the electric potential on the mid-surface of the piezoelectric layer. The electrodes on each piezoelectric layer are short-circuit. It is to be noted that the assumed potential function has satisfied the boundary conditions in which electric potential vanishes at the internal surfaces $z = \pm h$ and the external surfaces $z = \pm(h + h_1)$.

3 ANALYSIS OF PIEZOELECTRIC COUPLED CIRCULAR PLATE

Based on the displacement field model described by Eqs. (1)-(3), and electric potential field model of equations, the associated governing equations for the piezoelectric coupled circular plate can be deduced. Using Eq. (18) the components of electric field E and electric displacement 'D' is written as

$$E_r = -\frac{\partial \phi}{\partial r} = \left[1 - \left(\frac{2z - 2h - h_1}{h_1}\right)^2\right]\frac{\partial \varphi}{\partial r} \tag{20}$$

$$E_\theta = -\frac{\partial \phi}{\partial \theta} = 0 \tag{21}$$

$$E_z = -\frac{\partial \phi}{\partial z} = \frac{8(z - h - h_1/2)}{h_1^2}\varphi \tag{22}$$

$$D_r = \overline{\Xi}_{11}E_r = -\overline{\Xi}_{11}\left[1 - \left(\frac{2z - 2h - h_1^2}{h_1}\right)^2\right]\frac{\partial \varphi}{\partial r} \tag{23}$$

$$D_\theta = \overline{\Xi}_{11}E_\theta = 0 \tag{24}$$

$$D_z = \overline{\Xi}_{33}E_z + \overline{e}_{31}(\varepsilon_r + \varepsilon_\theta) = \frac{8\overline{\Xi}_{33}(z - h - h_1/2)}{h_1^2}\varphi + \overline{e}_{31}^2 z\nabla^2 w \tag{25}$$

where $\overline{\Xi}_{11}$ and $\overline{\Xi}_{33}$ are reduced dielectric constants of the piezoelectric layer for the plane stress problem, which are given by $\overline{\Xi}_{11} = \mathrm{E}_{33}, \overline{\mathrm{E}}_{33} = \mathrm{E}_{33} + \left(e_{33}^2/C_{33}^E\right)$; E_r, E_θ and E_z are the electric field intensity in the r, θ and z directions, respectively; D_r, D_θ and D_z are the corresponding electric displacement; Ξ_{11} and Ξ_{33} are the dielectric constants of the piezoelectric layer; ∇^2 is the Laplace operator and given as:

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$

In order to obtain the governing differential equation of the coupled circular plate, we begin with resultant moment's components

$$M_r = \int_{-h-h_1}^{h+h_1} z\sigma_r dz = \int_{-h}^{h} z\sigma_r^1 dz + 2\int_{h}^{h+h_1} z\sigma_r^2 dz = -\left[(D_1 + D_2)\frac{\partial^2 w}{\partial r^2} + \left(\upsilon D_1 + \frac{\overline{C}_{12}^E}{\overline{C}_{11}^E}D_2\right)\frac{\partial w}{r\partial r} + \frac{4}{3}h_1\overline{e}_{31}\varphi\right] \tag{26}$$

$$M_\theta = \int_{-h-h_1}^{h+h_1} z\sigma_\theta dz = \int_{-h}^{h} z\sigma_\theta^1 dz + 2\int_{h}^{h+h_1} z\sigma_\theta^2 dz = -\left[\left(\upsilon D_1 + \frac{\overline{C}_{12}^E}{\overline{C}_{11}^E}D_2\right)\frac{\partial^2 w}{\partial r^2} + (D_1 + D_2)\frac{\partial w}{r\partial r} + \frac{4}{3}h_1\overline{e}_{31}\varphi\right] \tag{27}$$

where the thickness of the main plate is $2h$; the piezoelectric layer extends from $z = h$ to $z = h + h_1$ and $D_1 = 2Eh^3/[3(1- \upsilon^3)]$, $D_2 = \frac{2}{3}h_1\left(3h^2 + 3hh_1 + h_1^2\right)\overline{C}_{11}^E$ also, the resultant shear force is herein written as:

$$Q_r = \frac{dM_r}{dr} + \frac{M_r - M_\theta}{r} \tag{28}$$

Substituting Eq. (26) into Eq. (27) and Eq. (28) and substituting the final results into the governing equation for the Kirchhoff plate,

$$\frac{\partial Q_r}{\partial r} + \frac{Q_r}{r} - \left(\int_{-h}^{h}\rho_1\frac{\partial^2 u_z}{\partial t^2}dz + 2\int_{h}^{h+h_1}\rho_2\frac{\partial^2 u_z}{\partial t^2}\right) + f = 0. \tag{29}$$

where $f = f(r,t)$ is the external load. This will result in the equation for the piezoelectric coupled circular plate as follows:

$$(D_1 + D_2)\nabla^4 w(r,t) + \frac{4}{3}h_1\overline{e}_{31}\nabla^2\varphi(r,t) + 2(\rho_1 h + \rho_2 h_1)\frac{\partial^2 w(r,t)}{\partial t^2} = f(r,t) \tag{30}$$

Where 'ρ_1' and 'ρ_2' are material densities of the main plate and piezoelectric layer, respectively. Note that all electrical variables, primarily, must satisfy the Maxwell's equation which requires the divergence of the electric flux density to vanish at any point within the media as follows [23]:

$$\int_{h}^{h+h_1}\left(\frac{\partial D_r}{\partial r}+\frac{D_r}{r}+\frac{\partial D_z}{\partial z}dz\right)dz=0. \tag{31}$$

Substituting Eqs. (22), (23) and (24) into above equation yield

$$\frac{h_1^2\overline{\Xi}_{11}}{12\overline{\Xi}_{33}}\nabla^2\varphi(r,t)-\varphi(r,t)+\frac{h_1^2\overline{e}_{31}^2}{8\overline{\Xi}_{33}}\nabla^2 w(r,t)=0. \tag{32}$$

The problem is therefore formulated to find the solution of two coupled partial differential equations (30) and (32) in association with the following mechanical and electrical boundary conditions and symmetry constraints for the clamped, fully grounded piezoelectric coupled plate

$$r=a:w=dw/dr=0 \tag{33}$$

$$r=a:\varphi=0.,\ z=\pm h,\pm(h+h_1):\varphi=0. \tag{34}$$

4 SOLUTION METHOD

The forcing function, $f(r,t)$, is assumed to be harmonic, with frequency 'ω', as:

$$f(r,t)=F(r)e^{i\omega t} \tag{35}$$

So, the steady state solutions of Eqs. (29) and (31) are assumed to be in the form

$$w(t,r)=W(r)e^{i\omega t} \tag{36}$$

where $W(r)$ is the displacement amplitude in the 'z' direction as a function of radial displacement only, and in order to obtain appropriate solution for $\varphi(r,t)$ we assume

$$\varphi(r,t)=\varphi(r)e^{i\omega t} \tag{37}$$

Substituting Eqs. (35) and (36) into Eqs. (30), (32) yields:

$$(D_1+D_2)\nabla^4W(r)+\frac{4}{3}h_1\overline{e}_{31}\nabla^2\varphi(r)-2(\rho_1 h+\rho_2 h_1)\frac{\partial^2W(r)}{\partial t^2}\omega^2=F(r) \tag{38}$$

$$\frac{h_1^2\overline{\Xi}_{11}}{12\overline{\Xi}_{33}}\nabla^2\varphi(r)-\varphi(r)+\frac{h_1^2\overline{e}_{31}^2}{8\overline{\Xi}_{33}}W(r)=0 \tag{39}$$

The complete solution is composed of two solutions: (i) for the plate under the action of $F(r)$ and (ii) for the plate under the action of a radial bending moment applied at the edge. The complete solution is a suitable superposition of the two solutions, ensuring that the condition $dw/dr = 0$ holds at $r = a$. The solution is therefore written in the form

$$W(r)=\widetilde{W}(r)+\overline{W}(r) \tag{40}$$

$$\varphi(r)=\widetilde{\varphi}(r)+\overline{\varphi}(r) \tag{41}$$

To find the first solution, we assume

$$\widetilde{W}(r)=\sum_{j=1}^{\infty}A_jJ_0(\alpha_j r) \tag{42}$$

$$\widetilde{\varphi}(r)=\sum_{j=1}^{\infty}B_jJ_0(\alpha_j r) \tag{43}$$

Meanwhile, $F(r)$ is expanded into a series in terms of Bessel functions, as:

$$q_j=\frac{2}{a^2}\frac{1}{J_1^2(\alpha_j a)}\int_0^a rq(r)J_0(\alpha_j r)dr\ ,\ j=1,2,... \tag{44}$$

In Eqs. (42), (43) and (44) , $J_0(\alpha r)$ and $J_1(\alpha r)$ are Bessel functions of first kind of order zero and one, respectively,[28], and α_j are the positive roots of

$$J_0(\alpha a)=0. \tag{46}$$

Substituting Eqs. (42), (43) into Eq. (42) yields:

$$B_j=\frac{-p_0p_2\alpha_j^2}{1+\alpha_j^2p_1}A_j \tag{47}$$

Substituting Eqs. (42), (43) and (44) into Eq. (37) yields:

$$A_j = \frac{q_j}{D\left(\alpha_j^4 - \lambda^4 + \dfrac{p_0 p_2 \alpha_j^4}{1+\alpha_j^2 p_1}\right)} \tag{48}$$

where $D = D_1 + D_2$, and

$$p_0 = \frac{4h_1\bar{e}_{31}}{3D}\ ,\ p_1 = \frac{h_1^2\Xi_{11}}{12\Xi_{33}}\ ,\ p_2 = \frac{h_1^2\bar{e}_{11}}{8\Xi_{33}}\ ,$$
$$\lambda^4 = \frac{2(\rho_1 h + \rho_2 h_1)}{D}\omega^2$$

The solutions of $\tilde{W}(r)$ and $\overline{W}(r)$ satisfy the boundary condition of $W(r = a) = 0$. On the other hand they don't satisfy the boundary condition of $dW/dr = 0$ at $r = a$ separately, but sum of them satisfy both conditions. Let the second solution, $\overline{W}(r)$ and $\overline{\varphi}(r)$ have the form:

$$\overline{W}(r) = C(r^2 - a^2) + W_1(r) \tag{49}$$

where constant 'C' and function $W_1(r)$ have to be determined. Substituting Eq. (49) into Eqs. (37) and (38) with $F(r) = 0$, it is found that $W_1(r)$ and $\bar{\phi}(r)$ must satisfy the following equations:

$$\nabla^4 W_1(r) + p_0\nabla^2\overline{\varphi}(r) - \lambda^4\left(C(r^2 - a^2) + W_1(r)\right) = 0 \tag{50}$$

$$p_0\nabla^2\overline{\varphi}(r) - \overline{\varphi}(r) + p_2\nabla^2\overline{W}(r) = 0 \tag{51}$$

The second solutions are expanded in the similar way as the first solutions, viz.

$$W_1(r) = \sum_{j=1}^{\infty} F_j J_0(\alpha_j r)\ ,\ \overline{\varphi}(r) = \sum_{j=1}^{\infty} G_j J_0(\alpha_j r) \tag{52}$$

Following similar steps than those used to derive 'A_j' and 'B_j', one obtains

$$F_j = \frac{\left(\dfrac{p_0 p_2 \bar{q}_j \alpha_j^2}{1+\alpha_j^2 p_1} + c_j\right)C}{\left(\alpha_j^4 - \lambda^4 + \dfrac{p_0 p_2 \alpha_j^4}{1+\alpha_j^2 p_1}\right)} \tag{53}$$

$$G_j = \frac{p_2\left(C\bar{q}_j - \alpha_j^2 F_j\right)}{1+\alpha_j^2 p_1} \tag{54}$$

with

$$\bar{q}_j = \frac{8}{a\alpha_j J_1(\alpha_j a)}\ ,\ c_j = \frac{\lambda^4 J_2(\alpha_j a)}{a\alpha_j J_1(\alpha_j a)}\ , j = 1,2,\ldots \tag{55}$$

The second solution, $\tilde{W}(r)$ and $\tilde{\varphi}(r)$ satisfies Eqs. (36) and (37) with $F(r) = 0,$ and (32), (33) ,except the boundary condition $dw/dr = 0$ at $r = a.$ Substituting $\tilde{W}(r)$ and $\overline{W}(r)$ in this boundary condition

$$r = a:\ \frac{d\left[\tilde{W} + \overline{W}\right]}{dr} = 0 \tag{56}$$

Results in a linear, algebraic equation for 'C', whose value can easily be determined. Hence, the function $W(r) = \tilde{W}(r) + \overline{W}(r)$ and $\varphi(r) = \tilde{\varphi}(r) + \overline{\varphi}(r)$ are known and describe the exact, complete solution for the clamped boundary condition. Moreover, the final values of $w(r,t)$ and $\varphi(r,t)$ are

$$w(r,t) = \left(\sum_{j=1}^{\infty}(A_j + F_j)J_0(\alpha_j r) + C(r^2 - a^2)\right)e^{i\omega t} \tag{57}$$

$$\varphi(r,t) = \left(\sum_{j=1}^{\infty}(B_j + G_j)J_0(\alpha_j r)\right)e^{i\omega t} \tag{58}$$

4 NUMERICAL RESULTS AND DISCUSSION

In this section, steady state response of clamped circular plate subjected to a harmonically varying uniform pressure all over the surface area is considered. The amplitude of external load is 10Pa and its frequency, 'ω', is 100(rad/s). The material parameters and geometric size for the structure used in this paper are listed in table 1. In this example the thickness ratio of the piezoelectric layer and main plate ($h_1/2h$) is 1/20. Fig. 2 shows the radial distribution of the deflection of plate at time t = 0.2(s).

The curve in Fig. 2 shows that boundary conditions at $r = a : w = dw/dr = 0$ are satisfied. Since there were no published results for forced vibration of compound piezoelectric plate, we decided to verify the validity of the obtained results with those obtained from Abaqus results. Fig. 3 shows the vibration of compound piezo-plate centre with respect to time. As seen from Fig. 2 and Fig. 3, there is a good adaption between the curve obtained from our method and the curve obtained from FEM.

Table 1 Material properties and geometric size of the piezoelectric coupled plate [20]

Main Structure(Steel)	*E = 205(GPa)*	*ρ =7800(Kg/m³)*
PZT4	$C_{11}^{E} = 132(GPa)$	$C_{12}^{E} = 71(GPa)$
	$C_{33}^{E} = 115(GPa)$	$C_{13}^{E} = 73(GPa)$
	$e_{33} = 14.1(C/m^2)$	$e_{31} = -4.1(C/m^2)$
	$\Xi_{11} = 7.124\times10^{-9}(F$	$\Xi_{33} = 5.841\times10^{-9}(F/n$
		$\rho_2 = 7500(Kg/m^3)$
Geometry	*a = 600(mm) , h = 10(mm) , h_1 = 2(mm)*	

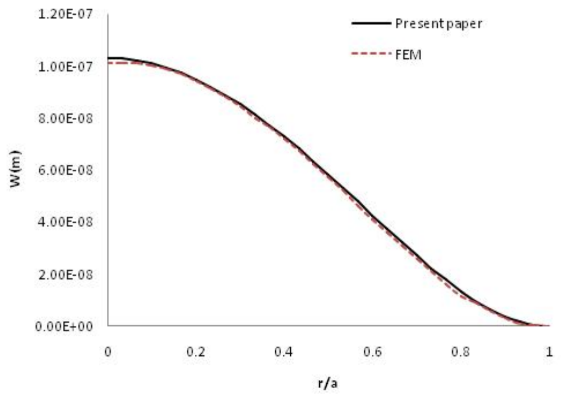

Fig. 2 Radial distribution of deflection of plate at t = 0.2(s)

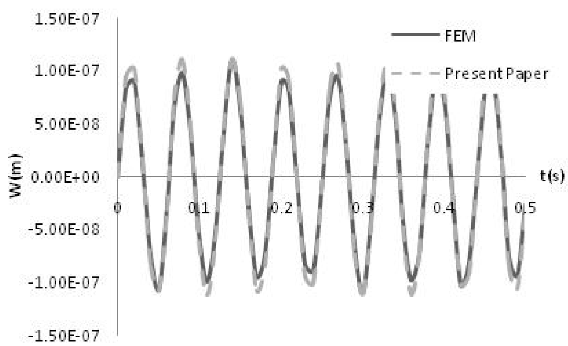

Fig. 3 Dynamic deflection of plate at r = 0

Fig. 4 shows the dynamic deflection of centre of piezoelectric coupled structure for piezoelectric layers of different thicknesses. The piezo-effect is obvious when a thicker piezoelectric layer is created on the main material. It can be seen that by increasing the value of 'h_1', the deflection of system decreases under the same value of transverse harmonic load. If $h_1 = 0$, there is no piezo-layer on the plate and so, we have a pure structure. Note that as the forcing frequency, 'ω', approaches the *j*th natural frequency of vibration of the plate, $\left[\frac{D}{2(\rho_1 h + \rho_2 h_1)}\left(1+\frac{p_0 p_2}{1+\alpha_j^2 p_1}\right)\right]^{1/2} \alpha_j^2$, in Eqs. (46) and (51) the deflection of the plate $W(r) \to \infty$, thereby causing resonance.

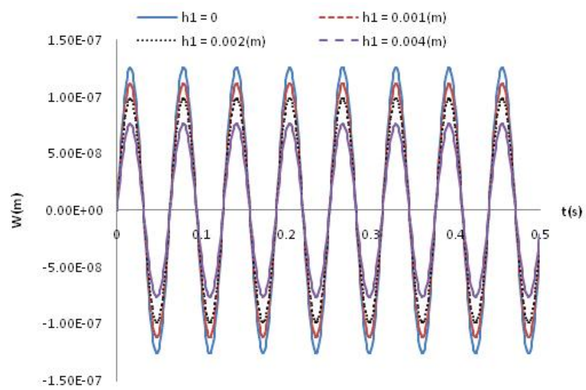

Fig. 4 Effect of piezo-layer on dynamic deflection of plate

Fig. 5 shows the effect of thickness of piezoelectric layer on the centre deflection of the piezoelectric coupled structures at time t = 0.7(s). As seen from Fig. 5 by increasing the thickness of piezo-layer linearly the deflection decreases non-linearly.

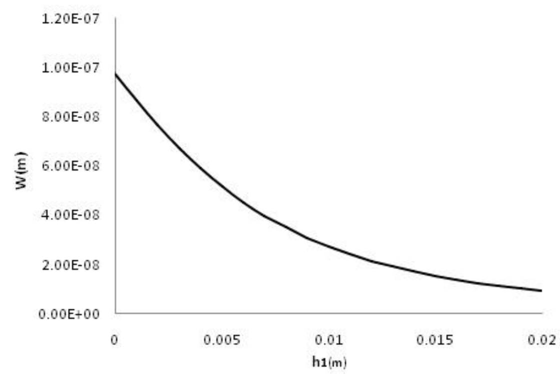

Fig. 5 Effect of h_1 on the deflection of plate at r = 0 and t = 0.7(s)

By letting $\bar{e}_{31} = 0$ in Eqs. (36) and (37) the electrical coupling disappears and the solution reduces to that of a transversely isotropic circular plate. Also the solution $\widetilde{W}(r)$ given in Eq. (40) shows that each term in the series is the product of a mode function $J_0(\alpha_j r)$ and a coefficient of magnitude A_j. Eq. (46) shows that $\frac{p_0 p_2 \alpha_j^4}{1+\alpha_j^2 p_1}$ is positive and increases by increasing

$|\bar{e}_{31}|$ .Thus, an increase in $|\bar{e}_{31}|$ results in decrease in $|A_j|$, which, in consequence, reduces the global deflection $\widetilde{W}(r)$.

Since the total deflection of the plate is the sum of $\widetilde{W}(r)$ and $\overline{W}(r)$, a more elaborate evaluation is required to enable a comprehensive conclusion on the effect of $\bar{e}_{31}$ but in this example $|\bar{e}_{31}|$ has not any significant effect on the deflection of the plate and the effect of $|\bar{e}_{31}|$ can be neglected. As demonstrated in table (2) increasing $|\bar{e}_{31}|$, doesn't change at the same value of transverse mechanical load.

Table 2 Material properties and geometric size of the piezoelectric coupled plate [20]

	W(μm)			
r/a	$\lvert\bar{e}_{31}\rvert = 0$	$\lvert\bar{e}_{31}\rvert = 20$	$\lvert\bar{e}_{31}\rvert = 50$	$\lvert\bar{e}_{31}\rvert = 100$
0	0.083	0.083	0.083	0.082
0.2	0.077	0.076	0.076	0.075
0.4	0.059	0.058	0.058	0.057
0.6	0.034	0.034	0.034	0.033
0.8	0.011	0.010	0.010	0.010
1	0	0	0	0

5 CONCOLUSION

A model for the analysis of a piezoelectric coupled circular plate structure is proposed. The equation of motion is achieved based on the Kirchhoff plate model for harmonic forced vibration. The solutions are given in terms of elementary Bessel functions. The model is validated using the results from the present method and those from finite-element analysis. It is shown that the thickness of piezo-layer has a significant effect on the deflection amplitude. It is also demonstrated that the piezoelectric constant $\bar{e}_{31}$ controls the electro-mechanical coupling and the effect of $|\bar{e}_{31}|$ on plate deflection can be neglected.

REFERENCES

[1] Bailey, T., and Hubbard, J. E., "Distributed piezoelectric polymer active vibration control of a cantilever beam", J. Guid. Control Dyn. 8, Jr 1985, pp. 605-11.

[2] Lee, C. K., "Theory of laminate piezoelectric plates for the design of distributed sensors/actuators, part 1: governing equations and reciprocal relationships", J. Acoust. Soc. Am. 87, 1990, pp. 1144-58.

[3] Wang, B. T., and Rogers, C. A., "Laminate plate theory for spatially distributed induced strain actuators", J. Comp. Mater. 25, 1991, pp. 433-52.

[4] Ha, S. K., Keilers, C., and Chang, F. K., "Finite element analysis of composite structures containing distributed piezoceramic sensors and actuators", AIAA J. 30, 1992, pp. 772-80.

[5] Wang, Q., Liew, K. M., and Wang, D. J., "Some issues of control of structures using piezoelectric actuators", Proc. SPIE 2921, 1997, pp. 425-30.

[6] Kim, J., Varadan, V. V., Varadan, V. K., and Bao, X. Q., "Finite element modeling of a smart cantilever plate and comparison with experiments", Smart Mater. Struct. 5, 1996, pp. 65-70.

[7] Tzou, H. S., and Tseng, C. I., "Distributed piezoelectric sensor/actuator design for dynamic measurement/control of distributed parameter system", J. Sound Vib. 138, 1990, pp. 17-34.

[8] Robinson, D. H., and Reddy, J. N., "Analysis of piezoelectrically actuated beams using a layer-wise displacement theory", Comput. Struct. 41, 1991, pp. 265-79.

[9] Wang, S., and Park, H. C., "Finite element modeling of piezoelectric sensors and actuators", AIAA J. 31, 1993, pp. 930-937.

[10] Saravanos, D. A., and Heyliger, P. R., Coupled layer-wise analysis of composite beams with embedded piezoelectric sensors and actuators", J. Intell. Mater. Syst. Struct. 6, 1995, pp. 350-363.

[11] HKS Inc 1993 ABAQUS User's Manual (version 5.2) (Providence, RI: Hibbitt, Karlsson and Sorenson Inc).

[12] Swanson Inc 1993 ANSYS User's Manual (version 5.2) (Houston, PA: Swanson Inc.).

[13] Crawley, E. F., and de Luis, J., "Use of piezoelectric actuators as elements of intelligent structures", AIAA J. 25, 1987, pp. 1373-85.

[14] Crawley, E. F., and Anderson, E., "Detailed model of piezoelectric actuation of beams", Proc. 30th AIAA/ASMA/SAE Conf. on Structures, Structural Dynamics, and Material, Washington, DC, April 1989, pp. 2000-10.

[15] Leibowitz, and Vinson, "The use of hamilton's principle in laminated piezoelectric and composite structures", Adaptive Structures and Material Systems AD- 35, 1993.

[16] Ding, H. J., Chen, B., and Liang, J., "General solutions for coupled equations for piezoelectric media", Int. J Solids Struct. 33, 1996, pp. 2283-98.

[17] Sun, C. T., and Zhang, X. D., "Use of thickness-shear mode in constructing adaptive sandwich structure", Smart Mater. Struct. 4, 1995, pp. 202-206.

[18] Zhang, X. D., and Sun, C. T., "Formulation of an adaptive sandwich beam", Smart Mater. Struct. 5, 1995, pp. 814-23.

[19] Zhang, X. D., and Sun, C. T., "Analysis of sandwich plate containing a piezoelectric core", Smart Mater. Struct. 8, 1999, pp. 31-40.

[20] Han, J. H., and Lee, L., "Analysis of composite plates with piezoelectric actuators for vibration control using Layerwise displacement theory Composites B 29", 1998, pp. 621-632.

[21] Sungsoo, N., and Libresco, L., "Oscillation control of cantilevers via smart materials technology and optimal feedback control: actuator location and power

consumption issues", J. Smart Mater. Struct. 7, 1998, pp. 833-842.
[22] Wang, Q., and Quek, S. T., "Flexural vibration analysis of Sandwich beam coupled with piezoelectric actuator", J. Smart Mater. Struct. 9, 2000, pp. 103-109.
[23] Wang, Q., Quek, S. T., Sun, C. T., and Liu, X. "Analysis of piezoelectric coupled circular plate", J. Smart Mater. Struct. 10, 2001, pp. 229-239.
[24] Timoshenko, S., and Woinowsky-Krieger, S., "Theory of plate and shells, second ed., Mc Grow –Hill", New York, 1959.
[25] He L.H., "Axisymmetric response of circular plates with piezoelectric layers: an exact solution", Int. J Mechanical Science 12, 1998, pp. 1265-79.
[26] Din, H., Xu, R., Chi, Y., and Chen, W., "Free axisymmetric vibration transversely isotropic piezoelectric circular plates", Int. J. Solids and Structures 36, 1999, pp. 4629-52.
[27] Zhang, X. Z., Veidt, M., and Kitipornchai, S., "Transient bending of a piezoelectric circular plate", Int. J Mechanical Sciences 46, 2004, pp. 845-1859.
[28] Abramowitz, M., and Stegun, I. A., "Handbook of mathematical functions", Washington, DC: National Bureau of Standards, 1964.