

Trajectory Path Planning of Cable Driven Parallel Manipulators, Considering Masses and Flexibility of the Cables

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Abstract: Cable driven parallel manipulator (CDPM) is a special class of parallel manipulator in which the rigid extensible links are replaced by actuated cables. It is necessary to take into consideration the cable dynamics, i.e.; its mass, flexibility and curved shape for manipulating a long-span CDPM. These terms complicate governing equation of motion in a way that special tactic are applied for simulation and solving this problem. Flexibility and mass of cables impose vibration and error in path trajectory planning. Effect of varying stiffness in precise performance of CDPM is surveyed. The cables are modelled, in ADAMS software to illustrate the dynamical behaviours of the manipulator for comparison with the simulated results. Moreover, an algorithm is developed to study the effects of velocity and acceleration of the end-effector on the dynamics of CDPMs. Moreover it is shown that the evolutionary computing algorithms are so effective in solving complicated nonlinear dynamic path trajectory planning. Simulations for different trajectories of two CDPMs are included to demonstrate the efficiency of the proposed algorithm.

Keywords: Cable Driven Parallel Manipulators, Dynamic Modelling, Trajectory Path planning, Under Constraint CDPM

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1 INTRODUCTION

Cable driven parallel manipulators (CDPMs) are a class of parallel manipulators (PMs) which utilize multiple actuated cables to manipulate objects. They offer some advantages over the conventional PMs; Since the rigid links have been replaced by cables in their structures, they offer some valuable characteristics such as; overall mass and inertia of the manipulator are reduced tremendously, they are cheaper and easier to build, their ability to store significant lengths of cables on winches provide them the possibility of very large workspaces and their payload to weight ratio increase. Moreover, they are also suitable for high acceleration applications; their transportability and re-configurability by changing the positions of the motors are improved. Therefore they have recently attracted the attention of researchers from both academia and industries. They found many applications, such as NIST RoboCrane which is a large-workspace CDPM which was developed to be used in shipping ports [1], McDonnell-Douglas Charlotte that had been designed for use in the International Space Station [2], Sky cam which is a remote-controlled aerial camera suspended being used in sport facilities [3], haptic devices [4], receiver support systems for large radio telescopes [5], rehabilitation systems [6], robots for assembly-disassembly operations [7] and interaction with hazardous environment [8]. Moreover, CDPMs have been employed for service robots [9], long range positioning devices [10] and high speed manipulation [11]. The most important limitation of a CDPM is that, the cables cannot support the compressive load and they must be in tension in the whole workspace. Based on this property, CDPMs can be classified into two categories; namely under constrained and fully constrained ones [12-14].

In the general case of fully constrained manipulator, the end-effector (EE) should be connected to the base by at least seven cables [15]. While, for under constrained CDPM, the cables are six or less; therefore, the cables do not fully constrain the EE [16]. In most of the existing literatures, the cables are assumed to be mass-less linear elements that can only work in tension and therefore their tensions are assumed constant along their entire lengths. Upon this simplification, the dynamics of a CDPM is reduced to the dynamics of a single rigid body, i.e. the EE with several external forces acting from the cables to the EE. Therefore, dynamical characteristics of the cables, such as the mass, flexibility and vibration are neglected. The kinematic, workspace and dynamic analysis, and control of this simplified CDPM have been studied extensively in the literature [17-20]. However, neglecting the masses of the cables is not justified in general. In fact, when the weight of a cable is relatively

significant with respect to the weight of the EE, it has significant effects on the foregoing analyses and the stability, accuracy and control of the manipulator. Moreover, cables are usually flexible which lead to pose errors of the EE. Vibration of a CDPM may be a concern for some applications which require high bandwidth or high stiffness of the system. Reported studies on the effect of cable flexibility on modelling, optimal design and control of such manipulators are very limited [21]. In some studies, cables were modelled by mass-less springs which lead to vibration of the EE [22], [23]. Some assumed the cables as straight line members with time varying lengths [24]. Moreover, considering cable mass and flexibility, some derived the equations of motion but some complicated terms in equations were ignored [25].

This paper deals with the dynamics of CDPM considering the mass, flexibility and the curvature shapes of the cables in 3-dimensional space. In a long span CDPM, especially when one deals with high velocity or acceleration, the effects of the foregoing terms are significant. Including these terms lead to complicated dynamic governing equation of motion and demand special tactic for simulation and solving. Currently, a few works have focused on the dynamic characteristics of such cables and their effects on precision of the trajectory performance.

In this paper, dynamic analysis of CDPMs with long span workspace is addressed, focusing on a foregoing complicity in model involving cables mass, curvature shape, flexibility and time-varying lengths. Here, dynamics partial differential equations of a cable are derived and converted into the ordinary differential equations through a spatial finite difference discretization method. Then, the governing dynamics equations of the EE motion are derived and solved along with the former equations. Moreover, an algorithm is developed to study the effects of velocity and acceleration of the end-effector on the dynamics of CDPMs. Finally, some simulations for different trajectories of two CDPMs are included to demonstrate the efficiency of the proposed algorithm.

2 DYNAMICS OF CDPM

A CDPM consists of an EE platform connected by n parallel cables to the base, as depicted in Fig. 1. Therefore, a CDPM is mainly composed of two kinds of sub-systems, i.e. an EE and some cables. In this section the models of these sub-systems are considered, and then the dynamics of the CDPM system, as a whole are derived.

Dynamics of EE

Consider an inertial coordinate frame f^n in base O^n with $\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$ unit vectors, the moving frame f^b with the unit vectors $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ attached to the mass center of the EE, O^b . Moreover, the pose of the EE is specified by the position vector of O^b in f^n and its Euler angles $\mathbf{x} = [x_b \ y_b \ z_b \ \psi \ \theta \ \varphi]^T$. In which φ, θ and ψ are the Euler angles. The governing equations for the dynamic analysis of the EE can be written in the following general form [20].

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{c}(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{g}(\mathbf{x}) = \mathbf{J}^T \mathbf{t} \tag{1}$$

$$\mathbf{M} = \begin{bmatrix} m\mathbf{I}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_P \end{bmatrix}, \mathbf{c}(\mathbf{x}, \dot{\mathbf{x}}) = \begin{bmatrix} \mathbf{0}_{3 \times 1} \\ \mathbf{I}(\dot{\mathbf{P}}\dot{\mathbf{o}}) + (\mathbf{P}\dot{\mathbf{o}}) \times \mathbf{I}(\mathbf{P}\dot{\mathbf{o}}) \end{bmatrix}$$

$$\mathbf{J}^T = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \\ \mathbf{r}_1 \times \mathbf{u}_1 & \mathbf{r}_2 \times \mathbf{u}_2 & \mathbf{r}_3 \times \mathbf{u}_3 \end{bmatrix}$$

$$\dot{\mathbf{o}} = [\dot{\psi} \ \dot{\theta} \ \dot{\varphi}]^T \text{ and } \mathbf{P} = \begin{bmatrix} 1 & 0 & -\sin \theta \\ 0 & \cos \psi & \sin \psi \cos \theta \\ 0 & -\sin \psi & \cos \psi \cos \theta \end{bmatrix}$$

Where \mathbf{I}_3 is 3*3 identity matrix, \mathbf{I} is the inertia matrix for the system, $\mathbf{g}(\mathbf{x})$ is the gravity vector, m is the mass of the platform, $\mathbf{0}_{3 \times 1}$ is 3*1 zero vector, \mathbf{t} is tension vector of the cables, \mathbf{u}_i is unit vector in the direction of the i th cable and \mathbf{r}_i is the vector from the origin of the EE to the i th cable attachment point to the EE. Now, it is easy to convert the foregoing equation into a state-space form equation.

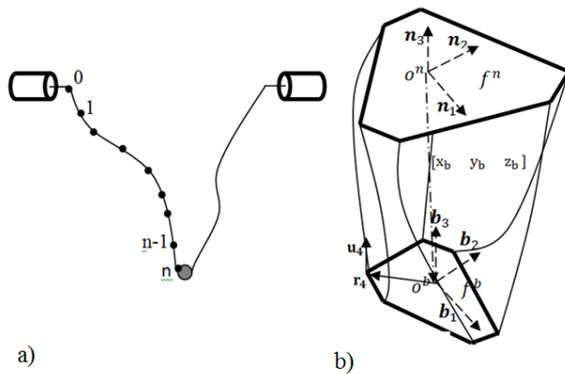


Fig. 1 a) Planar CDPM b) six degrees of freedom CDPM

Dynamics of the cables

The cables are considered here as flexible elements with uniform masses, curved element with negligible bending and torsional stiffness. Consider a flexible cable of an un-

stretched length L . The un-stretched curve length along the connected base point to a point P on the cable is denoted by s , and the corresponding stretched length by s_e . Thus the axial strain of the cable is given as:

$$\varepsilon = \frac{ds_e - ds}{ds} \tag{2}$$

Where

$$ds_e^2 = dx^2 + dy^2 + dz^2 \tag{3}$$

In which x, y and z are the global coordinates of point P in the cable. Substituting the value of ds_e from Eq. (3) into Eq. (2), upon simplification leads to:

$$\varepsilon = \sqrt{(\partial x / \partial s)^2 + (\partial y / \partial s)^2 + (\partial z / \partial s)^2} - 1 \tag{4}$$

Moreover, the tangent unit vector at each point of the cable is given as:

$$\mathbf{b}(s, t) = \frac{\partial s}{\partial s_e} \frac{\partial \mathbf{r}}{\partial s} = \frac{1}{1 + \varepsilon(s, t)} \frac{\partial \mathbf{r}}{\partial s} \tag{5}$$

Where $\mathbf{r}(s, t) = [x(s, t) \ y(s, t) \ z(s, t)]^T$ is the position vector of point P in the cable. Therefore, the equations of motion of the cable can be described as the following form [25].

$$\rho \frac{d^2 \mathbf{r}(s, t)}{dt^2} = \frac{\partial}{\partial s} [EA \varepsilon(s, t) \mathbf{b}(s, t)] + \rho \mathbf{g} \tag{6}$$

Where ρ is the linear density of the un-stretched cable, E is the Young's modulus and A is the cross-sectional area of the cable. Substituting the value of $\mathbf{b}(s, t)$ from Eq. (5) into Eq. (6), upon simplification leads to:

$$\rho \frac{d^2 \mathbf{r}(s, t)}{dt^2} = \frac{\partial}{\partial s} \left[\frac{EA \varepsilon(s, t)}{1 + \varepsilon(s, t)} \right] \frac{\partial \mathbf{r}}{\partial s} + EA \frac{\varepsilon(s, t)}{1 + \varepsilon(s, t)} \frac{\partial^2 \mathbf{r}}{\partial s^2} + \rho \mathbf{g} \tag{7}$$

Moreover, one can write

$$\frac{d\mathbf{r}}{dt} = \frac{\partial \mathbf{r}}{\partial t} + \frac{\partial \mathbf{r}}{\partial s} \dot{s} \tag{8}$$

$$\frac{d^2 \mathbf{r}}{dt^2} = \frac{\partial^2 \mathbf{r}}{\partial t^2} + 2 \frac{\partial^2 \mathbf{r}}{\partial s \partial t} \dot{s} + \frac{\partial^2 \mathbf{r}}{\partial s^2} \dot{s}^2 + \frac{\partial \mathbf{r}}{\partial s} \ddot{s} \tag{9}$$

Now, one can solve Eq. (7) by a finite difference method in which the cable is divided into n equal parts from $s = 0$ to $s = L$. Where point 0 is attached to a very small pulley and point n is connected to the EE, as depicted in

3 SIMULATION

In this sub-section, two examples and simulation are included to justify the dynamic modeling. It is noted that only for a positive axial strain of whole cables nodes, the cables are in tension and therefore the CDPM is controllable.

Two cables planar CDPM

The model of a planar CDPM consists of a concentrated mass EE actuated by two cables as shown in Fig. 1-a. Based on the proposed modeling, dynamic analysis is performed for this manipulator. Upon knowing the position of the EE, the problem is to determine the cables lengths, for $i=1,2$ by solving the inverse kinematic problem. It is noted that a flexible cable can take a curved shape under its own weight.

Therefore, the effect of cable sag should be considered in the calculation [26]. The time varying cable length for a tracking path is derived from quasi static configurations, then with specified time interval is feed to the manipulator and the effect of this condition is assessed in the dynamic modeling. Considering the static posture of the manipulator, the cables lengths to satisfy point to point establishment are considered and resulted in specified time feed to the cable robot. With cable robot dynamic simulation, error in the trajectory can be calculated, as well.

Moreover cable model shown in Fig. 2 is created using the cable demo in Adams software to illustrate the dynamical behaviors of the manipulator for comparison with the simulated results. The cable is discretized with appropriate parts, joints and forces (mass, rod-formulation-based longitudinal stiffness). There is also a revolute joint between each cable part.

When the EE moves from an initial pose to a final pose, with a constant velocity of $1m/s$, the desired straight line quasi-static trajectory, dynamic analysis from the data of quasi-static analysis and Adams software results are depicted in Fig. 3, for different cases. The simulation results show that the EE moves along and vibrates around its desired quasi-static trajectory.

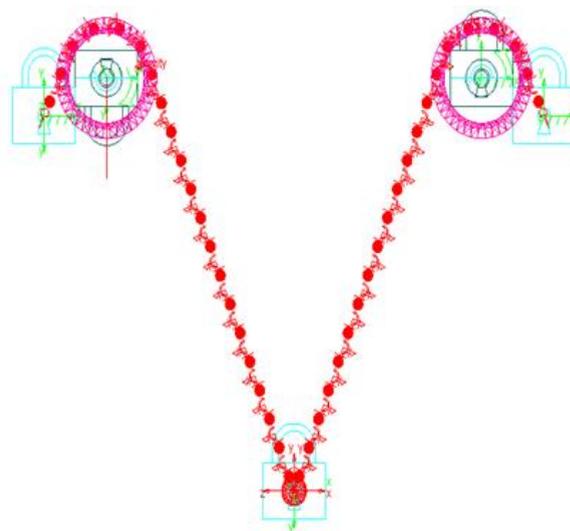


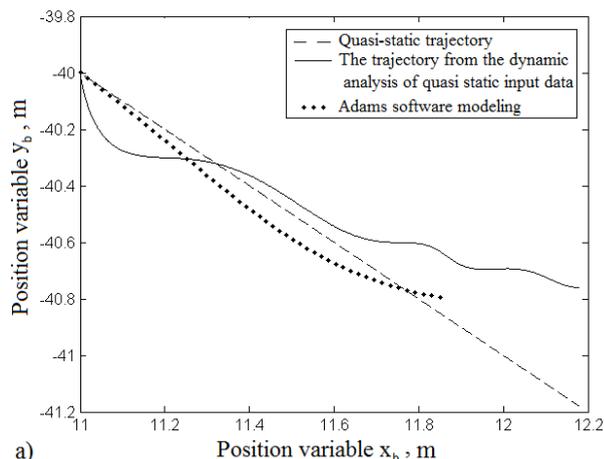
Fig. 2 Adams modeling of planar CDPM

Effects of the cables stiffness on reduction of the vibration amplitude are shown. When someone applies a high stiffness cables in CDPM construction, the oscillations of EE decrease around its desired path. In Fig. 3-b. the ratios of the maximum deviation from the desired quasi-static path to the average cables lengths are 0.14% for Adams model and 0.13% for the dynamic model introduced in this paper. This shows matching of results between models. Also dynamic model in this paper illustrates oscillation nature of system clearly compared to Adams software model.

Table 1 The data of the cables and the EE

Name	Value
The density for unit length	$\rho = .1 \text{ kg/m}$
The EE mass	$m = 100 \text{ kg}$
The cross section times the Young modulus	$AE = 10^6, AE = 10^7, AE = 10^8, AE = 10^9 \text{ N}$
Distance between cable pulleys	Distance=30 m

The platform poses are calculated by computation of the dynamic behavior of the manipulator, imposed by varying cables lengths. The data of the cables and the EE are given in table 1.



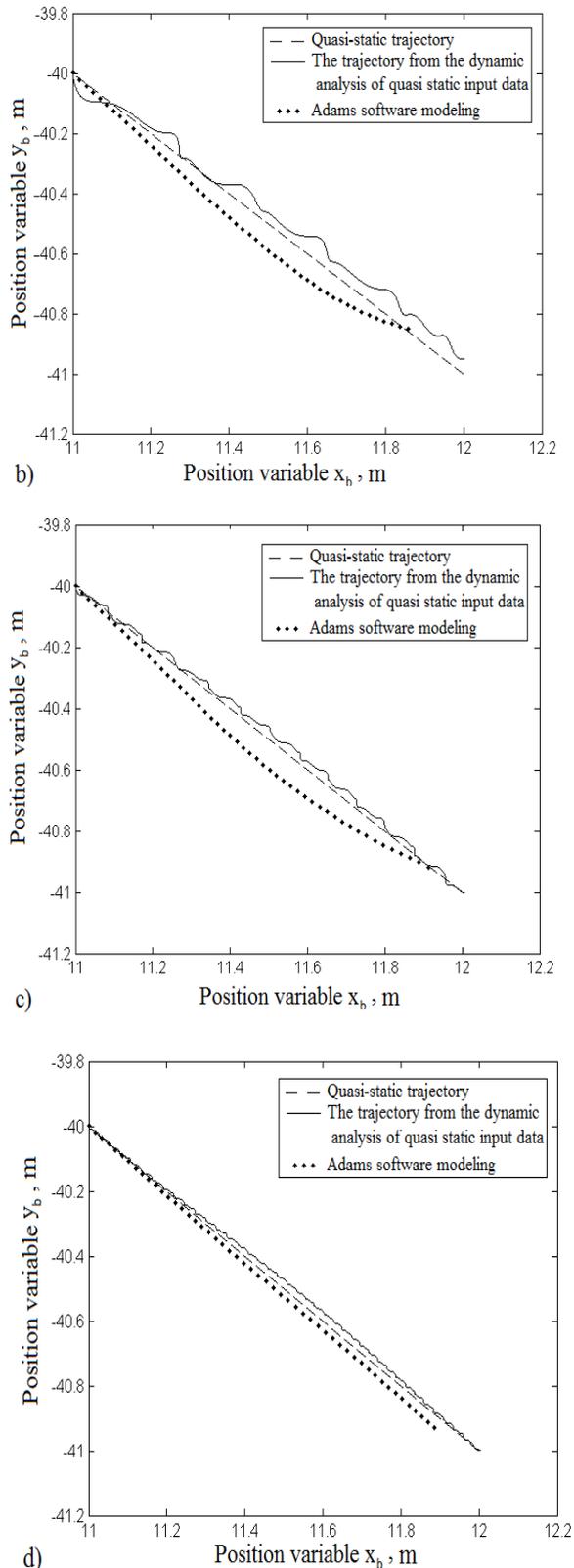


Fig. 3 Quasi static trajectory, the trajectory from the dynamic analysis of quasi-static input data and Adams software modeling trajectory. a) $AE=10^6 N$, b) $AE=10^7 N$, c) $AE=10^8 N$ and d) $AE=10^9 N$.

The desired and the generated trajectories from dynamic analysis of quasi-static input data are depicted in Fig. 4, for different velocities, as well. If the EE is excited by increasing traveling velocity, the vibration amplitude will increase noticeably.

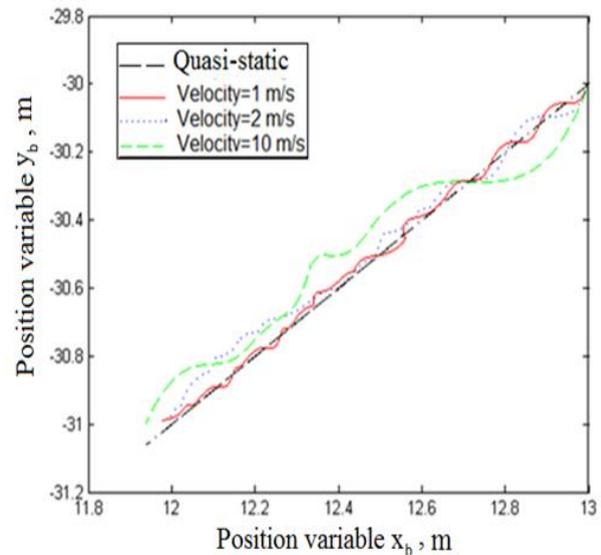


Fig. 4 Path trajectory planning for different velocities

Table 2 cable and EE specification

Name	Value
The inertia matrix of EE	$I = \text{diag}[237 \ 237 \ 314] \text{kg.m}^2$
Angular position between \mathbf{n}_1 axis and cables attachment points in base	$\gamma = [45, 75, 165, 195, 285, 315] \text{deg}$
Angular position between \mathbf{b}_1 axis and cables attachment points in EE	$\beta = [15, 105, 135, 225, 255, 345] \text{deg}$
Mass of the EE	$m = 100 \text{ kg}$
Cross section cable area to Young modulus	$AE = 10^7 N$
Unit length density of cable	$\rho = .1 \text{kg/m}$
Velocity of central mass of EE	$V=3 \text{m/s}$

Six DOF CDPM

The simulations for a six under-actuated CDPM are given in this sub-section. The manipulator consists of an EE connected to the base by six cables. The base points of the manipulator are all contained within the same fixed

plane as shown in Fig. 1-b. The cables attachment points are placed at the same radial distance $r_n = 30\text{m}$ from the base coordinate system f^n that is located at the base center. The moving platform has a set of connection points located at a distance of $r_b = 1\text{m}$ from the moving coordinate frame f^b attached to the platform center. The cables and the EE specifications for the simulation are given in table 2. For two different trajectories, the desired quasi-static and the generated trajectories from dynamic analysis of quasi-static input data of the EE for the fixed Euler angles $\psi = 3\text{deg}$, $\phi = 7\text{deg}$ and $\theta = 10\text{deg}$ are illustrated in Figs. 5 and 6. It is apparent that the maximum deviations from the desired path are 0.052m and 0.027m for these examples.

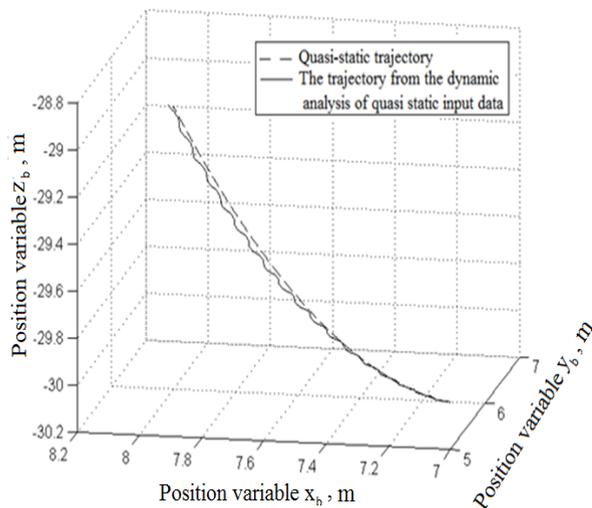


Fig. 5 Trajectory path planning quasi-static compared to Dynamic analysis

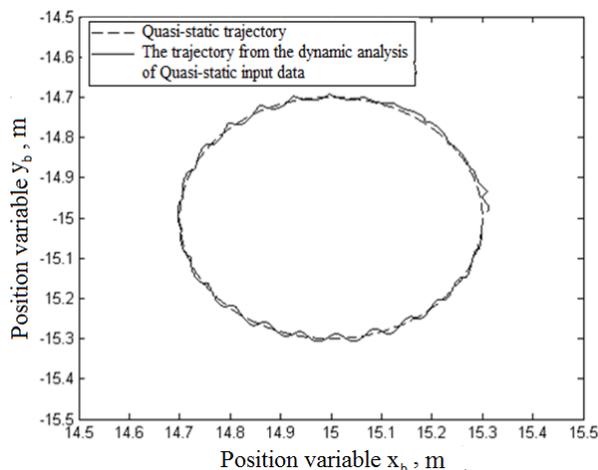


Fig. 6 Circle path planning in dynamic model

4 CONCLUSION

In this paper dynamic modelling of cable driven parallel manipulator (CDPM), considering the mass, flexibility and curvature shape of the cables has been derived. Moreover, an algorithm has been developed to study the complications of these effects in the dynamics of CDPM. Simulations for different trajectories of two CDPMs have been included to demonstrate the efficiency of the algorithm. It is shown that the evolutionary computing algorithms are so effective in solving complicated nonlinear dynamic path trajectory planning. The results reveals that mass and flexibility of the cables have considerable effects on the trajectory, therefore they should be considered in the dynamic analysis of long span CDPMs. Moreover, it has been shown that higher velocities cause poor overall performances for CDPMs.

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