

# Multi-Objective Suspension Optimization of a 5-DOF Vehicle Vibration Model Excited by Random Road Profile

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**Abstract:** The vehicle driving comfort has become one of the important factors of vehicle quality and receives increasing attention. In this paper, optimal points of vehicle suspension parameters are generated using modified non-dominated sorting genetic algorithm (NSGA-II) for Pareto optimization of 5-degree of freedom vehicle vibration model considering three conflicting functions simultaneously. In this way random profile is considered for the road excitation. Objective functions are vertical acceleration of seat, relative displacement between sprung mass and forward tire and relative displacement between sprung mass and rear tire. The results are compared with the previous works, where it indicates satisfactory behaviour of the optimum design points proposed in this work.

**Keywords:** Multi-Objective Optimization, NSGA-II, Random Vibration, Vehicle 5-DOF Model

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## 1 INTRODUCTION

Vehicle vibration and dynamics analysis has been a hot research topic due to its important role in ride comfort, road holding and overall vehicle performance. Ride comfort and road holding are two conflicting goals. Many researches showed that trade-off between these two objectives is hard to achieve[1]. In recent years genetic algorithm application is being more common in optimization problems due to its capabilities, flexibility and processing speed. Generally, there are three types of suspension system, namely, passive suspension, active suspension and semi-active suspension. These three types of suspension systems were studied by Bouazara in his PhD thesis for 5 and 8-DOF vehicle models[2].

In this paper, all vehicle performance parameters were taken under consideration to achieve a certain objecting function using weighting coefficients. Then vertical acceleration of seat and relative displacement between sprung mass and tires were used as a comfort and road holding criteria respectively. Due to result extreme dependence on weighing coefficients, this method is not a suitable way to solve the multi-optimization problems. In further research, self-tuning PID and fuzzy controller were used to optimize the vertical motion of vehicle based on genetic algorithm[3]. Alkhatib et al. used genetic algorithm to optimize a linear 1-DOF vibration isolator mount[4]. In further research, this method used to optimize the linear vibration of a quarter car model. Nariman-zadeh et al. used multi-objective optimization on 5-DOF active suspension system with sinusoidal double bump excitation[5].

The road profile has a significant effect on the vehicle's vibration. Non-stationary random vibration analysis of a quarter-car model was studied in [6]. Li-Xin Guo and Li-Ping Zhang worked on half-car model random vibration analysis in changeable speed [7]. In this paper, modified non-dominated sorting genetic algorithm (modified NSGA-II) is used for multi-objective optimization of a 5-DOF vehicle model which excited with random road profile. The conflicting objective functions which have been considered are vertical acceleration of seat relative displacement between sprung mass and forward tire  $d_f$  and relative displacement between sprung mass and rear tire  $d_r$ . The design variables used in this optimization are seat damping coefficient  $c_s$ , seat stiffness coefficient  $k_s$ , forward vehicle damping coefficient  $c_f$ , rear vehicle damping coefficient  $c_r$ , forward vehicle stiffness coefficient  $k_f$  and rear vehicle stiffness coefficient  $k_r$ . As a result, 5-objective optimization is used to select the optimal parameters.

Finally, selected points are compared with those which have been chosen by [5].

## 2 SIMULATION OF ROAD ROUGHNESS

The road profile is the only input of the passive suspension system which has a significant effect on the vehicle's vibration. Hence in order to generate a suitable road profile, it was investigated very carefully [8]. The road profile can be represented by a PSD function. The power spectral densities of roads show a characteristic drop in magnitude with the wave number [9]. Random road profiles can be approximated by a PSD in the form of

$$\phi(\Omega) = \phi(\Omega_0) \left( \frac{\Omega}{\Omega_0} \right)^{-\omega} \quad (1)$$

Where  $\Omega = \frac{2\pi}{\lambda}$  in rad/m denotes the angular spatial frequency,  $\lambda$  is the wavelength,  $\phi_0 \triangleq \phi(\Omega_0)$  in  $m^2/(\text{rad}/m)$  describes the value of the PSD at the reference wave number  $\Omega_0 = 1 \text{ rad}/m$ ,  $\omega$  is the waviness, for most of the road surface,  $\omega = 2$ . In [10] and [11] the road roughness PSD distribution is modified as :

$$\psi(\omega) = \frac{2\alpha V \sigma^2}{\omega^2 + \alpha^2 V^2} \quad (2)$$

Where  $\sigma^2$  denotes the road roughness variance and  $V$  the vehicle speed, whereas  $\alpha$  depends on the type of the road surface. Since the spectral density of the road profile can be factored as:

$$\psi(\omega) = \frac{2\alpha V \sigma^2}{(\alpha V - j\omega)(\alpha V + j\omega)} = H(\omega) \psi_\omega H^T(-\omega) \quad (3)$$

Where  $H(\omega) \triangleq \frac{1}{(\alpha V + j\omega)}$  is the frequency response

function of the shaping filter,  $\psi_\omega = 2\alpha V \sigma^2$  is the spectral density of a white noise process. Hence, if the vehicle runs with constant velocity  $\frac{ds}{dt} = V$ , then the road profile signal,  $z_R(t)$ , would be obtained from the differential equation below:

$$\frac{d}{dt} z_R(t) = -\alpha V z_R(t) + \omega(t) \quad (4)$$

Where  $\omega(t)$  is a white noise process with the spectral

density,  $\psi_\omega$ . It can be shown that:

$$z_R(t) = e^{-\alpha V t} z_R(0) + \int_0^t e^{-\alpha V(t-\tau)} \omega(\tau) d\tau \tag{5}$$

Table 1 shows the road roughness standard deviation for A to E road classes.

**Table 1** Road roughness standard deviation

Road class	$\sigma(10^{-3} m)$	$\phi(\Omega_0)(10^{-6} m^3)$	$\alpha(rad / m)$
		$\Omega_0=1$	
A (very good)	2	1	0.127
B (good)	4	4	0.127
C (average)	8	16	0.127
D (bad)	16	64	0.127
E (very bad)	32	256	0.127

The equation above can be solved by MATLAB SIMULINK shown as Fig. 1.

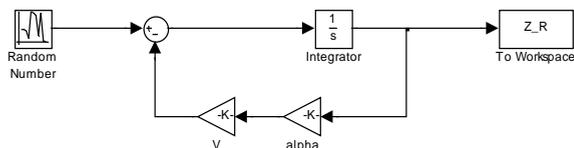


Fig. 1 Simulation of road roughness by Matlab Simulink

Figure 2 shows the sample  $z_R$  for Grade C road.

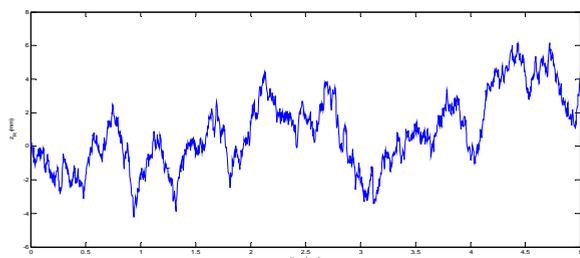


Fig. 2 Sample  $z_R$  for Grade C road

### 3 MULTI OBJECTIVE OPTIMIZATION

Multi-objective optimization (or multi-objective programming or "Pareto optimization"), [12] and [13] also known as multi-criteria or multi-attribute optimization, is the process of simultaneously optimizing two or more conflicting objectives subject

to certain constraints. In mathematical terms, the multi-objective problem can be written as:

$$\begin{aligned} & \min_x [\mu_1(x), \mu_2(x), \dots, \mu_n(x)]^T \\ & s.t. \\ & g(x) \leq 0 \\ & h(x) = 0 \\ & x_1 \leq x \leq x_u \end{aligned} \tag{6}$$

Where  $\mu_i$  is the  $i$ -th objective function,  $g$  and  $h$  are the inequality and equality constraints, respectively, and  $x$  is the vector of optimization or decision variables. The solution to the above problem is a set of Pareto points. Thus, instead of being a unique solution to the problem, the solution to a multi-objective problem is a possibly infinite set of Pareto points. A design point in objective space  $\mu$  is termed Pareto optimal if there does not exist another feasible design objective vector  $\mu^*$  such that  $\mu_i \leq \mu_i^*$  for all  $i \in \{1, 2, \dots, n\}$ , and  $\mu_j < \mu_j^*$  for at least one index of  $j$ ,  $j \in \{1, 2, \dots, n\}$ .

There are many ways and methods for finding a solution to a multi-objective optimization problem. The objective way of characterizing multi-objective problems, by identifying multiple Pareto optimal candidate solutions, requires a Pareto-compliant ranking method, favoring non-dominated solutions, as seen in current multi-objective evolutionary approaches such as NSGA-II and SPEA2. In recent years, the Pareto-based approach of NSGA-II has been used in a wide range of problems due to its simplicity and fast process.

NSGA-II proposed by Dep [14], has some drawback in its main program and crowding factor subprogram which are completely discussed by [15]. In this work, modified NSGA-II, proposed by [16] and [17] is used to find optimal characteristics of a 5-DOF vehicle model.

### 4 VEHICLE HALF MODEL

A 5-DOF linear mechanical model of a vehicle is shown in Fig. 3. In this figure,  $m_c$  is the mass of driver and chair and  $m_s$  is the mass of vehicle structure.  $m_f$  and  $m_r$  are the unsprung mass of front and rear suspensions, respectively.  $k_s$  and  $c_s$  are rigidity coefficient and damping coefficient of front and rear suspensions, respectively.  $z_{Rf}$  and  $z_{Rr}$  are the road excitation displacements at front and rear wheels,

respectively.  $r$  is the distance between chair and vehicle mass center.  $a$  and  $b$  are the distances from the vehicle mass center to front and rear wheel axles, respectively.

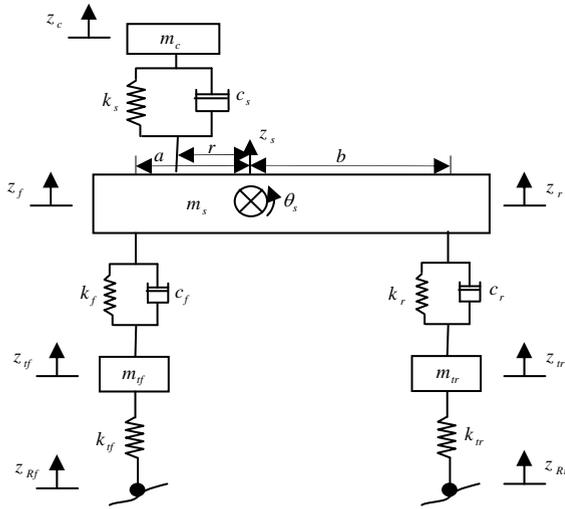


Fig. 3 Half vehicle model with five DOF

Table 2 Constant parameter of the model

Parameter	Value
$a$	1.011 m
$b$	1.803 m
$r$	0.279 m
$m_{tf}$	40 kg
$m_{tr}$	35.5 kg
$m_c$	75 kg
$m_s$	730 kg
$I_s$	1230 kg.m <sup>2</sup>
$k_{tf}$	175500 N/m
$k_{tr}$	175500 N/m

By using Newton's law, the mathematical model of the Figure 3 can be written as below:

$$[M]\{\ddot{Z}\} + [C]\{\dot{Z}\} + [K]\{Z\} = [F] \quad (7)$$

Where  $[M]$ ,  $[C]$ ,  $[K]$  and  $[F]$  are mass, damping coefficient, spring stiffness and force matrixes respectively. These matrixes are represented in the Appendix.

## 5 MULTI-OBJECTIVE OPTIMIZATION OF 5-DOF VEHICLE MODEL WITH RANDOM ROAD PROFILE

It is well-known that the results of multi-objective optimization would be a set of non-dominated optimized points, called Pareto set. These points offer the wide range of parameters to the designer to choose the optimum point depending on his designing conditions. There are always conflicting objective functions in vehicle designing which improvement in one function may have unfavorable influence on other functions.

In this chapter, multi-objective optimization for all 3-objective functions is done simultaneously. It is supposed that the vehicle moves at constant velocity  $v = 20 \text{ m/s}$  over a Grade C road. It is assumed that the rear tire moves at the same road profile as forward tire with a time delay of  $\Delta t = \frac{a+b}{v}$ . Table 2 shows the

constant parameters of the model presented in [2]. In this work,  $k_s$ ,  $C_s$ ,  $k_f$ ,  $C_f$ ,  $k_r$ , and  $C_r$  are considered to be design variables with the following relations:

$$\begin{aligned} 50000 < K_s (N/m) < 150000 \\ 1000 < C_s (Ns/m) < 4000 \\ 10000 < K_f (N/m) < 20000 \\ 500 < C_f (Ns/m) < 2000 \\ 10000 < K_r (N/m) < 20000 \\ 500 < C_r (Ns/m) < 2000 \end{aligned} \quad (8)$$

These variables should be optimally selected by multi-objective optimization of three conflicting objective functions namely, vertical acceleration of seat  $\ddot{z}_c$ , relative displacement between sprung mass and front tire  $d_f$  and relative displacement between sprung mass and rear tire  $d_r$  in which all of them should be minimized. To achieve this purpose RMS method is used to assign a certain value for every objective function against every set of design variables. Finally, optimum point can be easily achieved by mapping of the values of objective functions of all non-dominated points into interval 0 and 1. Using the sum of these values for each non-dominated points, the design point, represented by this work, would be the minimum of those values.

To solve the multi-objective optimization using genetic algorithm, a population of 80 individuals with a crossover probability of 0.9, mutation probability of 0.1, elimination criteria of  $10^{-6}$  in  $\epsilon$ -elimination algorithm and a chromosomes length of 56 has been used in 240 generations. Table 3 shows the optimum points represented in [2], [5], and this paper.

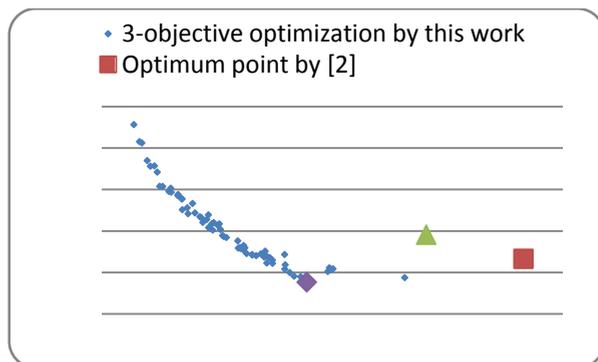
**Table 3** Optimum points represented in [2],[5], and this paper

Road class	[2]	[5]	This paper
$k_s \left( \frac{N}{m} \right)$	105630	144902	51254.84
$C_s \left( \frac{Nm}{s} \right)$	1956	2788.4	1577.85
$k_r \left( \frac{N}{m} \right)$	14958	10000	10110.98
$C_r \left( \frac{Nm}{s} \right)$	1956	1294.12	1986.95
$k_f \left( \frac{N}{m} \right)$	14985	10196.1	10881.84
$C_f \left( \frac{Nm}{s} \right)$	1967	1082.35	1075.68

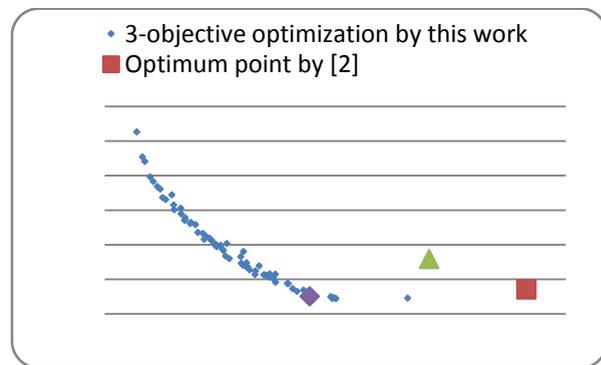
Figure 4 and 5 show the results of 3-objective optimization in the plane of  $(\dot{z}_c - d_f)$  and  $(\dot{z}_c - d_r)$ , respectively. As it can be seen from these figures, the optimum point which reported by this work have better response to the random input than points represented in [2] and [5]. RMS values of the objective functions of the design variables represented in the Table 3, are shown in Table 4.

**Table 4** RMS values of the objective functions of the optimum points in [2], [5], and this paper

Road class	[2]	[5]	This paper
RMS of $\dot{z}_c$	0.0040114	0.003548	0.00297782
RMS of $d_f$	6.16E-05	6.45E-05	5.88E-05
RMS of $d_r$	3.85E-05	4.29E-05	3.75E-05

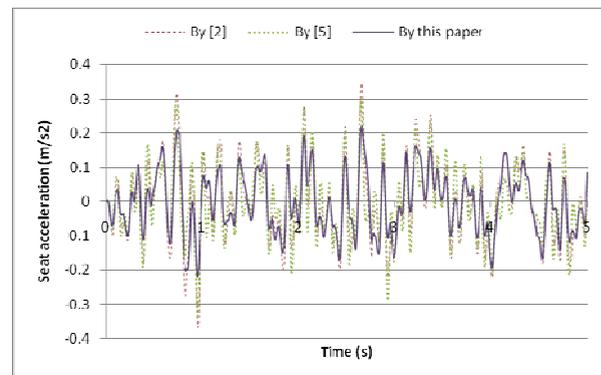


**Fig. 4** Pareto of RMS of vertical acceleration of seat and RMS of relative displacement between sprung mass and forward tire in 3-objective optimization

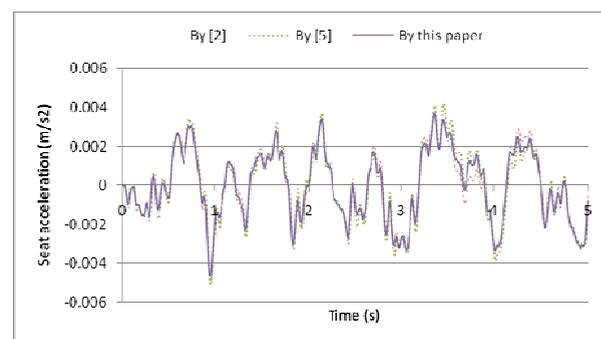


**Fig. 5** Pareto of RMS of vertical acceleration of seat and RMS of relative displacement between sprung mass and rear tire in 3-objective optimization

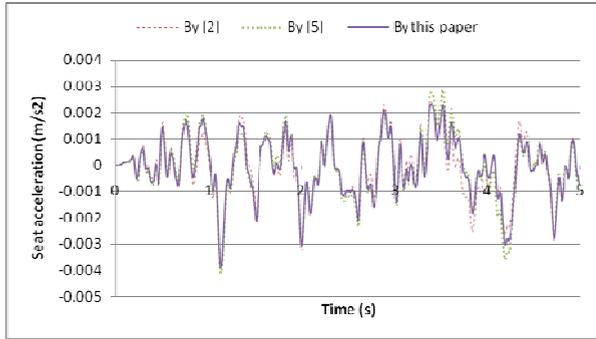
Figure 6 to 8 show the time responses of the objective functions based on design variables represented in Table 3. From these figures, it can be seen that all objective functions have better behavior in comparison with [2] and [5] by choosing design variables which represented in this paper.



**Fig. 6** Time responses of seat acceleration of optimum points represented in Table 3



**Fig. 7** Time responses of relative displacement between sprung mass and forward tire of optimum points represented in Table 3



**Fig. 8** Time responses of relative displacement between sprung mass and rear tire of optimum points represented in Table 3

## 6 CONCLUSION

Modified non-dominated sorting genetic algorithm II (NSGA-II) has been used for multi-objective optimization of a 5-DOF vehicle model which excited with random road profile. The conflicting objective functions which have been considered were vertical acceleration of seat  $\ddot{z}_c$ , relative displacement between sprung mass and forward tire  $d_f$ , and relative displacement between sprung mass and rear tire  $d_r$ .

The design variables have been used in this optimization were seat damping coefficient  $c_s$ , seat stiffness coefficient  $k_s$ , forward vehicle damping coefficient  $c_f$ , rear vehicle damping coefficient  $c_r$ , forward vehicle stiffness coefficient  $k_f$  and rear vehicle stiffness coefficient  $k_r$ .

By using random road excitation, much more reliable design variables would be reported. In this work, new optimum points have been represented by considering a random road profile instead of simple double bump which had been used in the literature and finally, the superiority of these optimum points have been shown in comparison with those represented in the literature.

## 7 APPENDIX

By using Newton's law for each mass, following relations could be obtained:

$$M = \begin{bmatrix} m_c & 0 & 0 & 0 & 0 \\ 0 & m_s & 0 & 0 & 0 \\ 0 & 0 & I_s & 0 & 0 \\ 0 & 0 & 0 & m_{ff} & 0 \\ 0 & 0 & 0 & 0 & m_{rr} \end{bmatrix} \quad (9)$$

$$C = \begin{bmatrix} c_s & -c_s & r c_s & 0 & 0 \\ -c_s & c_s + c_f + c_r & -r c_s - a c_f + b c_r & -c_f & -c_r \\ r c_s & -r c_s - a c_f + b c_r & r^2 c_s + a^2 c_f + b^2 c_r & a c_f & -b c_r \\ 0 & -c_f & a c_f & c_f & 0 \\ 0 & -c_r & -b c_r & 0 & c_r \end{bmatrix} \quad (10)$$

$$K = \begin{bmatrix} k_s & -k_s & r k_s & 0 & 0 \\ -k_s & k_s + k_f + k_r & -r k_s - a k_f + b k_r & -k_f & -k_r \\ r k_s & -r k_s - a k_f + b k_r & r^2 k_s + a^2 k_f + b^2 k_r & a k_f & -b k_r \\ 0 & -k_f & a k_f & k_f + k_{ff} & 0 \\ 0 & -k_r & -b k_r & 0 & k_r + k_{rr} \end{bmatrix} \quad (11)$$

$$F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ k_{ff} z_{ff} & 0 \\ 0 & k_{rr} z_{rr} \end{bmatrix} \quad (12)$$

$$\{Z\} = \{z_c \quad z_s \quad \theta_s \quad z_{ff} \quad z_{rr}\}^T \quad (13)$$

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