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Optimum Design and Construction of Hydraulic Sections of Parabolic Water Transmitting Channels using the Harris Hawks Optimization Algorithm

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Abstract: Channels have various types of cross-sectional shapes, including trapezoidal, rectangular, semi-circular, parabolic, chain-curved, semi-cubic parabolic, egg-shaped, and circular as the most common shapes. A channel designer has many design options in different conditions, including hydraulic, economic, and hydrological conditions, leakage, etc. Among the above-mentioned sections, the first two have a horizontal bottom while the other sections are curve-shaped with bottom curvature. The primary goal in the design of hydraulic channels is to achieve the maximum flow capacity considering the minimum channel construction cost. A variety of studies has been conducted on the different types of hydraulic channels so far, each dealing with the subject from a certain perspective. However, most of the studies have focused on circular, rectangular and trapezoidal channels. This study has focused on the parabolic channel. Genetic algorithm (GA) and particle swarm optimization (PSO) or GRG algorithms and their combination are usually used for optimization. However, this research adopts a novel and updated meta-heuristic algorithm, namely the Harris Hawks Optimization (HHO) algorithm, to optimize the parabolic channel with a fixed roughness coefficient and determine the optimal dimensions of the channel with different flow rates. This channel uses different flow rates, namely 50, 100, 150, 200, 250, and 300 m^3/s to solve the optimization problem. Finally, it was found that the lowest construction cost and the highest efficiency for water supply is achieved with a roughness coefficient of 0.015 and a flow rate of 100 m^3/s.

Keywords: Harris Hawks, Meta-Heuristic Algorithm, Optimization, Parabolic Channel

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Research paper

1 INTRODUCTION

The flow of water in an open channel is a familiar sight, whether it is a natural channel like that of a river, or an artificial channel like that of an irrigation ditch. Its movement poses a difficult problem when everything is considered especially with the variability of natural channels, but in many cases the major features can be expressed in terms of only a few variables, whose behavior can be described by a simple theory. The principal forces at work are those of inertia, gravity and viscosity, each playing an important role [1].

Open-channel flow occurs when a liquid flowing due to gravity is only partially enclosed by its solid boundary. The flowing liquid has a free surface that meets the atmosphere and is not subjected to any pressure other than that caused by its own weight and atmospheric pressure. The force affecting the open-channel flow is gravity. This type of flow can be seen in rivers, gravity sewer systems, drains, irrigation canals and many other examples in nature. Flow in open channels, or conduits where water has a free surface, differs from that in pipes because the pressure at the free surface is constant (usually atmospheric) and does not vary from point to point in the direction of flow as does pressure in a pipeline. Another difference is that the cross-section is not controlled by fixed boundaries, because the depth can change from one section to another without restrictions [2].

Investigations into open-channel flow types have many functions in civil engineering and some other branches of engineering, e.g. chemical and mechanical. Open-channel flow can be described and classified in various ways according to the change in flow depth with respect to time and space. They also have various types of cross-sectional shapes, including trapezoidal, rectangular, semi-circular, parabolic, chain-curved, semi-cubic parabolic, egg-shaped, and circular as the most common shapes. A channel designer has many design options in different conditions including hydraulic, economic, and hydrological conditions, leakage, etc. Researchers have considered the advantages of curved sections as follows: 1). Sharp angles that cause stress concentration are hardly found in them. 2). Since the amount of lateral slope in the channel surface is always lower than that in the water level in these channels, the curved channels are physically more stable. 3). Non-lined channels and irrigation grooves tend to approximate their cross-sectional shape as curves, which makes them hydraulically stable. 4). Normally, channels with curved sections have a higher water flow capacity than those with trapezoidal and rectangular sections [3].

A quadratic parabolic channel was analyzed by [4], and the hydraulic optimal section was specified as a section with 0.514 as the slope coefficient. Reference [5] presented an approach for calculating slope coefficient and parabolic exponent was considered as a variable for designing a power-law exponential parabolic channel section with an acceptable hydraulic efficiency. In [6], the relation between the specific energy of a parabolic channel section and the depth of the channel section was considered and the supercritical and subcritical depths were obtained in an analytic manner through finding a solution for a quartic Equation. Using the undetermined Lagrange multiplier optimization algorithm and a complex function technique, the hydraulic optimal section of a cubic parabola was theoretically solved and the superior hydraulic performance of the cubic parabolic section was indicated [3], [7]. Accordingly, Gaussian hypergeometric functions were used for determining the section parameters of the power-law exponential parabolic channel and finding the hydraulic optimal section. Reference [8] determined the hydraulic optimal section of a power-law exponential parabolic channel. For this purpose, the practical economic section was calculated. Reference [9] considered the lining thickness, and for an arch foot trapezoidal channel, developed a mechanical model. Reference [10] used the other channels' mechanical models in order to develop a mechanical model for quadratic parabolic channels.

Reference [11] discussed the optimal design of circular channels with fixed and variable roughness scenarios using machine learning models. They used two machine learning (ML) models named artificial neural networks (ANN) and genetic programming (GP) to determine optimum channel geometries for trapezoidal-family cross sections. Reference [12] investigated the optimal hydraulic cross-section in two-section parabolic channels. They tried to find the optimal hydraulic cross-section of a two-stage channel with a parabolic cross-section and a flat-width parabola. Therefore, by considering the constant value of the area and the wetted perimeter as the objective function, they obtained the optimal hydraulic geometric parameters for two types of channels and achieved the Equations of the best hydraulic section of each type.

As previously stated, it is of great importance to explore channels in order to achieve high efficiency and low cost. A great deal of research has been done in this field and valuable results have been produced by applying mathematical methods and old algorithms. However, this article examines the parabolic channel using the novel HHO algorithm. This algorithm is utilized in many cases, including weld cross-section optimization, gear design, etc. Nevertheless, this article marks the first time that this algorithm is applied for optimum design of channel cross-section. The results of this algorithm are highly consistent with and even more optimal than those of previous research.

It can be seen that the calculation methods related to optimal hydraulic sections have matured. However, optimal hydraulic design is often not suitable due to hydraulic and economic considerations for the construction of long channels. This study investigated a channel with a parabolic cross-section, trying to optimize its parameters using the meta-heuristic HHO algorithm. Among the various meta-heuristic algorithms proposed so far, the HHO algorithm was chosen due to its higher power and speed in convergence. The rest of the article is organized as follows: The second section deals with the optimal design of the channel with a parabolic cross-section and its governing fundamental and mathematical Equations. The third section discusses the optimization algorithm. The fourth and fifth sections present the results and the conclusion, respectively.

2 OPTIMUM DESIGN OF CHANNELS WITH PARABOLIC CROSS-SECTION

Another cross-section that can be used as a high-efficiency cross-section, especially in high-efficiency flows, is a parabolic cross-section. The figure below shows a schematic of a parabolic cross-section. In parabolic channels, the design variables are more than the circular cross-section, and the length and width of the cross-section are also added to them. All the considered specifications can be seen in the "Fig. 1". In the present work, the most generalized form was adopted for the cost function in [13]. This cost function assumes that the channel section's upper surface is the surface of the ground [14]. The excavation cost is assumed to be simply the digging cost.

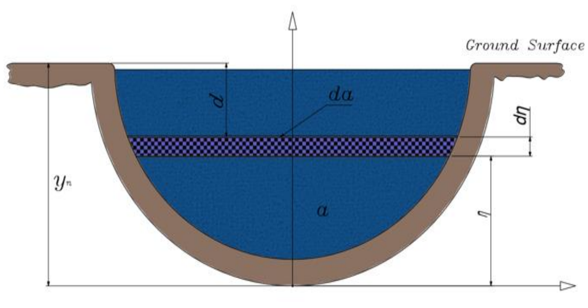

Fig. 1 A schematic of a parabolic channel section.

According to this cost function, there are two main costs included in the total construction cost per unit length of channel: (1) the lining cost, and (2) the earthwork cost. The earthwork cost includes two components: (1) the earthwork operation cost per unit area (β_E) and (2) the increment in the earthwork cost with depth under the ground surface (β_A). The additional earthwork vost accounts for the supporting costs in deep excavations and overload pressure on deeper soil strata [15]. As a result of this cost, different costs of earthwork occur that are different at varying depth levels. Lastly, it is possible to formulate the channel sum as below:

$$C = \beta_L P + \beta_E A + \beta_A \int_0^{y_n} adn \tag{1}$$

Where C is the total construction cost per length of a lined channel section, L is the cost of lining per length, P is the wetted perimeter, A is the cross-sectional area of the channel, y_n is the normal water depth, a is the flow area at height η, and dη is the unit length of earthwork at height η, where η represents the vertical axis of the channel geometry ("Fig. 1"). To make a relationship between the channel geometry and hydraulic parameters, a resistance Equation, like Manning's Equation, can be used. This resistance Equation, which is commonly used in open-channel flows, guarantees that the final optimization results can be applied from a hydraulic perspective. It is possible to write Manning's Equation in SI units as in the following:

$$Q - \frac{1}{n} A R^{\frac{2}{3}} \sqrt{S} \tag{2}$$

Where Q is the flow rate of the channel, n is Manning's roughness coefficient, R is the hydraulic radius and S is the bottom slope of the channel. We used dimensionless variables to extend the applicability of the solution to a wide range of possible values for the involved parameters. The dimensional parameters were converted to dimensionless using a new parameter, the length scale (λ), is presented in the following Equation:

$$\lambda = \left(\frac{Qn}{\sqrt{S}}\right)^{\frac{3}{8}} \tag{3}$$

All dimensional hydraulic variables can be converted to dimensionless ones using (λ). These parameters include: (1) total cost (C), (2) excavation cost per unit (β_E), (3) additional soil cost (β_A), (4) cost of lining per length (β_L), (5). The area of the excavated channel, i.e. $A = 0.5r^2(\theta - \sin(\theta)$.

Where θ is the water depth angle, (6) wetted perimeter (p=θr), (7) water depth (yn) and (8) channel radius (r). These parameters can be converted into their dimensionless forms using λ and β_E. The new dimensionless variables, marked with an asterisk, are presented in the following Equations:

$$C_* = \frac{c}{\beta_E \lambda^2} \tag{4}$$

$$\beta_{A*} = \frac{\beta_A \lambda}{\beta_E} \tag{5}$$

$$\beta_{L*} = \frac{\beta_L}{\beta_E \lambda} \quad (6)$$

$$A_* = \frac{A}{\lambda^2} \quad (7)$$

$$P_* = \frac{P}{\lambda} \quad (8)$$

$$y_{n*} = \frac{y_n}{\lambda} \quad (9)$$

$$r_* = \frac{r}{\lambda} \quad (10)$$

"Table 1" lists the objective function and constraints of the optimization problem above. The following relation is mainly aimed at minimizing the value of C*.

Table 1 Objective function and constraint of the problem

Objective Function:	$C_* = \beta_{L*}P_* + A_* + \frac{\beta_{A*}\int_0^{y_n} adn}{\lambda^3}$
Constraint of the Problem:	$1 - A_*^{\frac{5}{3}} P_*^{\frac{2}{3}} = 0$

2.1. Algorithm and the Optimization Method

In 1997, Louis Lefebvre proposed an approach to measure the "intelligence quotient" of birds. According to his studies, hawks can be classified among the most intelligent birds in nature. Harris hawks are well-known prey-hunting birds found in relatively stable flocks in the southern half of Arizona, United States. The HHO algorithm is a population-based and gradient-free optimization method. Hence, it can be applied to any optimization problem with a suitable formulation. The figure below shows all the steps of the algorithm, which will be described in detail in the next sections [16].

In the HHO algorithm, according to the nature of Harris hawks, it can be said that these birds can track and recognize the prey with their powerful eyes, but sometimes the prey is not easily seen. Hence, hawks wait and observe and monitor the area to detect the prey after a few hours.

In the HHO algorithm, Harris hawks are candidate solutions, and the best candidate solution at each step is considered the optimal or near-optimal prey. Harris hawks randomly sit and wait in places. If we consider the chance q for each sitting strategy, the prey is detected based on two strategies:

* Hawks sit and wait based on the location of other hawks and rabbits (q < 0.5).

* They sit and wait on high trees randomly (a random place near the group house) (q >= 0.5).

$$X(t+1) = \begin{cases} X_{rand}(t) - r_1 |X_{rand}(t) - 2r_2 X(t)| & q \geq 0.5 \\ (X_{rabbit}(t) - X_m(t)) - r_3(LB + r_4(UB - LB)) & q < 0.5 \end{cases} \quad (13)$$

Where X(t+1) is the location vector of the hawks at iteration t, Xrabbit(t) is the location of the rabbit, X(t) is the current location vector of the hawks, r1, r2, r3, r4 and q are random numbers in the (0,1) range, which are updated in each iteration, LB and UB represent the upper and lower bounds of the variables, Xrand(t) is the location of a random hawk from the current population, and Xm is the average location from the current population of hawks. The pseudocode of the HHO algorithm is given in "Fig. 2".

Inputs: The population size N and maximum number **of** iterations T
Outputs: The location **of** rabbit and its fitness value
Initialize the random population Xi(i = 1, 2, . . . ,N)
while (stopping condition is not met) **do**
Calculate the fitness values **of** hawks
Set Xrabbit as the location **of** rabbit (best location)
for (each hawk (Xi)) **do**
Update the initial energy E0 and jump strength J ▷ E0=2rand()-1, J=2(1-rand())
Update the E using
if (|E| >= 1) **then** ▷ Exploration phase
Update the location vector using
if (|E| < 1) **then** ▷ Exploitation phase
if (r >= 0.5 and |E| >= 0.5) **then** ▷ Soft besiege
Update the location vector using
else if (r >= 0.5 and |E| < 0.5) **then** ▷ Hard besiege
Update the location vector using
else if (r <0.5 and |E| >= 0.5) **then** ▷ Soft besiege with progressive rapid dives
Update the location vector using
else if (r <0.5 and |E| < 0.5) **then** ▷ Hard besiege with progressive rapid dives
Update the location vector using
Return Xrabbit

Fig. 2 Flowchart of the HHO algorithm.

"Table 2" presents the specifications considered for the optimization in the parabolic channel.

Table 2 Important parameters of channels in simulation

Parabolic Channel	
Manning's roughness coefficients	0.015
Bottom longitudinal slope	0.001
Volume flow rate	50-300

2.2. Optimization Results

The results were obtained for the parabolic channel with flow rates of 50, 100, 150, 200 and 300 m^3/s, with a fixed Manning's roughness coefficient of 0.015 and the bottom longitudinal slope of 0.001. The results related to the construction cost using the meta-heuristic HHO algorithm are given below.

As can be seen in “Fig. 3”, the execution of 100 iterations for the HHO algorithm led to the channel cost of 12.76, which was constant in the 78th iteration, after which there was no improvement in the results. In a simple comparison, we can see a relatively large difference between the initial and optimized values, indicating that optimization can significantly reduce the cost of channel construction.

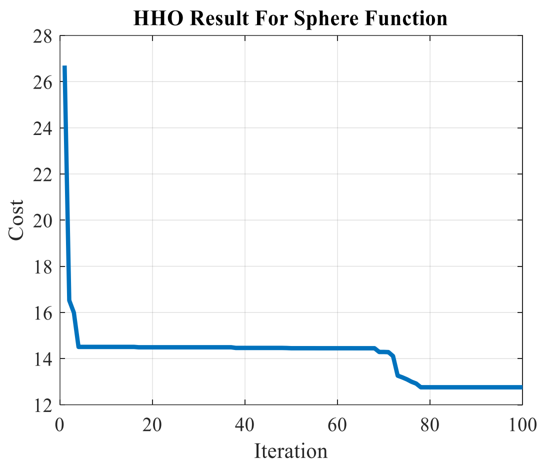

Fig. 3 Optimization of the construction cost of a parabolic channel with a flow rate of 50 m^3/s.

The curve in “Fig. 4” shows the amount of cost calculated for a parabolic channel with a flow rate of 100 m^3/s, a fixed Manning’s roughness coefficient of 0.015, and a bottom longitudinal slope of 0.001. The construction cost initially started from 56 in the first iteration, but it changed to 15.62 after several iterations. A comparison between two flow rates of 50 and 100 indicates that the cost of channel construction increases following an increase at the flow rate when other parameters are the same, and that the flow rate is an effective parameter in optimization. The same is the case about the flow rate of 150; an increase at the flow rate also leads to an increase in the channel construction cost. The noteworthy point is that the value shown for the cost is a number to compare the flow rate changes and construction cost in a channel, and its real value can be determined with appropriate coefficients.

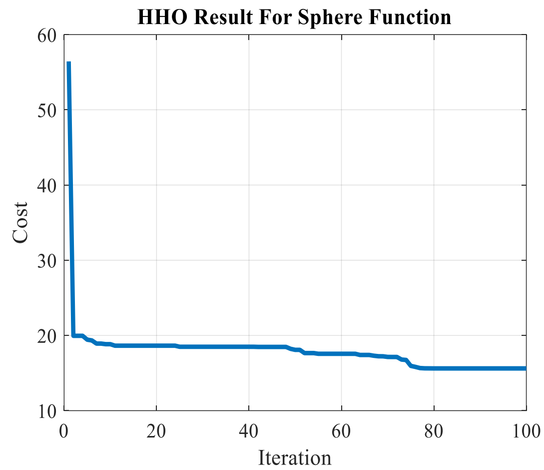

Fig. 4 Optimization of the construction cost of a parabolic channel with a flow rate of 100 m^3/s.

The construction cost of the channel was 18.27 at a flow rate of 150 m^3/s, which indicates an increase in the construction cost by 1.17 times compared to the channel with a flow rate of 100 m^3/s, (See Fig. 5).

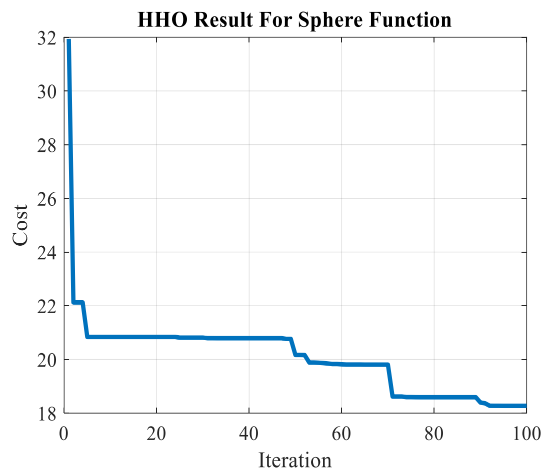

Fig. 5 Optimization of the construction cost of a parabolic channel with a flow rate of 150 m^3/s.

In addition, the optimized value was improved during several stages and successive iterations so that the value obtained in the first iteration was about 32 (note that these values are dimensionless) but it was reduced to 18.27 in the next iterations. Therefore, it can be concluded that the construction cost before optimization is about 1.75 times that after optimization, which indicates the need for optimization before the channel construction. The channel construction cost reached 19.39 at the flow rate of 200 m^3/s, which has increased by 1.06 compared to the flow rate of 150, (See Fig. 6)

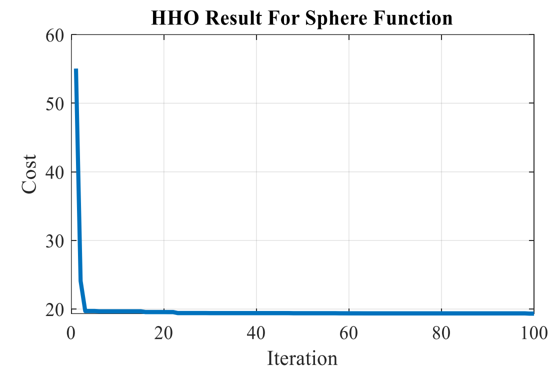

Fig. 6 Optimization of the construction cost of a parabolic channel with a flow rate of 200 m^3/s.

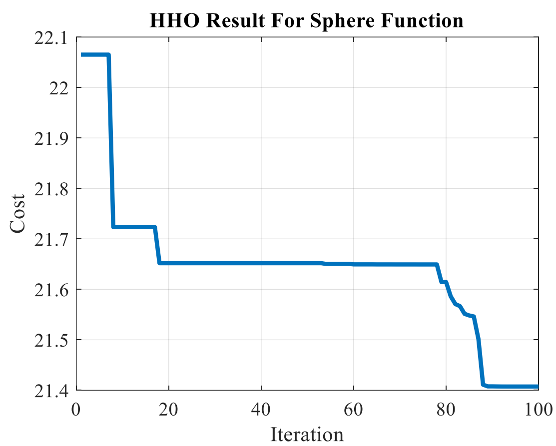

Fig. 7 Optimization of the construction cost of a parabolic channel with a flow rate of 300 m^3/s.

Figure 7 shows the optimization function of the cost function for a volume flow rate of 300 m^3/s in 100 iterations. As can be seen, the convergence curve in this volume flow rate was obtained as 21.41, which is the same as the channel optimal construction cost. This time, all the settings are the same as in the previous cases, with a fixed roughness coefficient of 0.015. In the first iteration, the displayed cost value is about 55, but the HHO algorithm gradually improved the results and reduced the construction cost value to 21.41 in the next iterations. "Table 3" summarizes the optimized results for 5 different flow rates, whose convergence curves were given before. These results indicate the upward trend of the construction cost, which was naturally expected.

Table 3 Results of the construction cost of a parabolic channel under different flow rate conditions

Row	Flow rate (m^3/s)	Construction cost (dimensionless)
1	50	12.76
2	100	15.62
3	150	18.27
4	200	19.39
5	300	21.41

Figure 8 shows the changes between the construction cost and the flow rate for a channel with a parabolic cross-section. As can be seen, the construction cost increases following an increase at the flow rate, which indicates the need to optimize the flow rate and construction cost. The figure below shows that there is a linear relationship between the increased volume flow rate and the construction cost. In other words, a linear relationship can be observed between the volume flow rate of the parabolic channel and the construction cost.

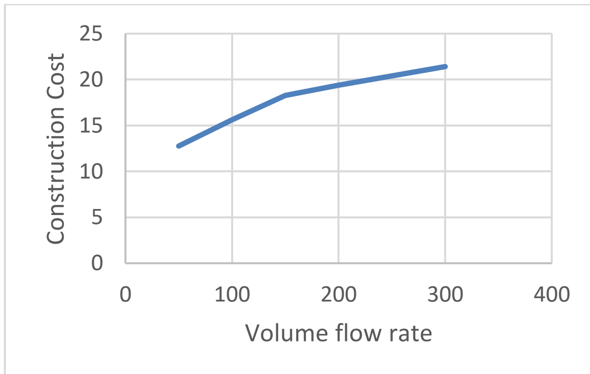

Fig. 8 Changes in the construction cost of the channel with changes in the volume flow rate.

The following "Figs. 9, 10, and 11" illustrate the dimensionless radius and depth values for the parabolic channel. As you can see, the horizontal axis (x-axis) is the dimensionless unit cost, which is known by the parameter β_l^* and includes values less than 1 to those greater than 1. The vertical axis (y-axis) also shows the values related to the radius and depth of the channel. For each curve, a value was first considered for β_A^*, and then the β_l^* parameter increased from 0 to 5, thus the radius and depth values were obtained. We considered the three values of 0, 0.5 and 1 for the β_A^* parameter to be able to examine the radius and depth changes following a change in the β_A^* parameter. In addition, the roughness parameter for the parabolic channel was considered constant in all cases.

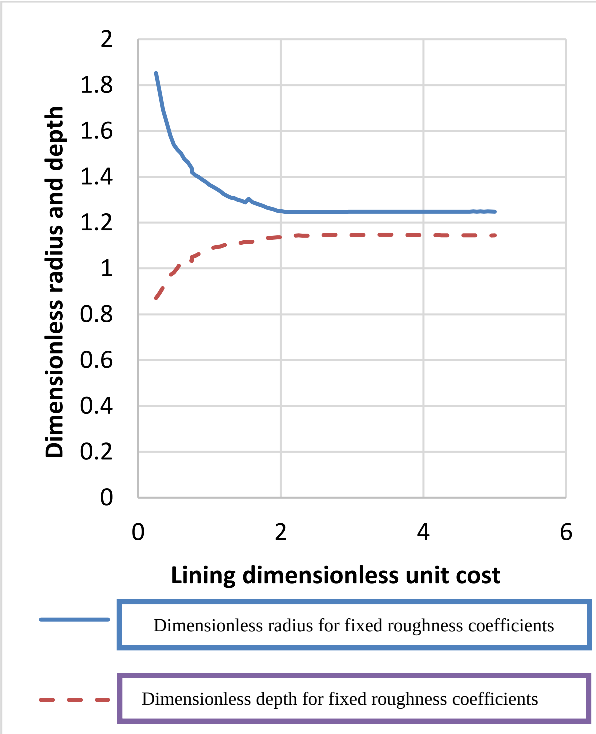

Fig. 9 Dimensionless radius and depth for different β_l^* values and β_A^* =0.

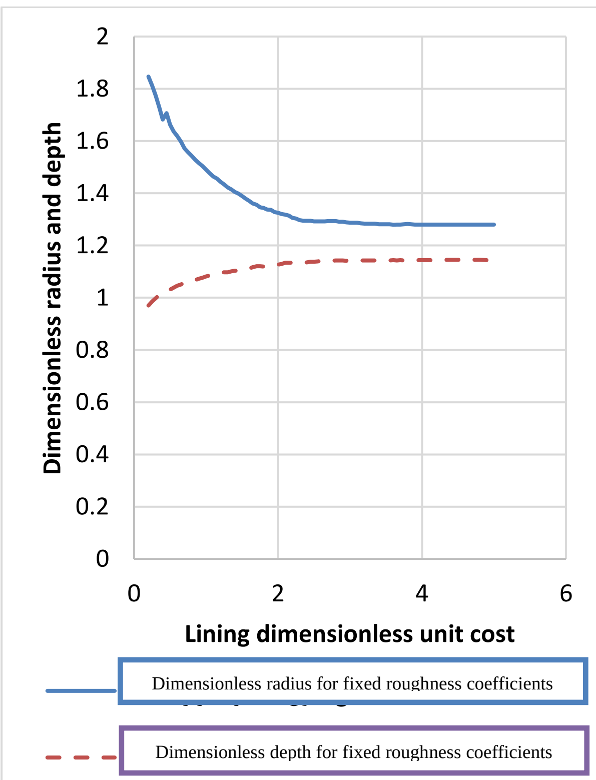

Fig. 10 Dimensionless radius and depth for different β_l^* values and $\beta_A^* = 0.5$.

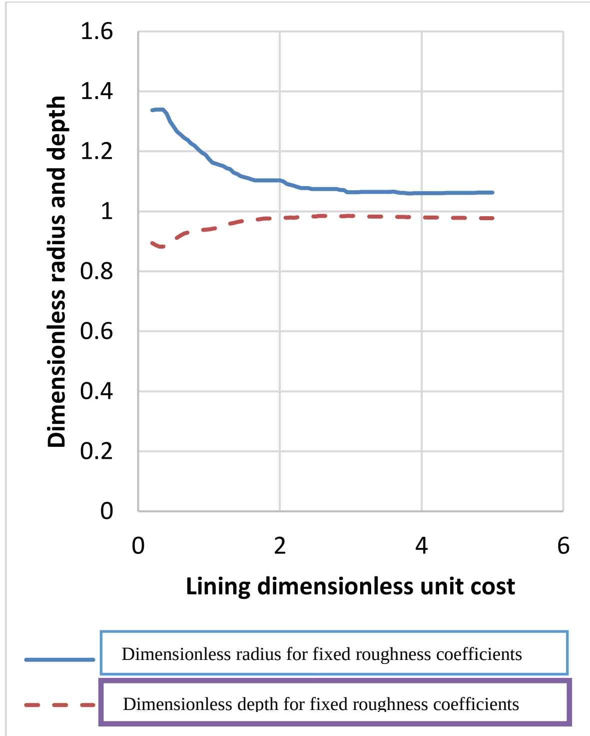

Fig. 11 Dimensionless radius and depth for different β_l^* values and $\beta_A^* = 1$.

The results for the parabolic section with the cost objective function and the constraint function of the Manning's Equation are presented in the "Table 4". l is the length scale for dimensionless direction, whose unit is meter. In this case, the flow rate is equal to 100 m^3/s, the bottom longitudinal slope of the channel is equal to 0.0002, and Manning's roughness coefficient is 0.015.

Table 4 Optimal results for the parabolic channel

Parabolic Cross-section	
Cost	y
15.62	6.25

3 CONCLUSIONS

This study investigated the optimization of hydraulic channel with parabolic cross-section. It used the meta-heuristic HHO algorithm to optimize the parameters. The settings of the research were based on a fixed roughness coefficient, and the results of changes in radius and depth of the optimized channel due to changes in β_l^* and β_A^* were presented. The overall result shows that the HHO algorithm has a very high speed and accuracy in optimization.

The investigations indicated that the flow rate has a direct effect on the cost of the constructed channel in addition to the dimensional parameters of the channel. The construction cost increases with increasing flow rate. It was also determined that a flow rate of 100 m3/s is suitable for constructing a channel and meeting the need for water supply assuming a fixed roughness coefficient and longitudinal slope. “Table 4” lists the optimized cost to construct the channel along with its dimensions.

It was also found that the HHO algorithm is highly accurate; therefore, it is expected to be applied also in other sectors in order to optimize other problems.

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