Time-Dependent Reliability Analysis of Mechanical Structures using an Analytical Approach

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Abstract: Time-dependent reliability analysis of mechanical structures is concerned by the use of an outcrossing approach. In this approach the so-called outcrossing rate plays a critical role and thus it is important to estimate it as straight ward as possible. Despite the availability of a variety of methods to estimate this rate in the literature, still more general and at the same time less sophisticated approaches are desired. In this paper, an analytical method is proposed to evaluate the required outcrossing rate in which the basics of "Parallel System Reliability Formulation" in the framework of directional simulation are used. To indicate the accuracy and efficiency of the method, it is applied to carry out the reliability analysis of a hydrokinetic turbine blade. Since the random variables/processes involved in this analysis possess a set of extremely different variances (and thus make a so-called "non-proportional space); it is shown that the proposed method is also capable to satisfactorily employ a technique of directional importance sampling in order to prohibit massive computations, normally required in such spaces. The results of the analysis show that the proposed method could be successfully applied for the circumstances whose involving processes may be non-stationary and whose space of random variables/processes is extremely non-proportional.

Keywords: Directional simulation, Hydrokinetic turbine blade, Importance sampling, Outcrossing rate, Reliability analysis

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1 INTRODUCTION

Reliability analysis of large and expensive mechanical structures is of prime importance in order to be ensured that they function as desired during their lifetimes. Hydrokinetic turbine blades may be categorized as a part of these structures whose appropriate performance under any extreme load condition is desirable. Since the loads acting on these structures are mainly due to the ocean or river currents whose velocities are uncertain and vary with time, generally time-variant methods are implemented to carry out reliability analysis.

These methods normally require the initial probability of failure (the probability of structural failure at t = 0) and the outcrossing rate to be both known. While the former is usually estimated by the use of well-established techniques (i.e. First/Second Order Reliability Methods, FORM/SORM, or simulation), the latter needs more sophisticated approaches.

Different precise solutions have been given in the literature to treat various problems of structural reliability including either continuous or discrete processes (see for example [1] and [2]). They are, however, restricted to many constraints which limit their implementation. For instance, stationarity and Gaussianity of the involving stochastic processes and linearity of the existing limit state functions are usual provisions which are required for the close-form solutions to be directly used. Although a large amount of efforts has been made to relax the limitations and to give more general solutions, still better and less sophisticated techniques are desired.

Among efforts made to achieve such an objective, there is an approach which is used, in principle, to reduce the current time-dependent reliability problem to an equivalent time-independent one using either "a parallel system reliability" or "a series reliability system" formulation and to compute the outcrossing rate using the conventional methods such as FORM or SORM. Hagen and Tvedt [3] were the first to propose employing a parallel system reliability concept to compute the outcrossing rate. This idea was more developed and applied to a variety of engineering problems by others including Der Kiurehian, Li and Vijalapura et al. ([4-8]). The idea was further developed by Andrieu-Renaud et al., [9] and Sudret [10] by proposing a method called PHI2.

Based on this method any pre-specified limit state of the problem is considered at two time points t and $t+\Delta t$; then using the basic definition of the outcrossing rate, intersection probability of two events namely un-violation of the limit state at time t (i.e. being in the safe domain) and violation of the limit state at time $t+\Delta t$ (i.e. being in the failure domain) during the time interval Δt is sought. Within the framework of FORM, this could be simply estimated using the joint cumulative distribution function of a bivariate Gaussian vector at two time points already mentioned and dividing the result by Δt. Recently, a basically similar analytical approach was suggested by Jiang et al., [11] and Zhang et al., [12] in which "a series system reliability formulation" is applied.

Based on this method, rather than probability of failure, system reliability is calculated using intersection probability of a number of events which are defined as survival of the system during a pre-specified small time interval Δt.

As it is obvious, in both methods pointed out above, attempt is made to discretize the (structural) system lifetime into a number of small time interval Δt within which the involving stochastic processes are assumed stationary. By the use of this technique, generally, non-stationary problems may also be addressed.

The results given by the above methods are, however, mostly dependent to the time-increment (i.e. Δt) taken in the analysis. Despite the efforts carried out by Sudret [10] to overcome the problem by using some further analytical derivations, still instabilities in the results may occur when Δt tends to zero. Moreover, since the methods use the conventional FORM method, less precise results may be obtained if the involving limit state function is highly-nonlinear.

In this paper attempt is made to use the main idea given in the application of a parallel system reliability, but now in the space of $\boldsymbol{X}(t) - \dot{\boldsymbol{X}}(t)$ (i.e. instead of the space $\boldsymbol{X}(t)$-$\boldsymbol{X}(t+\Delta t)$). This makes the analysis less dependent to the time increment value Δt if it is taken sufficiently small. Herein, furthermore, the idea is applied in the framework of directional simulation; the method which now is well-established (see e.g. [13–16]).

This method enables the analysis to cover a broad range of structures including hydrokinetic turbine blades whose governing limit state functions may be highly nonlinear and also whose space of random variables/processes may be highly non-proportional. The latter condition may occur when the involving random variables/processes are highly correlated or their variances are significantly different in magnitude.

This paper is organized as follows. Section 2 presents the basics of time-variant reliability analysis. It is followed by the review of PHI2 method in Section 3. In Section 4, an alternative approach to calculate the outcrossing rates is given. In Section 5 reliability analysis of a hydrokinetic turbine blade is investigated and finally the paper will be ended by Sections 6 and 7 in which some points are discussed and concluding remarks are given.

2 BASIS OF TIME-VARIANT RELIABILITY ANALYSIS

Let the structural reliability problem be assumed to contain a *n*-component vector ***Q****(t)* (i.e. $Q_1(t)$, $Q_2(t)$, ...$Q_n(t)$) which usually defines the loading acting on the structure and a *m*-component vector ***Y****(t)* (i.e. $Y_1(t)$, $Y_2(t)$, ...$Y_m(t)$) which normally describes structural system resistance. Although ***Q****(t)* components are always assumed to be stochastic processes, ***Y****(t)* components could be either a set of random variables or deterministically time-dependent functions. Let also the limit state function $G[\boldsymbol{X}(t)]$ be defined such that its positive values implies safe domain and inversely, its non-positive values defines failure domain and ***X****(t)* gathers all components of ***Q****(t)* and ***Y****(t)*. In the context of time-dependent reliability, the probability of failure over the time interval [*0, T*] is defined as follows [1]:

$$P_{f_T} = \mathbb{P}(\exists\ t\epsilon[0,T], G[\boldsymbol{X}(t)] \leq 0) \tag{1}$$

It is well-known that an upper bound to P_{f_T} needs:

$$P_{f_T} \leq P_{f_0} + E[N^+(0,T)] \tag{2}$$

In which P_{f_0} is the probability of failure at *t=0* and generally requires the following integration to be carried out (see e.g. [15]):

$$P_{f_0} = \int_D f_{\boldsymbol{X}}(\boldsymbol{x})dx \tag{3}$$

Where $f_{\boldsymbol{X}}(\boldsymbol{x})$ is the joint probability density function of ***X****(t)* components at time *t*=0 (i.e. all random variable and stochastic processes at time *t*=0) and D is the failure space in which $G[\boldsymbol{X}(t=0)] \leq 0$**.** *E [.]* in (2) is the expected number of outcrossings out of the safe domain, defined above, during time interval *[0, T]*. The latter may be evaluated from the outcrossing rate being defined as follows [1]:

$$\upsilon^+(t) = \lim_{\Delta t\to 0} \frac{1}{\Delta t}\mathbb{P}\{[G[\boldsymbol{X}(t)] > 0] \cap [G[\boldsymbol{X}(t+\Delta t)] \leq 0]\} \tag{4}$$

Where $\upsilon^+(t)$ is the outcrossing rate at time *t* and is related to *E[.]* in Eq. (2) as follows:

$$E[N^+(0,T)] = \int_0^T \upsilon^+(t)\, dt \tag{5}$$

Note that in the special case of stationarity, $\upsilon^+(t)$ is a constant value and the above integration reduces to $\upsilon^+ T$ in which υ^+ is the outcrossing rate independent of time. In an important special case, where only a scalar process *Q(t)* is involved and the relevant limit state function is *G[Q(t), Y(t)] = Y(t) - Q(t)* with *Y(t)* being a given deterministically time-dependent level function, $\upsilon^+(t)$ may be recast as follows:

$$\upsilon^+(t) = \lim_{\Delta t\to 0} \frac{1}{\Delta t}\left\{F_{Q(t)}[Y(t)]\, F_{Q(t),Q(t+\Delta t)}\big(Y(t), Y(t\Delta t)\big)\right\} \tag{6}$$

In which the functions $F_{Q(t)}$ and $F_{Q(t),Q(t+\Delta t)}$ are the Cumulative Distribution Function (CDF) of *Q(t)* and joint CDF of $Q(t)$ and $Q(t+\Delta t)$ respectively.

Eq. (6) corresponds to the probability content of shaded area indicated in Fig. 1 drawn in the $\boldsymbol{X}(t) - \boldsymbol{X}(t+\Delta t)$ space. $\upsilon^+(t)$ may be then derived if $f_{Q(t),Q(t+\Delta t)}$ is accessible as follows:

$$\upsilon^+(t) = \lim_{\Delta t\to 0} \frac{1}{\Delta t}\left\{\int_{-\infty}^{Y(t)}\left[\int_{Y(t+\Delta t)}^{\infty} f_{Q(t),Q(t+\Delta t)}(u,v)dv\right]du\right\} \tag{7}$$

Furthermore, the space $\boldsymbol{X}(t) - \dot{\boldsymbol{X}}(t)$ ($\dot{X}(t) = \frac{dX(t)}{dt}$ being time derivative process of *X(t)*) may be preferred to work in. In this space, letting $\dot{Q}(t)$ be defined as $\dot{Q}(t) \approx \frac{Q(t+\Delta t)-Q(t)}{\Delta t}$, Eq. (7) will be rewritten as follows:

$$\upsilon^+(t) = \int_{\dot{Y}(t)}^{\infty}\left[\frac{1}{\Delta t}\int_{[Y(t)+\dot{Y}(t)\Delta t]-\dot{Q}(t)\Delta t}^{Y(t)} f_{Q(t),\dot{Q}(t)}(q,\dot{q})dq\right]d\dot{q} \tag{8}$$

Where $\dot{Y}(t) \approx \frac{Y(t+\Delta t)-Y(t)}{\Delta t}$ and $f_{Q(t),\dot{Q}(t)} \approx \Delta t\, f_{Q(t),Q(t+\Delta t)}$ by mapping the variables from ***Q****(t)*-***Q****(t+* Δ**t)** space into $\boldsymbol{Q}(t) - \dot{\boldsymbol{Q}}(t)$ space. In this new space $\upsilon^+(t)$ in (7) corresponds to the probability content of the shaded area shown in Fig. 2. Note that this expression leads to the well-known Rice formula if the mean value theorem is applied to the inner integral above.

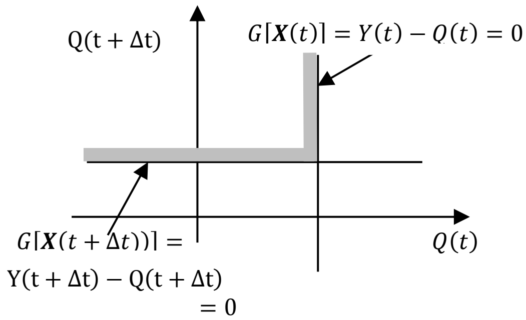

Fig. 1 Two limit states at time t and $t + \Delta t$ using ***X****(t)*-***X****(*$t + \Delta t$*)* space

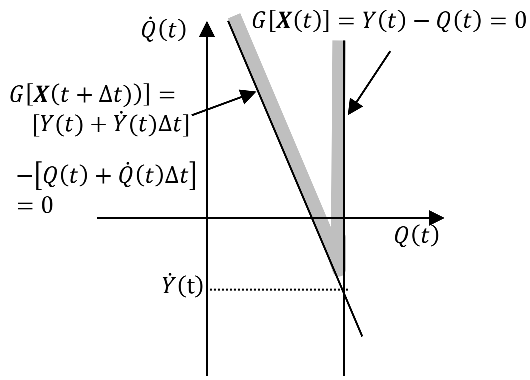

Fig. 2 Two limit states at time *t* and $t + \Delta t$ using $\boldsymbol{X}(t) - \dot{\boldsymbol{X}}(t)$ space

3 REVIEW OF THE PHI2 METHOD

As already mentioned, the basic idea is to use two-component parallel system reliability analysis to compute the outcrossing rate. Based on this idea, the intersection area defined in Eq. (4) and depicted in Fig. 1, is transferred to an equivalent standard Gaussian and then the probability content in this area is estimated using bi-normal cumulative distribution function $\Phi_2(\ )$ as follows [9]:

$$\upsilon^{+}_{PHI2} = \frac{1}{\Delta t}\, \Phi_2[\, \beta(t), -\beta(t+\Delta t); \rho(t, t+\Delta t)\,] \qquad (9)$$

Where $\beta(t)$, $\beta(t+\Delta t)$ are the reliability indices evaluated at two fix time points *t* and $t + \Delta t$. These two indices correspond to two limit states *G[**X**(t)]=0* and *G [**X**(*$t + \Delta t$*)]=0*, respectively shown in Fig. 1 which are now transferred to the standard Gaussian space ***Z****(t)*-***Z****(*$t + \Delta t$*)*. These two limit state functions are shown in the new space as $g(\boldsymbol{Z}(t)) = \beta(t) - \boldsymbol{\alpha}(t).\boldsymbol{Z}(t) = 0$ and $g(\boldsymbol{Z}(t+\Delta t)) = \beta(t+\Delta t) - \boldsymbol{\alpha}(t+\Delta t).\boldsymbol{Z}(t+\Delta t) = 0$ with $\boldsymbol{\alpha}(t)$ and $\boldsymbol{\alpha}(t+\Delta t)$ being the unit normal vectors associated to two design points respectively. Also, the correlation coefficient $\rho(t, t+\Delta t)$ corresponds to two events $g(\boldsymbol{Z}(t)) > 0$ and $g(\boldsymbol{Z}(t)) \le 0$; which is computed as shown below [9]:

$$\rho(t, t+\Delta t) = -\boldsymbol{\alpha}(t).\boldsymbol{\alpha}(t+\Delta t) \qquad (10)$$

The outcrossing rate calculated by the use of finite-difference concept reflected in Eq. (9), is highly dependent on the Δt value taken for calculation ([9], [10]). In order to overcome this difficulty, Sudret [10], by some further derivations, proved the following alternative relationships:

$$\upsilon^{+}_{PHI2} = \frac{\phi(\beta)}{\sqrt{2\pi}}\, \|\boldsymbol{\alpha}'(t)\| \text{(for stationary cases)} \qquad (11)$$

$$\upsilon^{+}_{PHI2} = \|\boldsymbol{\alpha}'(t)\|\phi[\beta(t)]\Psi\left[\frac{\beta'(t)}{\|\boldsymbol{\alpha}'(t)\|}\right] \text{(for non-stationary cases)} \qquad (12)$$

Where $\phi(\ )$ is the standard Gaussian probability density function, $\|\boldsymbol{\alpha}'(t)\|$ is the norm of the unit vector $\boldsymbol{\alpha}'(t)$ first derivative, $\beta'(t)$ is the first time-derivative of $\beta(t)$ and function $\Psi(\ )$ is defined as $\Psi(x) = \phi(x) - x\,\Phi(-x)$ with $\Phi(\ )$ being the standard Gaussian cumulative distribution function. It is obvious that in the stationary case, $\upsilon^{+}(t)$ and $\|\boldsymbol{\alpha}'(t)\|$ do not actually depend on time *t*. As is also clear, β is not time-dependent in that case.

4 ALTERNATIVE APPROACH FOR CALCULATION OF $\boldsymbol{\upsilon^{+}(t)}$

4.1 General concept

As pointed out, using the finite-difference version of Eq. (7) leads to unstable results since it is highly dependent on the value of Δt selected for computation. In order to remove this difficulty, and at the same time, to use the finite-difference concept to evaluate $\upsilon^{+}(t)$, it is proposed to use the $\boldsymbol{X}(t) - \dot{\boldsymbol{X}}(t)$ space to work instead of $\boldsymbol{X}(t) - \boldsymbol{X}(t+\Delta t)$. This requires that Eq. (8) is used and the probability content located in the shaded area shown in Fig. 2 is estimated. In this space, using the Expression $\dot{X}(t) \approx \frac{X(t+\Delta t) - X(t)}{\Delta t}$, all values of the processes at time $t + \Delta t$ are shown as $X(t+\Delta t) = X(t) + \dot{X}(t)\Delta t$. Using the $\boldsymbol{X}(t) - \dot{\boldsymbol{X}}(t)$ space, the involving limit state functions at times *t* and $t + \Delta t$ are now re-written as:

$$G[\boldsymbol{X}(t)] = \boldsymbol{Y}(t) - \boldsymbol{Q}(t) = 0$$
$$G[\boldsymbol{X}(t+\Delta t)] = \big(\boldsymbol{Y}(t) + \dot{\boldsymbol{Y}}(t)\Delta t\big) - \big(\boldsymbol{Q}(t) + \dot{\boldsymbol{Q}}(t)\Delta t\big) = 0 \qquad (13)$$

As it is clear, following this procedure the joint probability density function $f_{\boldsymbol{X}(t),\dot{\boldsymbol{X}}(t)}$ is basically required. This function corresponds to the joint pdf of $f_{\boldsymbol{X}(t),\boldsymbol{X}(t+\Delta t)}$ already used in the $\boldsymbol{X}(t)-\boldsymbol{X}(t+\Delta t)$ space. These two PDFs are related to each other through the relationship discussed already in Section 2.

4.2 Evaluation of $\boldsymbol{v^+(t)}$ using directional simulation

As was noted, in order to overcome the shortcomings usually involved by using FORM, and further, in order to make computations less dependent to the value of Δt in finite-difference estimation of $v^+(t)$, herein it is proposed to use the Directional Simulation Method (DSM) in the original space of $\boldsymbol{X}(t)-\dot{\boldsymbol{X}}(t)$ (see Fig. 3 for the case of only one process). This means that the probability content in the shaded area is directly calculated by DSM. As will be shown, the outcomes of the analysis are quite precise and stable if Δt is taken as sufficiently small (see below). The outcrossing rate $v^+(t)$, here, is approximated as:

$$v^+(t) \approx \frac{1}{\Delta t} \int_{\substack{unit\\sphere}} \left[\int_{s=s_1}^{s_2} f_{\boldsymbol{X}(t),\dot{\boldsymbol{X}}(t)}(s\boldsymbol{\alpha} + \boldsymbol{c}) \frac{s^{2n+m-1}}{f_{\boldsymbol{A}}(\boldsymbol{\alpha})}\, ds \right] f_{\boldsymbol{A}}(\boldsymbol{\alpha}) d\boldsymbol{\alpha}$$
$$= \frac{1}{\Delta t} E_{\boldsymbol{A}} \left[\int_{s=s_1}^{s_2} f_{\boldsymbol{X}(t),\dot{\boldsymbol{X}}(t)}(s\boldsymbol{\alpha} + \boldsymbol{c}) \frac{s^{2n+m-1}}{f_{\boldsymbol{A}}(\boldsymbol{\alpha})}\, ds \right] \tag{14}$$

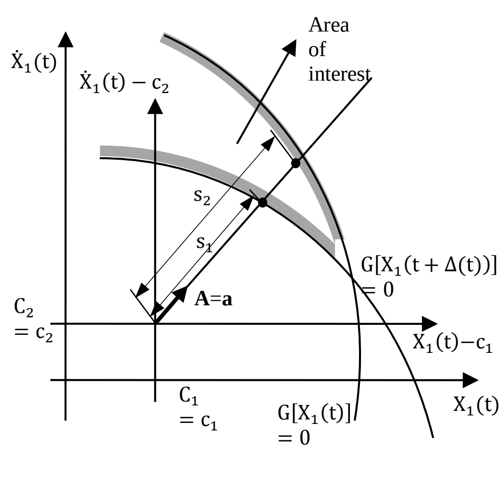

Fig. 3 Directional Simulation in $\boldsymbol{X}(t)-\dot{\boldsymbol{X}}(t)$ space (two-dimensional case is depicted)

In which the relationship $\boldsymbol{X}=S\boldsymbol{A}+\boldsymbol{c}$ is used to transfer the involving random variables/processes into the (hyper)-polar coordinate system S-$\boldsymbol{A}$; with S and $\boldsymbol{A}$ being radius and (cosine) direction unit vector, $\boldsymbol{c}$ is an arbitrary origin from which the directions are simulated, $f_{\boldsymbol{X}(t),\dot{\boldsymbol{X}}(t)}$ is the joint probability density function of $\boldsymbol{X}(t)$ and $\dot{\boldsymbol{X}}(t)$ and $f_{\boldsymbol{A}}$ is the pdf of $\boldsymbol{A}$ directions, $E_{\boldsymbol{A}}[\ ]$ is the expectation operator indicating that the simulations are taken from $\boldsymbol{A}$ and s_1 and s_2 are the radii defining boundaries of the area given in the Eq. (4) along the given direction $\boldsymbol{A}=\boldsymbol{a}$ (see Fig. 3).

It has been shown if directional simulation is carried out in a non-proportional spaces, in which the involving random variables/processes are highly correlated or their variances are significantly different, convergence of the calculations is normally slow and therefore a large amount of computations is required to achieve acceptable results. In such spaces, to make calculations more efficient, use of a directional importance sampling technique has been proposed ([13], [14], and [16]). If this technique is applied, Eq. (14) becomes:

$$v^+(t) \approx \frac{1}{\Delta t} E_{\boldsymbol{B}} \left[\int_{s=s_1}^{s_2} f_{\boldsymbol{X}(t),\dot{\boldsymbol{X}}(t)}(s\boldsymbol{b} + \boldsymbol{c}) \frac{s^{2n+m-1}}{h_{\boldsymbol{B}}(\boldsymbol{b})}\, ds \right] \tag{15}$$

In which directional samples are, now, taken using a new sampling function $h_{\boldsymbol{B}}(\boldsymbol{b})$ and $E_{\boldsymbol{B}}[\ ]$ is the expectation operator in terms of $\boldsymbol{B}$ vector. Derivation of the directional simulation Eqs. (14) and (15) and also details on how an appropriate function $h_{\boldsymbol{B}}(\boldsymbol{b})$ may be constructed is out of scope of this paper and may be found elsewhere (e.f. [16] and [17]).

It is necessary to note that the dimension of the $\boldsymbol{X}(t)-\dot{\boldsymbol{X}}(t)$ space in which simulation is carried out is $2n+m$ which covers n components of $\boldsymbol{Q}(t)$, n components of $\dot{\boldsymbol{Q}}(t)$ and m components of $\boldsymbol{Y}$. Obviously, if all or some of the $\boldsymbol{Y}$ components are stochastic processes, they would be treated analogous to those of the $\boldsymbol{Q}(t)$ components. The other point which needs to be addressed is the selection of $\boldsymbol{c}$, the point from which the directions are simulated. As pointed out above, selection of this point is rather arbitrary and ideally should be defined so as to facilitate the calculations. The mean point of $\boldsymbol{X}(t)$ is usually proposed in the literature (e.g. [15]).

5 RELIABILITY OF HYDROKINETIC TURBINE BLADE

In this section, time-variant reliability analysis of a hydrokinetic turbine blade is investigated which is adapted from [18] and [19]. The structure is considered to be exposed to a time-variant river flow loading. The river velocity, $v(t)$, due to its uncertain seasonal characteristics is taken here as a stochastic process whose mean, $\mu_{v(t)}$ and standard deviation, $\sigma_{v(t)}$ are both functions of time (i.e. and thus make the process as non-stationary). These have been found to be as follows ([18] and [19]):

$$\mu_{v(t)} = \sum_{i=1}^{4} a_i^m \, sin(b_i^m t + c_i^m) \quad (16)$$

$$\sigma_{v(t)} = \sum_{i=1}^{4} a_i^s exp\left\{-\left[\frac{t-b_i^s}{c_i^s}\right]^2\right\} \quad (17)$$

In which a, b and c are constants and are given by the followings:

a_1^m=3.815	b_1^m=0.2895	c_1^m=-0.2668
a_1^m=2.528	b_2^m =0.5887	c_2^m=0.9651
a_1^m=1.176	b_3^m =0.7619	c_3^m =3.116
a_1^m =-0.07856	b_4^m =2.183	c_4^m =-3.161
a_1^s = 0.7382	b_1^s = 6.456	c_1^s = 0.9193
a_2^s = 1.013	b_2^s = 4.075	c_2^s = 1.561
a_3^s = 1.875	b_3^s = 9.913	c_3^s = 6.959
a_4^s = 1.283	b_4^s = 1.035	c_4^s = 2.237

In addition, it has been found that the auto-correlation coefficient function could be expressed as:

$$\rho_v = cos[2\pi(t_2 - t_1)] \quad (18)$$

Fig. 4 and 5 indicate the turbine blade simplified cross-section and the blade under river flow respectively. It has been shown that one of the critical loading system under which the blade must be checked to be reliable in its lifetime, is the flap wise bending moment [20] created at the blade root (see Fig. 6). This moment could be expressed by the following relationship ([18], [19]):

$$M_{flap} = \frac{1}{2}\,\rho\, C_m\, v(t)^2 \quad (19)$$

Where $\rho = 10^3\ kg/m^3$ is the river water density, $C_m = 0.3422$ is the coefficient of moment which is obtained from the blade element momentum theory [20]. Now, the limit state function for the blade may be written as below:

$$\text{G}[\boldsymbol{X}(t)] = \text{M}_{\text{resist}} - \text{M}_{\text{flap}} = \frac{\varepsilon_a EI}{h_1} - \frac{1}{2}\,\rho\, C_m\, v(t)^2 \quad (20)$$

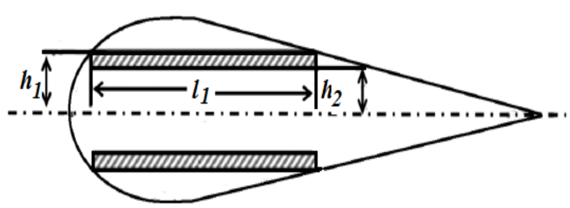

Fig. 4 Turbine blade cross section at its root area

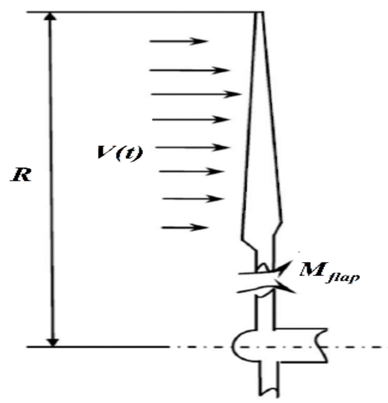

Fig. 5 Flow loading on turbine blade

In which ε_a is the allowable strain, $E = 14\ GPA$ is the Young modulus and $I = (2/3)l_1(h_1^3 - h_2^3)$ is the moment of inertia at the root of the blade with l_1, h_1 and h_2 being the dimension variables as depicted in Fig. 4. In Eq. (20), $v(t), l_1, h_1, h_2$ and ε_a are taken as random variables/processes whose probability properties are gathered in Table 1.

Table 1 Random Variables/processes and their probabilistic properties

Random Variable/Process	Mean	Standard Deviation	Distribution
v(t) (m/sec)	$\mu_{v(t)}$	$\sigma_{v(t)}$	Gaussian Process
l_1 (m)	0.22	0.002	Gaussian
h_1 (m)	0.025	0.00025	Gaussian
h_2 (m)	0.019	0.00019	Gaussian
ε_a	0.025	0.00025	Gaussian

Based on the theory given in this paper, in order to estimate the structural probability of failure, in addition to the limit state function (20) which representing the safe/failure area at time *t*, the corresponding limit state function at time $t + \Delta t$ is also required. This function could be simply written as follows:

$$G[\boldsymbol{X}(t + \Delta t)] = \frac{\varepsilon_a EI}{h_1} - \frac{1}{2}\,\rho\, C_m\,[v(t) + \dot{v}(t)\Delta t]^2 \quad (21)$$

As it is clear, $\dot{v}(t)$ is the second stochastic process, presented in the analysis and thus must be included in the **X**(t) components. Gathering all the involving random variables and stochastic processes, the vector ***X(t)*** may be set up as follows:

$$\boldsymbol{X}(t) = \boldsymbol{X}[v(t)\ \dot{v}(t)\ \ l_1\ \ h_1\ \ h_2\ \ \varepsilon_a] \quad (22)$$

The above six components will therefore construct the space in which directional simulation is to be carried out. To do so, probabilistic characteristics of $\dot{v}(t)$ itself, as well as its correlation to other variables/processes are also required. Using the Eqs. (16), (17) and (18), it may be shown that the mean $\mu_{\dot{v}(t)}$, variance $\sigma^2_{\dot{v}(t)}$ and covariance function $cov\,[\dot{v}(t), v(t)]$ are derived as follows:

$$\mu_{\dot{v}(t)} = \sum_{i=1}^{4} a_i^m . b_i^m \cos(b_i^m t + c_i^m)\,, \tag{23}$$

$$\sigma^2_{\dot{v}(t)} = \left(\frac{\partial^2}{\partial t_1 \partial t_2} cov\,[v(t_1), v(t_2)]\right)_{|t_1=t_2=t} = (2\pi)^2 \left\{\sum_{j=1}^{4} a_j^s \exp\left[-\left(\frac{t-b_j^s}{c_j^s}\right)^2\right]\right\}^2 + \left\{\sum_{j=1}^{4} \frac{-2a_j^s}{c_j^s} \left(\frac{t-b_j^s}{c_j^s}\right) \exp\left[-\left(\frac{t-b_j^s}{c_j^s}\right)^2\right]\right\}^2 \tag{24}$$

and:

$$cov\,[\dot{v}(t), v(t)] = \left(\frac{\partial}{\partial t_1} cov\,[v(t_1), v(t_2)]\right)_{|t_1=t_2=t} = \left\{\sum_{j=1}^{4} a_j^s \exp\left[-\left(\frac{t-b_j^s}{c_j^s}\right)^2\right]\right\} \times \left\{\sum_{j=1}^{4} \frac{-2a_j^s}{c_j^s} \left(\frac{t-b_j^s}{c_j^s}\right) \exp\left[-\left(\frac{t-b_j^s}{c_j^s}\right)^2\right]\right\}^2 \tag{25}$$

Having the data given above and having all ***X(t)*** components are known to be Gaussian distributed, the joint PDF $f_{\boldsymbol{X}}(\boldsymbol{x})$ required in Eq. (14) and (15) may be constructed. Now, the reliability analysis of the above hydrokinetic turbine blade may be carried out in the following steps:

5.1. Estimation of the probability of failure at *t=0*

Based on the theory given in this paper, the probability of failure at *t=0*, namely P_{f_0}, could be calculated by taking only the limit state function (20) into account and by evaluation of whole probability content placed in the failure domain. Clearly for these calculations, means and variances of *v(t)* are evaluated at *t=0* and directional simulation is performed in the five-dimensional space $[v(t)\; l_1\;\; h_1\;\; h_2\;\; \varepsilon_a\,]$ (i.e. $\dot{v}(t)$ needs not to be considered here).

By taking 100,000 directional simulation, P_{f_0} will converge to 0.1445×10^{-11}. As is seen, P_{f_0} is very small and as will be seen shortly comparing to $E[N^+(0,t)]$ in the subsequent times could be just ignored.

5.2. Estimation of expected number of outcrossings at time *t*

In order to perform time-variant reliability analysis, estimation of the expected number of outcrossings at a desired time *t* is required (see Eqs. (3) -(5)). Since the processes $v(t)$ and $\dot{v}(t)$ are not stationary, the outcrossings rate defined in (4) is not constant along time and thus the integration (5) for any time *t* should be evaluated approximately as:

$$E[N^+(0,t)] = \sum_{i=1}^{n} v^+(t_i)\;\Delta time \tag{26}$$

In which $\Delta time$ is the time increment considered to calculate $E[N^+(0,t)]$. Note that this time increment is not necessarily equal to that used already to evaluate $v^+(t)$. Further, in Eq. (26), $n = t/\Delta time$, $t_i = i\,.\Delta time$ and $v^+(t_i)$ is the instantaneous outcrossing rate evaluated at time t_i using the theory given in this paper (i.e. using Eq. (14) or (15) for directional simulation to derive $v^+(t_i)$ for any time t_i).

In the current reliability analysis, since variances of the involving random variables/processes are extremely different (see Table 1 for standard deviations/variances of l_1, h_1, h_2 and ε_a and note that typical values for $\sigma^2_{v(t)}$ and $\sigma^2_{\dot{v}(t)}$ are about 4.5 and 180.0 respectively at different times t), it becomes essential to employ directional importance sampling to achieve desirable results in an acceptable machine time duration. The experiences in this work have shown if the technique is not used, convergence is extremely low due to the non-proportional space in which the directional simulation is carried out.

In Fig. 6, variation of the blade probability of failure during [0, 12] months period against *Δt* is shown using the two methods FORM and DSM. From the figure it is seen that the results obtained by the use of DSM are always larger than those given by FORM. Since either methods use the same philosophy (i.e. the outcrossing approach) to calculate p_{f_T}, these differences are attributed to nonlinearity of the limit state used in the problem. Further, while the stability of p_{f_T} calculated by DSM is maintained even with very small *Δt*'s, the results given by PHI2 method suffer instability when *Δt* tends to zero. This indicates that, generally, some limitations must be applied to find out an appropriate *Δt* when PHI2 method is under consideration (see also [10]). However, as soon as *Δt* is sufficiently small, no additional limitation is required when DSM is employed.

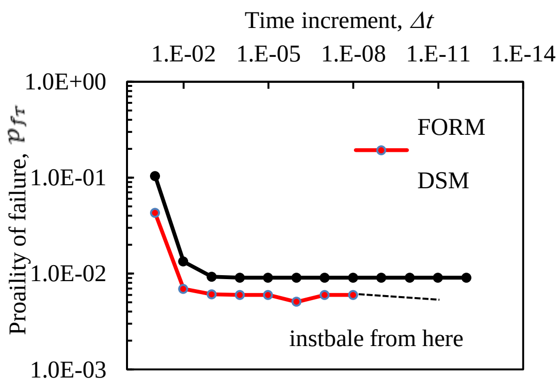

Fig. 6 Probability of failure versus *Δt*, using FORM and DSM methods (Note: *t* =12 months; $\Delta time = 0.1$)

In Fig. 7, the calculated instantaneous outcrossing rates and corresponding probability of failure using inequality (3) and Eq. (26) are depicted. In these calculations $\Delta t = 10^{-7}$ and $\Delta time = 0.1$ months are used and 10,000 directional simulations are taken to have the computational converged to acceptable results. From Fig. 7 it is seen that p_{f_T} is increased in time during a year (12 months) and reaches 9.052×10^{-3} at the end of year. This value is much more than that recorded earlier (i.e. $p_{f_T}(T = 12)$ that is recorded about 3.0×10^{-3} in [18] and [19]). The reason relies upon the theory of outcrossing approach used here which gives the upper bound for p_{f_T} and thus presents more conservative results.

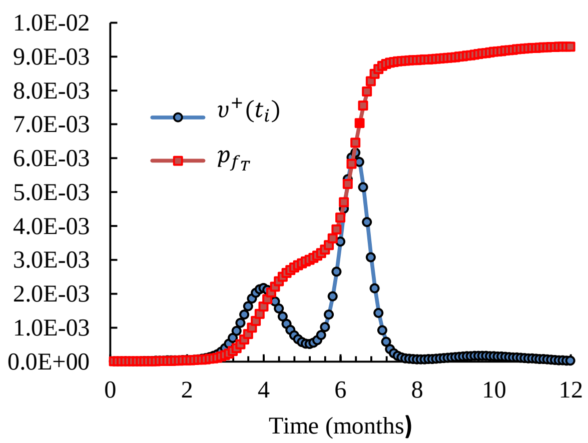

Fig. 7 Instantaneous outcrossing rates and probability of failure at different times

6 DISCUSSION

Obviously the time-dependent analysis proposed in this study by DSM in $\boldsymbol{X}(t) - \dot{\boldsymbol{X}}(t)$ space requires a working space which its dimension is more than that normally used in ***X(t)*** space (i.e. it is *2n+m*, comparing to *n+m* used in X(t) space). This will lead commonly to less efficient calculations and thus needs larger number of directional simulations to be carried out. Nevertheless, since the proposed numerical and finite-difference method is, in principle, applicable to a wider range of stochastic processes which could also be non-stationary, it appears worth to be employed in practical applications.

Using directional simulation in $\boldsymbol{X}(t) - \dot{\boldsymbol{X}}(t)$ space (rather than $\boldsymbol{X}(t) - \boldsymbol{X}(t + \Delta t)$) possesses the advantage of having commonly less correlation between $\boldsymbol{X}(t)$ and $\dot{\boldsymbol{X}}(t)$ components comparing to those of $\boldsymbol{X}(t)$ and $\boldsymbol{X}(t + \Delta t)$. This makes directional simulation commonly more efficient and thus less computations are required.

7 CONCLUSION

Estimation of the outcrossing rate in time-variant reliability problems by the use of a parallel system reliability concept was discussed. The so-called PHI2 method which uses this concept was reviewed first. Since this method is highly dependent to the time increment Δt, attempt was made to propose an alternative method with better computational properties. The main idea of the proposed method, in principle, was to work in $\boldsymbol{X}(t) - \dot{\boldsymbol{X}}(t)$ space rather than working in $\boldsymbol{X}(t) - \boldsymbol{X}(t + \Delta t)$ in the reliability analysis. Further, application of the above concept was suggested to be in the framework of directional simulation. This makes it possible to circumvent the conventional shortcomings inherently included in the FORM approach. In this paper, the proposed method was successfully employed for reliability analysis of a hydrokinetic turbine blade whose involving processes were non-stationary and whose space of random variables/processes was extremely non-proportional.

This paper showed that using the concept of parallel system reliability together with directional simulation may facilitate estimation of the outcrossing rate for extra-ordinary circumstances in which non-stationary processes and also non-proportional spaces may exist and time-dependent reliability analysis is carried out with acceptable preciseness and with less complex mathematics.

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