Optimal Robust Design of Sliding-mode Control Based on Multi-Objective Particle Swarm Optimization for Chaotic Uncertain Problems

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Abstract: The aim of this paper is to present an optimal robust Pareto design of sliding-mode control for chaotic uncertain problems. When designing and applying sliding mode control to challenging dynamic systems, it is crucial to gain optimal control effort and minimum tracking errors, simultaneously. In this regard, multi-objective particle swarm optimization (periodic CDPSO) benefiting from crucial factors such as divergence and convergence operators, the leader selection method, and the adaptive elimination technique is utilized to design the optimal control approach via obtaining the Pareto front of objective functions addressing the trade-off between the states errors and control effort. Afterward, the Pareto front acquired by the periodic CDPSO algorithm is contrasted with those obtained via other prominent algorithms in the literature including Sigma method, Modified NSGAII, and MOGA. Eventually, the numerical results elucidate the effectiveness of the proposed optimal control scheme in terms of optimal control effort and minimum tracking errors.

Keywords: Lorenz chaotic problem, Multi-objective Optimization, Optimal control, Robust control, Particle swarm optimization, Sliding-mode control

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1 INTRODUCTION

Due to the presence of unpredicted challenges in the dynamics of real problems in industry, the control of chaotic problems providing a comprehensive evaluation of the designed controller is of great interest to researchers [1-4]. In this regard, the Lorenz problem which benefits from a chaotic nature provides a real challenge to assess the performance of the designed controller [5-8]. Lately, to cite just a few, the Lorenz problem was controlled by using an optimal controller in both finite and infinite time with ensuring the asymptotic stability of desired states in both cases [9], via employing three strategies of dislocated feedback control to enhance the capability of the feedback control and its speed [10], and by benefiting from the robust method of fractional-order derivative to control the unstable equilibrium points of the fractional-order Lorenz chaotic system [11].

One approach to make control of a complicated nonlinear system straightforward is to eliminate the nonlinearity of a dynamic system. Hence, the nonlinearity of the Lorenz systems was cancelled in [12] by utilizing the time delay estimation. Since the time delay estimation enabled a very effective and efficient cancellation of nonlinearity and disturbances, the technique turned out to be simple and robust. Moreover, an adaptive controller of linear time invariant systems via a wavelet network was utilized to control the Lorenz chaos and to explore the mechanism of a wavelet controller through integrating the controller with linear time invariant systems [13].

Sliding-mode control is a robust nonlinear controller employed by a number of researchers in a variety of field of research, mainly in steering of vehicles [14-17], robots [18-21], and actuators [22]. Although the heuristic parameters of sliding-mode control are frequently identified by trial-and-error processes, it is scientifically crucial to gain them by means an optimization approach in order to enhance the efficiency of the control approach. One proper methodology to choose these factors is using smart optimization algorithms, such as particle swarm optimization, genetic algorithm, etc. [24].

In elaboration, it has been illustrated that particle swarm optimization presents a robust performance in the design of the challenging control problems [25]. To this end, multi-objective particle swarm optimization is utilized in the present study to eliminate the boring and repetitive trial-and-error process and find the parameters of sliding-mode control. Particle Swarm Optimization (PSO), which is one of the advanced robust heuristic algorithms in solving both single-objective optimization problems and multi-objective optimization problems [26], was presented first by Kennedy and Eberhart [27] and was progressed through the simulation of basic social systems. As an effectual optimization algorithm, researchers have reported successful applications of PSO in the following fields: industrial engineering [28-31], robotics [32-36], vehicle design [37-39] and gas industry [40-41]. This algorithm can generate a high quality solution with short calculating time and a more stable convergence characteristic in comparison with other evolutionary methods [42].

In the recent years, several approaches have been proposed to develop the PSO algorithm for dealing with multi-objective optimization problems. For instance, dynamic neighborhood PSO [43], dominated tree [44], Sigma method [45], dynamic multiple swarms [46], periodic CDPSO [47], [48] and others [49-55] have been proposed to address the multi-objective optimization problems. As elucidation of the applications of PSO over sliding mode control, some notable studies are as [56-58].

The present investigation develops significantly authors' previous work [59] as follows. In the present research, multi-objective periodic CDPSO [47-48] is utilized to design the parameters of sliding-mode control in order to control one of the challenging uncertain chaotic problems, the Lorenz problem, which resulted in the evaluation of several aspects of the proposed optimal control methodology. However, in the previous work [59], a controller with a different structure based on PID and sliding mode control optimized via multi-objective genetic algorithm was used for a biped robot walking in the lateral plane.

In elaboration of the present study, multi-objective periodic CDPSO [47-48] is involving the following steps. At the first step, PSO is combined with two convergence and divergence operators. At the second step, two mechanisms are utilized to produce the set of Pareto optimal solutions benefiting from good convergence, diversity, and distribution. At the first mechanism, a leader selection approach utilizing the periodic iteration and the concept of the number of the particle’s neighbors is defined.

At the second mechanism, an adaptive elimination approach is employed to confine the number of non-dominated solutions in the archive. In fact, the adaptive elimination approach influences the computational time, convergence and diversity of solutions. Lastly, multi-objective periodic CDPSO algorithm is employed to gain the parameters of the designed sliding mode control methodology for the Lorenz chaotic problem and the obtained result is compared to the results obtained by three robust multi-objective optimization algorithms including Sigma method, Modified NSGAII, and MOGA.

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2 THE SLIDING-MODE CONTROL FOR THE LORENZ CHAOTIC PROBLEM

Sliding-mode control is a variable structure control approach having a unique characteristic of robustness making it not be sensitive to parameter variations [60,61]. It is based on maintaining an appropriately chosen constraint with regard to the high-frequency control switching [62]. As applications of sliding mode control on chaotic systems, to cite just a few, a radial basis function sliding-mode controller was employed for the chaotic Lorenz system [63]. The sliding-mode control was used for chaotic systems based on LMI as well as establishing a feedback controller to guarantee the asymptotical stability of the chaotic systems based on the sliding-mode control theory [64]. A chatter free sliding-mode controller was designed for the chaos control and synchronization with the nonlinear uncertain chaotic systems by proposing a new sort of dynamical sliding-mode surfaces [65]. The dynamic equation of the Lorenz chaotic system with disturbances is regarded, as follows [66]:

$$\dot{x}_1 = -\sigma x_1 + \sigma x_2 \tag{1}$$

$$\dot{x}_2 = r x_1 - x_2 - x_1 x_3 + d + u \tag{2}$$

$$\dot{x}_3 = x_1 x_2 - b x_3 \tag{3}$$

in which $u \in R^1$ is an additive and scalar control input. Furthermore, d is the bounded disturbance by considering that $|d| \leq \delta$ as δ is the constant parameter. By regarding the target points, the control input u seeks to steer the trajectory to the equilibrium point $X_r = \left(\sqrt{b(r-1)}, \sqrt{b(r-1)}, r-1\right)$ which equals $(\sqrt{8/3(28-1)}, \sqrt{8/3(28-1)}, 28-1)$. To obtain the sliding surface, scalars S_1, S_2, and S_3 are defined as follows [66].

$$S_1 = C_1(x_2 - x_{2r} - z) \tag{4}$$

$$S_2 = C_2(x_1 - x_{1r}) \tag{5}$$

$$S_3 = C_3(x_3 - x_{3r}) \tag{6}$$

In which, C_1, C_2, and C_3 are the coefficients of the sliding surface variables S_1, S_2, and S_3, respectively. The control objective of u is changed from $x_2 = 0$ to $x_2 = x_{2r} + z$ according to Eq. (4). In this respect, the amount of x_2 is bounded due to designing the controller u. Moreover, z and Z_{upper} is regarded as follows [66]:

$$|z| \leq Z_{upper} \qquad 0 < Z_{upper} < 1 \tag{7}$$

In which, Z_{upper} represents the upper bound of $|z|$. $z = \mathrm{SOFL}(z_L) \times Z_{upper}$, $0 < Z_{upper} < 1$ in which, SOFL stands for Soft Limit Function.

$$sign(z_L) \cong SOFL(z_L) = \frac{z_L^2}{(1+z_L^2)} \times \tanh(z_L) \tag{8}$$

and z_L is defined as:

$$z_L = \begin{cases} -\frac{S_2}{\Omega_1} & if\ S_3 \leq |S_t| \\ \frac{S_3}{\Omega_2} & if\ \ S_3 > |S_t| \end{cases} \quad .. \tag{9}$$

In which, S_t represents the limitation of S_3. Moreover, both Ω_1 and Ω_2, which transfer both S_2 and S_3 to the appropriate span of x_2,are boundary layers of S_2 and S_3 to make z_L smooth. The soft limit function Eq. (8) is employed to approximate the $sign$ function. In addition, by regarding the fact that the range of Z_{upper} is less than one, z will be a decaying oscillation signal.

Remark 1. (Lyapunov's second method for an asymptotically stable system) The Lyapunov's second method, which is a solid approach, employs a Lyapunov function $V(x) = \frac{1}{2}S^2$ that presents an analogy to the potential function of classical dynamics. Hence, it is introduced for a system having a point of equilibrium at $x = 0$, as follows [67]:

Consider a function $V(x): R^n \to R$ such that

- $V(x) \geq 0$ with equality if and only if $x = 0$ (positive definite)
- $\dot{V}(x) = \frac{d}{dt}V(x) \leq 0$ with equality if and only if $x = 0$ (negative definite)

Then, $V(x)$ is called a Lyapunov function candidate and the system is asymptotically stable in the sense of Lyapunov if Eq. (10) is satisfied:

$$\dot{V}(x) = S_1 \dot{S}_1 < 0 \tag{10}$$

$$\begin{aligned} \dot{V}(x) &= [C_1 \times (x_2 - x_{2r} - z)] \times [C_1 \times (\dot{x}_2 - \dot{z})] \\ &= [C_1 \times (x_2 - x_{2r} - z)] \\ &\quad \times [C_1 \\ &\quad \times (r x_1 - x_2 - x_1 x_3 + d + u - \dot{z})] \\ &= C_1{}^2\, [r x_2 x_1 - x_2{}^2 - x_1 x_2 x_3 + d x_2 + u x_2 - \dot{z} x_2 - \\ &\quad x_{2r}\, r\, x_1 + x_{2r}\, x_2 + x_{2r}\, x_1\, x_3 - x_{2r}\, d - x_{2r}\, u + \\ &\quad x_{2r}\, \dot{z} - z r x_1 + z x_2 + z x_1 x_3 - z d - z u + z\dot{z}] \end{aligned} \tag{11}$$

Lyapunov function $V(x)$ and its derivative $\dot{V}(x)$ are shown in Fig. 1. Moreover, the sliding surface of the sliding mode controller and its derivative are illustrated in Figs. 2 and 3.

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By $\dot{S}_1 = 0$, the sliding mode equation will be obtained. To analyze the sliding mode equation, the equivalent control effort, i.e. Eq. (13) obtained by the sliding mode equation is shown in Fig. 4.

$$\dot{S}_1 = C_1 \times (\dot{x}_2 - \dot{z}) = C_1 \times (rx_1 - x_2 - x_1x_3 + d + u - \dot{z}) = 0 \tag{12}$$

Then, the equivalent control effort will be resulted, as follows:

$$u_{eq} = x_2 + x_1x_3 - rx_1 - d + \dot{z} \tag{13}$$

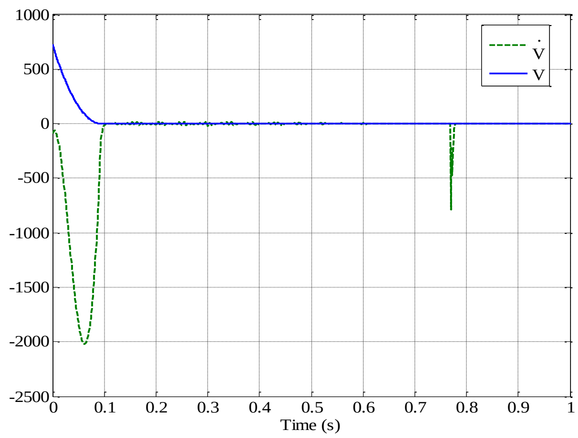

Fig. 1 Lyapunov function and its derivative to show the asymptotical stability of the system.

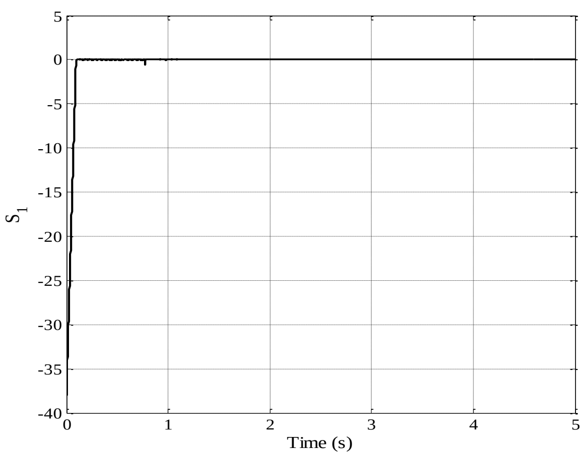

Fig. 2 The sliding surface of the sliding mode controller for the Lorenz chaotic problem.

Remark 2. (The state errors in finite time) Because sliding mode control laws are not continuous, it has the ability to drive trajectories to the sliding mode in finite time (i.e., the stability of the sliding surface is better than asymptotic). To ensure that the sliding mode is moving into finite time [67], Eq. (15) must be satisfied:

$$\dot{V}(x) \le -\mu(\sqrt{V})^{\alpha} \tag{14}$$

$$\dot{V}(x) + \mu(\sqrt{V})^{\alpha} \le 0 \tag{15}$$

Where $\mu > 0$ and $0 < \alpha \le 1$ are constants. The subspace for this system and the sliding surface $\{x\epsilon R^n : S(x) = 0\}$ is given by $\{x\epsilon R^n : S^T(x)\dot{S}(x) < 0\}$. That is, when initial conditions come entirely from this space, the Lyapunov function candidate $V(x)$ is a Lyapunov function and x trajectories approach the sliding mode surface where $V(x) = 0$. Moreover, if the reachability conditions are satisfied, the sliding mode will move into the region where $\dot{V}(x)$ is bounded and away from zero in finite time. Hence, the sliding mode $V(x) = 0$ will be attained in finite time.

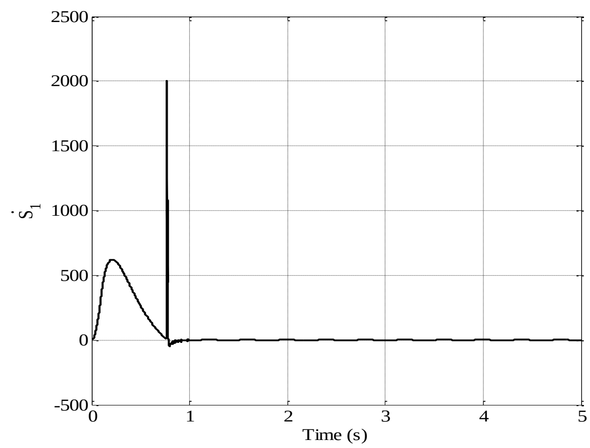

Fig. 3 The derivative of the sliding mode controller for the Lorenz chaotic problem.

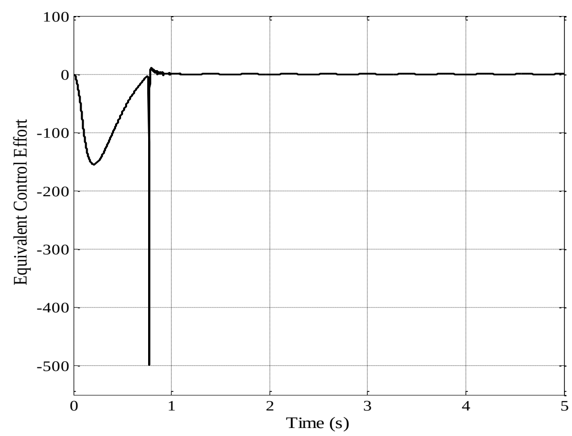

Fig. 4 The equivalent control effort of the sliding mode equation.

Fig. 5 is obtained for Eq. (15) by regarding $\mu = 0.5$ and $\alpha = 0.5$. The control effort of sliding-mode control is computed by the following formula.

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$$u = u_{eq} - k\ sat(S_1) \quad (16)$$

In Eq. (16), u_{eq} is the equivalent control effort that is obtained by $\dot{S}_1 = 0$, k is the design parameter of the sliding mode control, and sat is the saturation function. The choice of eight control coefficients $k, Z_{upper}, S_t, \Omega_1, \Omega_2, c_1, c_2$, and c_3 has major effects on the behavior in the transient state of the system. An appropriate choice of the sliding factors is necessary for achieving favorable transient response. In the present study, the particle swarm optimization is used to find these coefficients, properly.

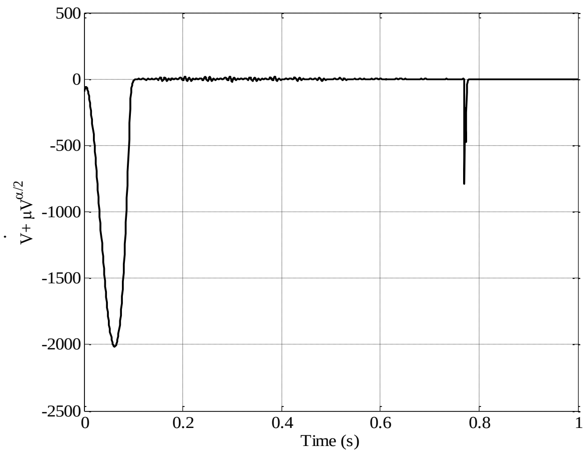

Fig. 5 $\dot{V}(x) + \mu(\sqrt{V})^{\alpha}$ is always negative and the state errors will be attained in finite time.

3 THE OPTIMIZATION ALGORITHM

The proper selection of parameters is one of the most important issues in the aspect of the optimal performance of controllers. Hence, multi-objective particle swarm optimization is applied in this paper to overcome this problem. Previous researchers illustrated that this optimization method could be used successfully to obtain the Pareto frontiers of non-commensurable objective functions in the design of linear state feedback controllers [48] and suspension systems for a vehicle vibration model [47]. This method is a combination of the particle swarm optimization, convergence, and divergence operators as well as implementing a periodic leader selection method and adaptive elimination technique to prune the archive in this algorithm. The algorithm has been named multi-objective periodic CDPSO. In the following, PSO, convergence divergence operator, periodic leader selection method, and adaptive elimination technique are described, briefly.

Particle swarm optimization: PSO is a population-based evolutionary algorithm which is inspired by the simulation of social behavior [68]. Even though PSO had been initially employed for balancing weights in neural networks [69], it turned out to be a popular global optimization algorithm, mostly for the problems with decision variables which are real numbers [70-71]. In PSO, each candidate solution is associated with a velocity [68], [72] and it is expected that the particles will approach superior solution areas. Mathematically, the particles are manipulated according to the following equations.

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{v}_i(t+1) \quad (17)$$

$$\vec{v}_i(t+1) = W\vec{v}_i(t) + C_1 r_1(\vec{x}_{Pbest_i} - \vec{x}_i(t)) + C_2 r_2(\vec{x}_{gbest} - \vec{x}_i(t)) \quad (18)$$

Where $\vec{x}_i(t)$ and $\vec{v}_i(t)$ denote the position and velocity of particle i at the time step (iteration) t. $r_1, r_2 \in [0,1]$ are random values. C_1 is the cognitive learning factor and represents the attraction that a particle has toward its own success. C_2 is the social learning factor and represents the attraction that a particle has toward the success of the whole swarm. It was elucidated that the best solutions were determined when C_1 is linearly decreased and C_2 is linearly increased over the iterations [72]. W is the inertia weight which influences the previous history of velocities with regard to the current velocity of particle i. Experimental results indicated that decreasing the inertia weight linearly over iterations enhances the PSO performance [68]. $\vec{x}_{Pbest_i}$ represents the personal best position of the particle i. $\vec{x}_{gbest}$ stands for the position of the best particle of the whole swarm.

Convergence operator: A novel convergence formula that involves four parent particles was proposed in [47-48], and also is used in this paper. Let $\rho \in [0,1]$ be a random number. If $\rho \leq P_{Convergence}$ ($P_{Convergence}$ is convergence probability), then one of the following operators should be performed to generate the new particle position $\vec{x}_i(t+1)$ from the old particle position $\vec{x}_i(t)$:

If fitness $\vec{x}_i(t)$ is smaller than fitness $\vec{x}_j(t)$ and fitness $\vec{x}_k(t)$ then:

$$\vec{x}_i(t+1) = \vec{x}_{gbest} + \sigma_1\left(\frac{\vec{x}_{gbest}}{\vec{x}_i(t)}\right)(2\vec{x}_i(t) - \vec{x}_j(t) - \vec{x}_k(t)) \quad (19)$$

If fitness $\vec{x}_j(t)$ is smaller than fitness $\vec{x}_i(t)$ and fitness $\vec{x}_k(t)$ then:

$$\vec{x}_i(t+1) = \vec{x}_{gbest} + \sigma_2\left(\frac{\vec{x}_{gbest}}{\vec{x}_i(t)}\right)(2\vec{x}_j(t) - \vec{x}_i(t) - \vec{x}_k(t)) \quad (20)$$

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If fitness $\vec{x}_k(t)$ is smaller than fitness $\vec{x}_j(t)$ and fitness $\vec{x}_i(t)$ then:

$$\vec{x}_i(t+1) = \vec{x}_{gbest} + \sigma_3(\frac{\vec{x}_{gbest}}{\vec{x}_i(t)})(2\vec{x}_k(t) - \vec{x}_j(t) - \vec{x}_i(t)) \quad (21)$$

In which, particles $\vec{x}_j(t)$ and $\vec{x}_k(t)$ are chosen from swarm by a uniformly selection approach. σ_1, σ_2, and σ_3 are random numbers chosen from [0,1] and $\vec{x}_{gbest}$ is the position of the best particle of the whole swarm. After computing Eqs. (19), (20), or (21), superior between $\vec{x}_i(t)$ and $\vec{x}_i(t+1)$ should be selected. If $\rho \geq P_{Convergence}$, then no convergence operation is performed for $\vec{x}_i(t)$.

Divergence operator: The divergence operator presents a feasible leap on some particles selected. Let $\vartheta \in [0,1]$ be a random number. If $\vartheta \leq P_{Divergence}$, ($P_{Divergence}$ is divergence probability) and particle $\vec{x}_i(t)$ was not enhanced by a convergence operator, then the following divergence operator is implemented to produce a new particle.

$$\vec{x}_i(t+1) = Norm\,rand(\vec{x}_i(t), S_D) \quad (22)$$

$Norm\,rand(\vec{x}_i(t), S_D)$ generates random numbers from the normal distribution with a mean parameter $\vec{x}_i(t)$ and standard deviation parameter S_D (S_D is a positive constant). If particle $\vec{x}_i(t)$ was enhanced by a convergence operator or $\vartheta \geq P_{Divergence}$, then no divergence operation will be performed. More details of this operator are mentioned in [47], [48].

Periodic leader selection method: This technique is based on the density measures, and a neighborhood radius $R_{\text{neighborhood}}$ is defined for leaders. Indeed, two leaders are regarded as neighbors if their Euclidean distance (measured in the objective domain) is less than $R_{\text{neighborhood}}$. Using this definition, the number of neighbors of each leader is computed in the objective function domain. The particle which has fewer neighbors is preferred as the leader. However, the leader position and its density will change after a number of iterations. Hence, the leader selection operation should be repeated and a new leader must be identified. To this end, the maximum iteration is divided into several equal periods and each period has the same iteration T.

In each period, the leader selection operation could be implemented and the non-dominated solution which has fewer neighbors is preferred as the leader. Moreover, if a particle dominates the leader in the beginning of the iteration in a period, then this particle will be considered as a new leader.

Adaptive elimination technique: This technique is utilized to prune the archive; and in this approach, the archive's members have an elimination radius which equals $\varepsilon_{\text{elimination}}$. If the Euclidean distance (in the objective function space) between two particles is less than $\varepsilon_{\text{elimination}}$, then one of them will be omitted. The following equation is introduced to determine the value of $\varepsilon_{\text{elimination}}$ that is named adaptive $\varepsilon_{\text{elimination}}$:

$$\varepsilon_{\text{elimination}} = \frac{t}{\zeta \times maximum\ iteration} \quad (23)$$

In which ζ is a positive constant, t is the current iteration number, and $maximum\ iteration$ is the maximum number of allowable iterations [47-48].

4 THE OPTIMAL PARETO OF THE SLIDING-MODE CONTROL FOR THE LORENZ CHAOTIC PROBLEM

Sliding-mode control is an approach to define asymptotically stable surfaces such that all system trajectories converge to these surfaces and slide along them until achieving the origin at their intersection [73]. Nevertheless, the heuristic sliding parameters require to be chosen, properly. Therefore, multi-objective periodic CDPSO is used to determine the proper parameters and to eliminate the tedious and repetitive trial-and-error process. Moreover, the performance of a controlled closed loop system is evaluated by a variety of goals [74-75]. Here, normalized summation of states errors and normalized control effort are regarded as the objective functions. These objective functions have to be minimized, simultaneously.

The vector $[k, Z_{upper}, S_t, \Omega_1, \Omega_2, c_1, c_2, c_3]$ is the vector of selective parameters of sliding-mode control. k is the design parameter. Z_{upper} is the upper bound of $|z|$. S_t is the threshold value of S_3. Ω_1, and Ω_2 are boundary layers of S_2 and S_3 to smooth z_L. c_1, c_2, and c_3 are the coefficients of the sliding surface variables. The normalized summation of states errors and the normalized control effort are functions of this vector's components. This means that changes will occur in the normalized summation of states errors and normalized control effort by the selection of various values for the selective parameters. Thus, this is an optimization problem with two objective functions (normalized summation of states errors and normalized control effort) and eight decision variables $(k, Z_{upper}, S_t, \Omega_1, \Omega_2, c_1, c_2, c_3)$. The block diagram to find the Pareto front of the sliding-mode control for the Lorenz chaotic problem based on the multi-objective periodic CDPSO algorithm is illustrated in Fig. 6.

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Based on extensive experiments, the regions of the selective parameters are chosen, as follows:

$10 \leq k \leq 100,\ 0 \leq Z_{upper} \leq 1,\ -1000 \leq S_t \leq -10,\ 10 \leq \Omega_1 \leq 1000,$
$1 \leq \Omega_2 \leq 100,\ 0.1 \leq c_1 \leq 10,\ -100 \leq c_2 \leq -1,$
$-100 \leq c_3 \leq -1.$

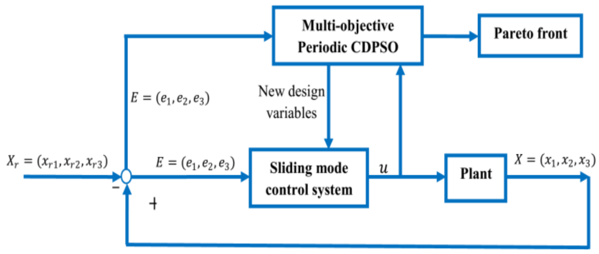

Fig. 6 The block diagram to find the Pareto front of the sliding-mode control for the Lorenz chaotic problem based on the multi-objective periodic CDPSO algorithm.

The parameters of multi-objective algorithm are chosen as follow. In each period, the inertia weight W is linearly decreased from $W_1 = 0.9$ to $W_2 = 0.4$, C_1 is linearly reduced from $C_{1i} = 2.5$ to $C_{1f} = 0.5$, and C_2 is linearly increased from $C_{2i} = 0.5$ to $C_{2f} = 2.5$, over the time. The related variables used in the convergence and divergence operators are: $P_{Convergence} = 0.1$, $P_{Divergence} = 0.1$, and $S_D = \frac{x_{max}-x_{min}}{2}$. The term $\vec{v}_i(t)$ is limited to the range $[-v_{ave}, +v_{ave}]$ in which $v_{ave} = \frac{x_{max}-x_{min}}{2}$. While the velocity violates this range, it will be multiplied by a random number between [0,1]. Furthermore, the positive constant for $\varepsilon_{\text{elimination}}$ is $\zeta = 300$, and the neighborhood radius for the leader selection is $R_{\text{neighborhood}} = 0.04$.

The number of iterations in a period is $T = 7$, the swarm size equals 150 and the maximum iteration is 300. Furthermore, three well-known versions of multi-objective optimization algorithms Sigma method [45], Modified NSGAII [76], MATLAB Toolbox MOGA are used to compare the performance of the periodic multi-objective CDPSO. The population size 150 and function evaluation 4500 are regarded for all algorithms and other details are illustrated in Table 1.

The Pareto fronts of this multi-objective problem are shown in Fig. 7. Indeed, this figure illustrates the feasibility and efficiency of proposed multi-objective algorithm in comparison with other algorithms.

Fig. 7 shows that the periodic CDPSO algorithm has more uniform and diverse feasible solutions. In Fig. 7, points A and C stand for the best normalized summation of state errors and normalized control effort, respectively.

Table 1 Used multi-objective optimization algorithms for comparison and their parameter configurations.

Algorithm	Pruning of archive	Crossover rate	Mutation rate
Sigma MOPSO	Archive size is fixed	-	0.1
Modified NSGAII	$\varepsilon -$ dominance $= 0.01$	0.8	0.1
MATLAB MOGA	Distance crowding	Scattered	Constraint dependent default

It can be observed from Fig. 7 that all the optimal points in the Pareto front are non-dominated and can be chosen by the designer as an optimal sliding-mode controller. In addition, choosing a better value for any objective function in the Pareto front causes a worse value for another objective. The corresponding decision variables (vector of sliding-mode controllers) of the Pareto front shown in Fig. 7 are the best possible optimal points.

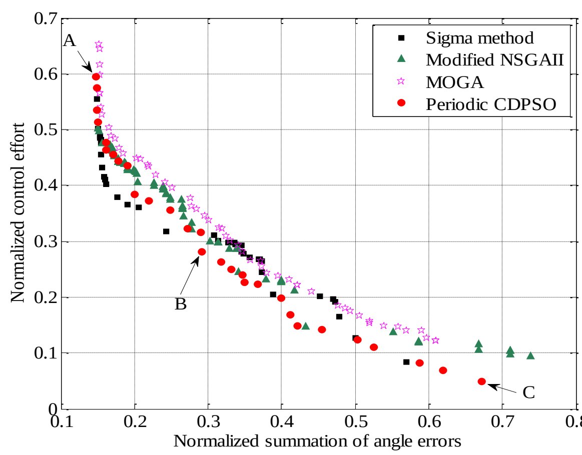

Fig. 7. The obtained Pareto fronts by using Sigma method [45], Modified NSGAII [76], MOGA (MATLAB Toolbox), and the proposed algorithm for the optimal control of the Lorenz chaotic problem.

As a matter of fact, if any other set of decision variables is chosen, the corresponding values of the pair of those objective functions locate an inferior point in the Pareto front. Such inferior area in the space of the two objectives is top/right side of Fig. 7. In Fig. 7, point B can be a trade-off optimum choice when minimum values of both the normalized summation of states errors and normalized control effort are considered. Design variables and objective functions corresponding to the optimal points A, B, and C are illustrated in Table 2. The block diagram of the optimum sliding-mode control system for the Lorenz chaotic problem is shown in Fig. 8. The time responses and control effort of the optimal points A, B, and C are shown in Figs. 9 through 12.

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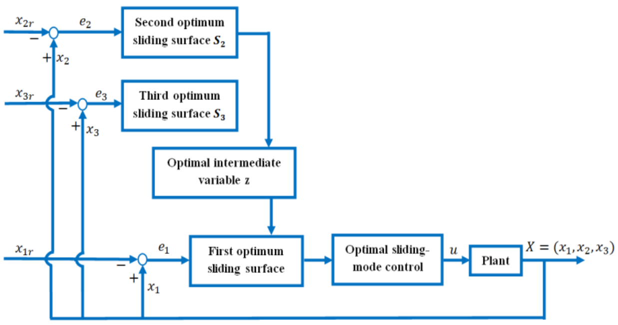

Fig. 8. The block diagram of the optimum sliding-mode control system for the Lorenz chaotic problem.

Table 2. The objective functions and their associated design variables for the optimum points of Fig. 7.

Optimum point	A	B	C
Normalized summation of states errors	1.48212×10^{-1}	2.91254×10^{-1}	6.71833×10^{-1}
Normalized control effort	5.94888×10^{-1}	2.81957×10^{-1}	4.83332×10^{-2}
Design variable k	9.99998×10^{1}	7.98928×10^{1}	5.17383×10^{1}
Design variable Z_{upper}	9.99998×10^{-1}	9.99976×10^{-1}	9.99958×10^{-1}
Design variable S_t	-7.04830×10^{2}	-7.82479×10^{2}	-9.82094×10^{2}
Design variable Ω_1	2.32966×10^{2}	3.21923×10^{2}	3.88847×10^{2}
Design variable Ω_2	1.00402×10^{0}	2.33312×10^{0}	2.77208×10^{0}
Design variable c_1	4.01359×10^{0}	9.30361×10^{-1}	8.80528×10^{-1}
Design variable c_2	-4.75068×10^{1}	-5.39727×10^{1}	-2.67425×10^{1}
Design variable c_3	-9.55806×10^{1}	-9.75049×10^{1}	-9.50384×10^{1}

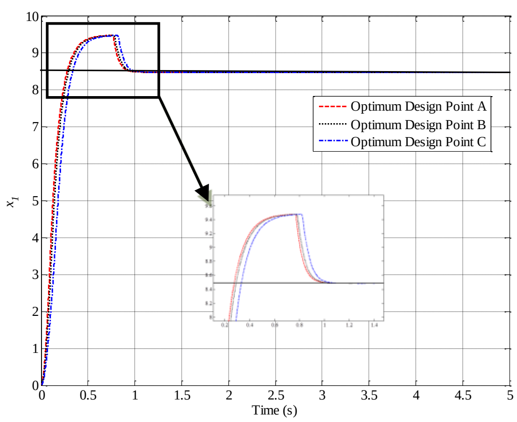

Fig. 9. State x_1 of the optimum points A, B, and C shown in the Pareto front.

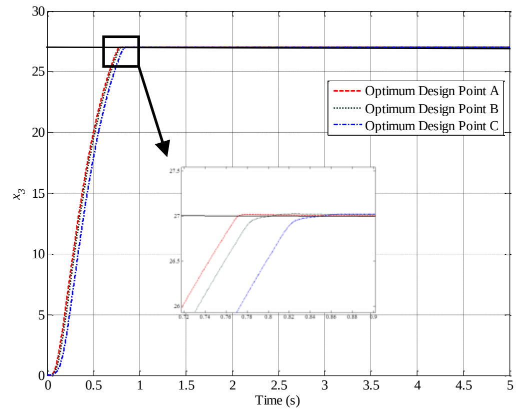

Fig. 10. State x_2 of the optimum points A, B, and C shown in the Pareto front.

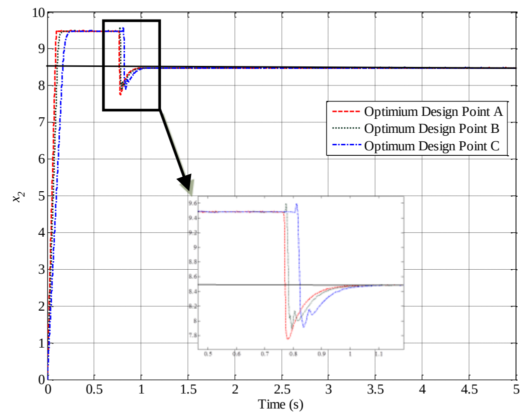

Fig. 11. State x_3 of the optimum points A, B, and C shown in the Pareto front.

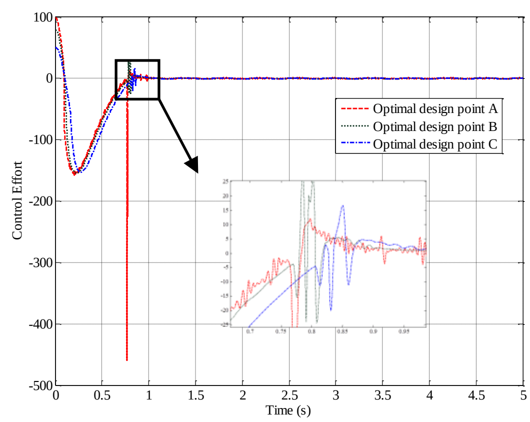

Fig. 12. Control effort of the optimum design points A, B, and C shown in the Pareto front.

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5 CONCLUSION

This paper presented a novel optimal robust sliding mode controller evaluated via an uncertain chaotic problem. When designing the control methodology, multi-objective periodic CDPSO benefiting from a number of crucial factors providing effective performance of the method was employed. Those factors involved are divergence and convergence operators, the periodic leader selection method, and the adaptive elimination technique. To design the sliding mode control, two conflicting objective functions, the normalized summation of states errors and normalized control effort, were regarded to optimize by multi-objective periodic CDPSO. Afterward, the obtained Pareto front was compared with the Pareto front of three prominent algorithms: Sigma method, Modified NSGAII, and MOGA. The Pareto front obtained via multi-objective periodic CDPSO provided superior optimal non-dominant points than that of the other three algorithms. Hence, it is presenting ample opportunities for the designers to come up with the best control methodology to control the chaotic uncertain problems. Finally, the presented methodology resulted in a better control performance in terms of providing optimal control effort and minimum states errors for challenging chaotic problems.

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