Investigation of Rail Track Vibration Due to Fractured Sleepers

H. Mortezaei*
Department of Mechanical Engineering,
Yazd University, Yazd, Iran,
E-mail: h.mortezaee@stu.yazduni.ac.ir
*Corresponding author

M. M. Jalili
Department of Mechanical Engineering,
Yazd University, Yazd, Iran,
E-mail: jalili@yazd.ac.ir

M. T. Ahmadian
Department of Mechanical Engineering,
Sharif University of Technology, Tehran, Iran,
E-mail: ahmadian@sharif.edu

Received: 8 July 2012, Revised: 30 July 2012, Accepted: 25 August 2012

Abstract: In this paper, the effects of fractured sleepers on rail track vibration under moving wheelset have been investigated. Two parallel rails of the track have been modeled as Euler-Bernoulli beams mounted on rail pads acting as elastic points. Moreover sleepers have been modeled as visco-elastic Euler-Bernoulli beams. It is assumed that, some sleepers under the rail track have been fractured and modeled by two beams. The wheelset has 5 DOF which are longitudinal, vertical and lateral movements plus roll and axial rotations. To determine normal contact force between the wheel and the rail, relative position of the wheel and the rail has been determined at each instant. Using the coordinate of each wheel point in the rail coordinate system, the penetration of the wheel into the rail has been determined. In order to investigate rail-wheel contact forces, Hertzian nonlinear contact theory and Kalker theory have been applied. A computer program has been developed that numerically solves the equations of motion of the system for different operating conditions using RungeKutta Cash-Karp computation method. Using this model the effects of wheel-set velocity, wagon weight, number of fractured sleepers and fracture location on rail track vibration have been investigated.

Keywords: Euler–Bernoulli Beam, Sleeper Defects, Visco-Elastic Foundation, Vertical Vibration

Reference: Mortezaei, H., Jalili, M. M., and Ahmadian, M. T, "Investigation of Rail Track Vibration Due to Fractured Sleepers", Int J of Advanced Design and Manufacturing Technology, Vol. 5/No. 4, 2012, pp. 65-70.

Biographical notes: H. Mortezaei received his MSc in Mechanical Engineering from Yazd University in 2012. **M. M. Jalili** received his PhD in Mechanical Engineering from Sharif University of technology, in 2009. He is currently Assistant Professor at Department of Mechanical Engineering, Yazd University, Yazd, Iran. **M. T. Ahmadian** received his PhD in Mechanical Engineering from University of Kansas, Kansas 1986. He is currently Professor at Department of Mechanical Engineering, Sharif University of technology, Tehran, Iran.

1 INTRODUCTION

Ride comfort is a prime concern for public transportation. Research has proven that insufficient railway maintenance, sleeper defects, passing over zigzag or curved routs and vertical and lateral rail irregularities are the most important parameters effecting wagon and track vibration.

The sleeper defect due to wheel impact from freight wagons is one of the most important concerns of railway companies. A large number of researches have been performed on this subject. For example sensitivity analysis of free vibration behaviors of an in situ railway concrete sleeper (standard gauge sleeper), incorporating sleeper/ballast interaction, subjected to variations of rail pad properties has been presented by Kaewunruen and Remennikov [1]. In the mentioned study Timoshenko-beam and spring elements have been used in the in situ railway concrete sleeper modeling. In addition experimental failure mode evaluation, flexural toughness, and energy absorption mechanisms for railway pre-stressed concrete sleepers under static and impact loadings have been investigated [2].

Analytical and experimental study of sleeper SAT S 312 in slab track Sateba system has been presented by Guigou-Cartera et al.[3].In the mentioned study, a simple prediction tool based on a two-dimensional model has been developed for a slab track system composed of two rails with rail pads, sleepers with sleeper pads, and a concrete base slab. The track and the slab are considered as infinite beams with bending stiffness, loss factor and mass per unit length.

In this paper, effects of sleepers defect on rail track vibration under moving train will be studied. Using the presented model in this article the effects of train velocity, wagon weight, number of fractured sleepers and fracture location on rail track vibration will be investigated.

2 MODELING

Considering the nature of this problem, the use of 3-dimensional wheel-set model is essential. Determination of the correct contact point between wheel and rail and the exact value of the contact force between the two members are the major issues in 3-D modeling of the wheel-set. There are two methods for deriving axle equations and determining contact forces between wheel and rail. In one method, the constraints against axle motion by the rail are used to calculate the normal forces at wheel rail contact point. The other method uses the elastic contact theory of wheel and rail in order to determine wheel-rail contact parameters.

In this method after determining the relative position of the wheel and the rail, Hertz contact theory is used to find the normal contact force between the two elements. Studies using this method are mostly two-dimensional. In the current article elastic contact theory and 3-D wheel-set model are used for wheel/rail contact modeling. Figure 1shows3-D model of the wheel-set, rails, sleepers and coordinate systems used in this research. The frame 'X', 'Y', 'Z' is the fixed coordinate system, where its origin being at the track center line. The axes$\acute{X}$, $\acute{Y}$, $\acute{Z}$make up the wheel-set body coordinate system, which has its origin at the wheel-set center of mass.

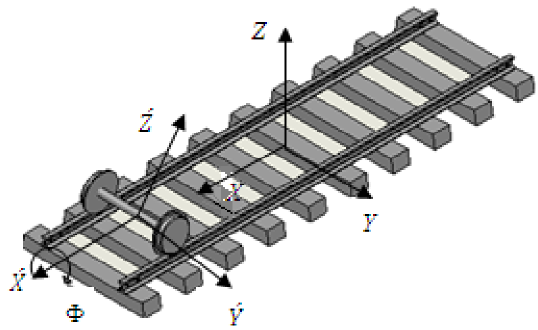

Fig. 1 3D view of wheel-set, rails and sleepers

Figure 2 shows the rail, sleeper and padmodels adopted in the present study. Sleepershave been modeled as visco-elastic Euler-Bernoulli beams. Alsorails are modeled as Euler-Bernoulli beams on elastic points as rail pads. The pads are modeled by springs and dampers.

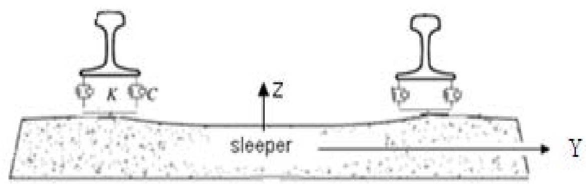

Fig. 2 Front view of rails, pads and sleeper model

3 EQUATIONS OF MOTIONS

3.1. Sleeper Equations of Motions

Describing the sleeper as visco-elastic Euler-Bernoulli beams**,** the equation of vertical vibration of the sleeper is given by:

$$E_s I_s \frac{\partial^4 w_s(x,t)}{\partial x^4} + \rho_s A_s \frac{\partial^2 w_s(x,t)}{\partial t^2} + K_b(w_s) + C_b(\dot{w}_s) = F_R \delta(x - x_r) + F_R \delta(x - x_l) \qquad (1)$$

where 'E_s', 'I_s', 'A_s', 'w_s' and 'ρ_s' are module of elasticity, second moment of area, cross section area, displacement and density of sleeper respectively. 'K_b' is the stiffness factor of the sleeperand 'C_b' is the damping factor of the sleeper. 'F_{Rr}' and 'F_{RL}' are vertical forces of right and left rails. Using Separation of variables, the solution can be expressed as:

$$w_s(x,t) = X_s(x)T_s(t) \tag{2}$$

where:

$$X_s(x) = x^2(ax^4 + bx^2 + c) \tag{3}$$

The boundary conditions are:

$$\begin{cases} \ddot{x}_s(x = -\frac{L_s}{2} = \frac{L_s}{2}) = 0 \\ \dddot{x}_s(x = -\frac{L_s}{2} = \frac{L_s}{2}) = 0 \end{cases} \tag{4}$$

where 'L_s' is the sleeper length. The solution can be determined as:

$$a = 4, \quad b = -5L_s^2, \quad c = 3.75L_s^4 \tag{5}$$

Substituting Eq.(2) into Eq.(1) yields:

$$E_sI_sT_sX_s^{(4)} + \rho_sA_sT_s^{(2)}X_s + C_bT_s^{(1)}X_s + K_bT_sX_s = F_{Rr}\delta(x-x_r) + F_{Rl}\delta(x-x_l) \tag{6}$$

Multiplying Eq. (6) by 'X_s' and then applying integral on the sleeper length, yields the second-order ordinary differential equation of the sleeper vibration in terms of the generalized coordinate 'T_s' as follows:

$$E_sI_sT\int_{-\frac{L_s}{2}}^{\frac{L_s}{2}} X_s^{(4)}X_s dx + \rho_sA_s\ddot{T}\int_{-\frac{L_s}{2}}^{\frac{L_s}{2}} X_s^2 dx + C_b\dot{T}\int_{-\frac{L_s}{2}}^{\frac{L_s}{2}} X_s^2 dx + K_bT\int_{-\frac{L_s}{2}}^{\frac{L_s}{2}} X_s^2 dx = F_rX(x_r) + F_lX(x_l) \tag{7}$$

thus:

$$\ddot{T} + \frac{C_b}{\rho_sA_s}\dot{T} + \left(\frac{E_sI_sB_1}{\rho_sA_sB_2} + \frac{K_b}{\rho_sA_s}\right)T = \frac{1}{\rho_sA_sB_2}(F_rX(x_r) + F_lX(x_l)) \tag{8}$$

in which:

$$B_1 = \int_{-\frac{L_s}{2}}^{\frac{L_s}{2}} X_s^{(4)}X_s dx \quad B_2 = \int_{-\frac{L_s}{2}}^{\frac{L_s}{2}} X_s^2 dx \tag{9}$$

3.2. Rails Equation of Motion:

Describing each rail as an Euler-Bernoulli beam, the vertical vibration of rail is given by:

$$E_RI_R\frac{\partial^4 w_{Rr}(x,t)}{\partial x^4} + \rho_RA_R\frac{\partial^2 w_{Rr}(x,t)}{\partial t^2} = F_{Rr} + F_e \tag{10}$$

In which

$$F_{Rr} = \sum_{i=1}^{N_s}[K_p(w_{rr}(x_i,t) - w_s(x_i,t)) + C_p(\dot{w}_{rr}(x_i,t) - \dot{w}_s(x_i,t))]\delta(x-x_i) \tag{11}$$

and

$$F_e = \sum_{w=1}^{4} F_w\delta(x - (v_wt + c_w)) \tag{12}$$

where 'E_R', 'I_R', 'A_R', 'w_{Rr}' and 'ρ_R' are module of elasticity, second moment of area, cross section area, displacement and density of rail respectively. 'K_p' is Fasten erstiffness of the pad and 'C_p' is Fasten erdamping of the pad. 'F_{Rr}' is the vertical force from sleeper to rail, and 'F_w' is vertical wheel/rail force. Also the same equations can be written for left rail. Boundary conditions are:

$$\begin{cases} w_{Rr}(0,t) = 0 \quad \frac{\partial^2 w_{Rr}(0,t)}{\partial x^2} = 0 \\ w_{Rr}(Lr,t) = 0 \quad \frac{\partial^2 w_{Rr}(L_R,t)}{\partial x^2} = 0 \end{cases} \tag{13}$$

in which L_R is rail length. The solution can be determined as:

$$w_{Rr} = \sum_{n=1}^{k} X_{R_n}(x)T_R(t) = \sum_{n=1}^{k}\sin(\frac{n\pi}{L_R}x)T_R(t) \tag{14}$$

Substituting Eq.(14) into Eq.(10) yields:

$$E_RI_R\sum_{n=1}^{k} X_R^{(4)}(x)T_R(t) + \rho_RA_R\sum_{n=1}^{k} X_R(x)\ddot{T}_{R_n}(t) = F_{Rr} + F_e \tag{15}$$

Multiplying Eq. (15) by "$X_R(x)$" and then applying integral on the raillength, yields the second-order ordinary differential equation of the rail vertical vibration in terms of the generalized coordinate "T_R" as follows:

$$\ddot{T}_R + \frac{E_R I_R}{\rho_R A_R}\left(\frac{n\pi}{L_R}\right)^4 T_{R_n} = \frac{-2}{\rho_R A_R L_R}\sum_{i=1}^{N_s}[K_p(w_{Rr}(x_i,t) - w_s(x_i,t)) +$$

$$C_p(\dot{w}_{Rr}(x_i,t) - \dot{w}_s(x_i,t))]X_{R_n}(x_i) + \frac{2}{\rho_R A_R L_R}\sum_{w=1}^{4} F_w X_R(x-(v_w t + x_w)) \quad (16)$$

3.3. Wheel-set Equations of Motion:

Using the components of the wheel-set forces and position vectors in the equilibrium coordinate system, the following equations yield:
Longitudinal equation:

$$m\ddot{x} = F_{Lx} + F_{Rx} + N_{Lx} + N_{Rx} \quad (17)$$

lateral equation:

$$m\ddot{y} = F_{Ly} + F_{Ry} + N_{Ly} + N_{Ry} \quad (18)$$

vertical equation:

$$m\ddot{z} = F_{Lz} + F_{Rz} + N_{Lz} + N_{Rz} - W_A \quad (19)$$

rolling equation:

$$I_{wx}\ddot{\phi} = R_{Ry}(F_{Rz} + N_{Rz}) - R_{Rz}(F_{Ry} + N_{Ry}) + R_{Ly}(F_{Lz} + N_{Lz}) - R_{Lz}(F_{Ly} + N_{Ly}) \quad (20)$$

spin equation:

$$I_{wy}\ddot{\beta} = R_{Rz}F_{Rx} - R_{Rx}(F_{Rz} + N_{Rz}) + R_{Lz}F_{Lx} - R_{Lx}(F_{Lz} + N_{Lz}) \quad (21)$$

where 'F_L' and 'F_R' are creep forces at the left and right contact points, 'N_L' and 'N_R' are normal forces at left and right contact points, 'R_L' and 'R_R' are position vectors at left and right contact points, 'W_A' is Weight of wheel-set, 'Φ' is Roll displacement about 'x'' axis, 'β' is Roll displacement about y' axis, and 'I_w' is the wheel second moment of area.
The creep forces, in general, are defined with respect to the contact plane. However, after the coordinate transformation, creep forces and creep moments are obtained in an equilibrium coordinate system. The creep force and creep moment components in equilibrium coordinate system are [4]:
Left wheel:

$$F_{LX} = -\left(\frac{f_{33}}{V}\right)\left[V\left[1 - \left(\frac{r_L}{r_0}\right)\right]\right]$$

$$F_{Ly} = -\left(\frac{f_{11}}{V}\right)[\dot{y} + r_L\dot{\phi}]\cos(\delta_L + \phi) + \left(\frac{f_{12}}{V}\right)[\Omega\delta_L]\cos(\delta_L + \phi) \quad (22)$$

right wheel:

$$F_{RX} = -\left(\frac{f_{33}}{V}\right)\left[V\left[1 - \left(\frac{r_R}{r_0}\right)\right]\right]$$

$$F_{Ry} = -\left(\frac{f_{11}}{V}\right)[\dot{y} + r_R\dot{\phi}]\cos(\delta_R + \phi) + \left(\frac{f_{12}}{V}\right)[\Omega\delta_R]\cos(\delta_R + \phi) \quad (23)$$

$$M_{Rz} = \left(\frac{f_{12}}{V}\right)[\dot{y} + r_R\dot{\phi}]\cos(\delta_R + \phi) + \left(\frac{f_{22}}{V}\right)[\Omega\delta_R]\cos(\delta_R + \phi)$$

where 'r_o' is the nominal wheel-set rolling radius, 'V' is the Wheel-set velocity, 'Ω' is the normal angular velocity and 'M_L' and 'M_R' are creep moments at the left and right contact points. Also, 'f_{11}', 'f_{22}', 'f_{33}' and 'f_{12}' are the creep coefficients, defined as:

$$\begin{aligned} f_{11} &= (ab)GC_{22} \\ f_{22} &= (ab)^2 GC_{33} \\ f_{12} &= (ab)^{\frac{3}{2}} GC_{23} \\ f_{33} &= (ab)GC_{11} \end{aligned} \quad (24)$$

where 'C_{ij}' is creepage and spin coefficients, 'G' is Modulus of rigidity and 'a' and 'b' are semi-axis of the contact ellipse in the longitudinal and lateral direction.

$$\begin{aligned} a &= m\left(3\pi N\left(K_1 + K_2\right)/4K_3\right)^{\frac{1}{3}} \\ b &= n\left(3\pi N\left(K_1 + K_2\right)/4K_3\right)^{\frac{1}{3}} \end{aligned} \quad (25)$$

in which N is the total normal force of the wheel:

$$N = \begin{cases} \dfrac{4G\left(\dfrac{R_1 R_1' R_2'}{R_1' - R_1}\right)^{\frac{1}{4}}}{(3 - 3\nu_{wr})}(Z_{wr})^{\frac{3}{2}} & Z_{wr}(t) \geq 0 \\ 0 & Z_{wr}(t) < 0 \end{cases}$$

$$K_1 = \frac{1 - \nu_w^2}{\pi E_w} \quad (26)$$

$$K_2 = \frac{1 - \nu_R^2}{\pi E_R}$$

$$K_3 = \frac{1}{2}\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_1'} + \frac{1}{R_2'}\right]$$

where 'υ_w' and 'υ_R' are poisson's ratio for the wheel and rail materials ,'E_w' is the wheel-set module of elasticity,'R_1' is the principal rolling radius of the wheel,'$\acute{R}_1$' is the principal transverse radius of curvature of the wheel profile, 'R_2' is the principal rolling radius of rail, and'$\acute{R}_2$' is the principal transverse radius of curvature of the rail profile. The coefficients 'm' and 'n' in Eq. (28) depend on the ratio'K_4/K_3' where 'K_4' is defined as follows:

$$K_4 = \frac{1}{2}\left[(\frac{1}{R_1} - \frac{1}{R_1'})^2 + (\frac{1}{R_2} - \frac{1}{R_2'})^2 + 2(\frac{1}{R_1} - \frac{1}{R_1'})(\frac{1}{R_2} - \frac{1}{R_2'}) \right]^{\frac{1}{2}} \quad (27)$$

The coefficients m and n are given in table 1 as a function of θ in which θ is defined as [4]:

$$\theta = \cos^{-1}(\frac{K_4}{K_3}) \quad (28)$$

Table 1 Coefficients *m* and *n*

θ (deg)	m	N	θ (deg)	m	n
0.5	61.4	0.1018	30	2.731	0.493
1	36.89	0.1314	35	2.397	0.53
1.5	27.48	0.1522	40	2.136	0.567
2	22.26	0.1691	45	1.927	0.604
3	16.5	0.1964	50	1.754	0.641
4	13.31	0.2188	55	1.611	0.678
6	9.79	0.2552	60	1.486	0.717
8	7.86	0.285	70	1.284	0.802
10	6.604	0.3112	80	1.128	0.893
20	3.813	1.4123	90	1	1

4 RESULTS AND DISCUSSION

In this section the effects of the sleeper defects on rail track vibration are studied. The main parameters of the wagon, rails and sleepers used in the simulation are listed in Tables 2-4.

Table 2 Main parameters of the sleeper

Parameter	Value	Parameter	Value
E_s[GPa]	25	d_s [m]	0.79
I_s [m^4]	1.85×10^{-5}	A_s [m^2]	22.5×10^{-3}
ρ_s [Kg/m^3]	2320	K_b [N/m]	23×10^6
L_s [m]	1.7	C_b [N.s/m]	50×10^3

Table 3 Main parameters of the rail

Parameter	Value	Parameter	Value
E_R [GPa]	207	A_R [m^2]	7.17×10^{-3}
I_R [m^4]	2.35×10^{-5}	K_p [N/m]	200×10^6
ρ_R [Kg/m^3]	7800	C_p [N.s/m]	70×10^3

Table 4 Main parameters of the wheel-set

Parameter	Value	Parameter	Value
E_w [GPa]	206	I_{wx} [m^4]	680
r_o [m]	0.49	I_{wy} [m^4]	73
P_w	0.3	I_{wz}[m^4]	680
M_w [Kg]	1200	G [GPa]	79.3

The complete system equations are obtained by combining the wheel-set, rail and sleepers equations. Using the Runge-Kutta method, these equations have been solved for different conditions. Effects of wheel-set weight, wheel-set velocity and defect position on maximum rail displacement have been illustrated in Figures 3-6.

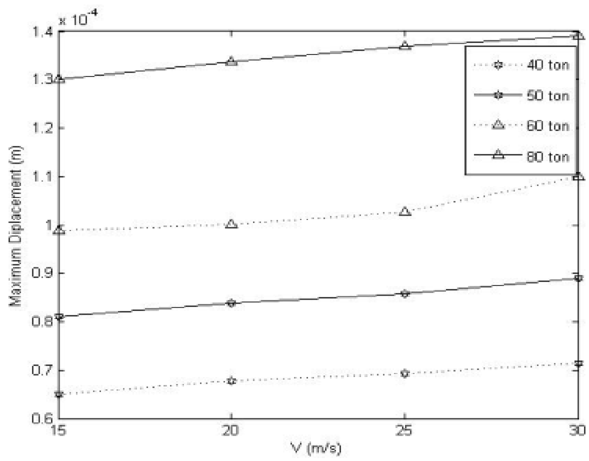

Fig. 3 Effects of wheel-set velocity on maximum displacement of the rail (perfect sleeper)

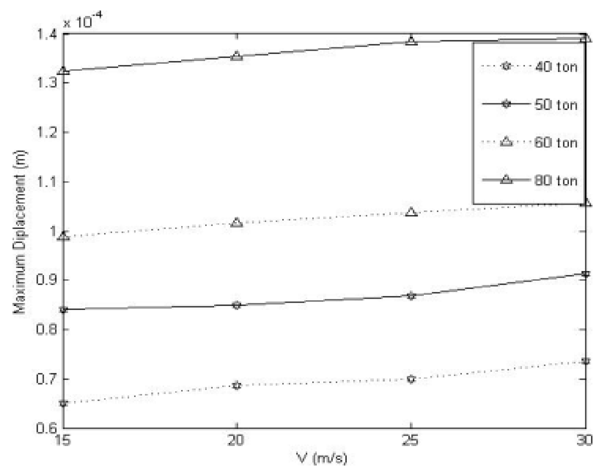

Fig. 4 Effects of wheel-set velocity on maximum displacement of the rail (defect position: x_s=0)

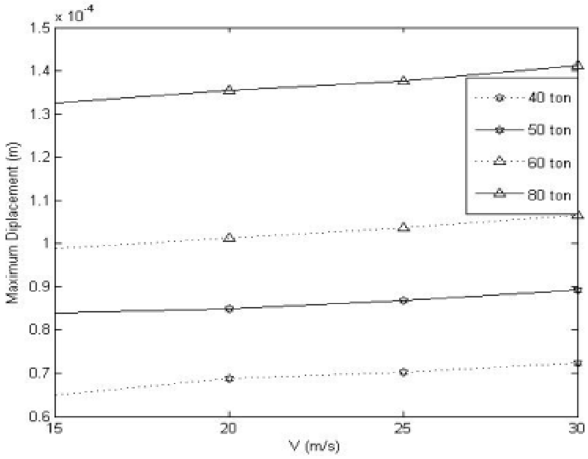

Fig. 5 Effects of wheel-set velocity on maximum displacement of the rail (defect position: $x_s=\frac{l_s}{3}$)

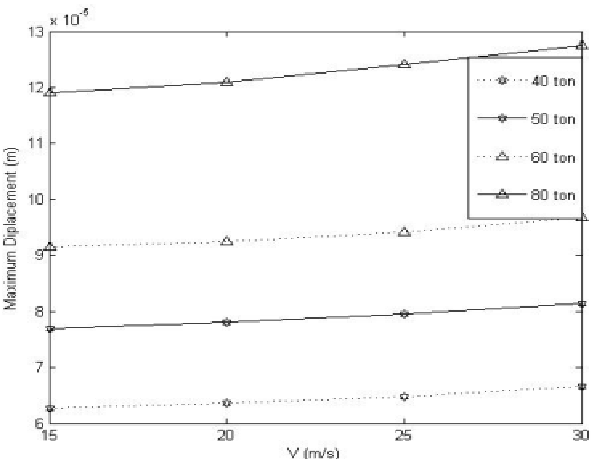

Fig. 6 Effects of wheel-set velocity on maximum displacement of the rail (defect position: $x_s=\frac{l_s}{4}$)

5 CONCLUSION

In this paper the effects of sleeper defects on rail track vibration under moving train have been investigated. The complete system equations have been obtained by combining the wheel-set, rails and sleeper equations and solved using the Runge-Kutta method. Using these equations, the effects of wheel-set weight, wheel-set velocity and defect position on maximum rail displacement have been investigated. According to these figures, defect position has not a significant effect on maximum rail displacement. The results presented in this article can be used in estimation of the appropriate time for railway repair.

REFERENCES

[1] Kaewunruen, S., and Remennikov, A. M., “Sensitivity analysis of free vibration haracteristics of an in situ Railway concrete sleeper to variations of rail padparameters”, Journal of Sound and Vibration,Vol. 298, No. 2, 2006, pp, 453- 461.

[2] Kaewunruen, S., and Remennikov, A. M., “Dynamicflexuralinfluenceonarailwayconcretesleeperintr ack Systemduetoasinglewheelimpact”, Engineering Failure Analysis,Vol. 16, No. 3, 2009, pp. 705-712.

[3] Guigou-Carter, C., and Villot, M., “Analytical and experimental study of sleeper SAT S 312 in slab Track Sateba system”, Journal of Sound and Vibration,Vol. 293, No.3, 2006, pp. 878-887.

[4] Garg, V. K., and Dukkipati, R. V., Dynamics of Railway Vehicle Systems, Canada Academic Press, 1984.