Optimal Swing up of Double Inverted Pendulum using Indirect Method

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Abstract: In this paper, optimal swing up of a double inverted pendulum (DIP) with two underactuated degrees of freedom (DOFs) is solved using the indirect solution of optimal control problem. Unlike the direct method that leads to an approximate solution, the proposed indirect method results in an exact solution of the optimal control problem, but suffers from its limited convergence domain which makes it difficult to solve. In order to overcome this problem, an inversion-based method is used to obtain the required initial solution for the indirect method. In the proposed methodology, dynamic equations are derived for a general inverted pendulum using Euler-Lagrange formulation. Then the necessary optimality conditions are derived for a DIP on the cart using the Pontryagin's maximum principle (PMP). The obtained equations establish a two-point boundary value problem (TPBVP) which solution results in optimal trajectories of the cart and pendulums. In order to demonstrate the applicability of the presented method, a simulation study is performed for a DIP. The simulation results confirm the superiority of the proposed method in terms of reduced effort.

Keywords: Boundary Value Problem, Inverted Pendulum, Optimal Swing up, Indirect Method

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1 INTRODUCTION

Pendulum systems represent a typical benchmark problem faced in the control field. Since it is an underactuated system with a highly nonlinear structure used to validate different nonlinear and linear controllers, it has found a wide range of applications such as those in rocket propellers, self-balancing robots, tank missile launchers, stabilization of ships, design of earthquake resistant buildings, etc. A vast deal of contributions has been presented on the stabilization and swing up of different types of inverted pendulums [1]. For the stabilization problem, there are numerous control techniques such as PI state feedback controller, LQR controller, feedback linearization, and many other control methods [2]. Beside the stabilization aspect, the swing up problem has gained an increasing deal of attention during the recent past. The entire deal of the research works dealing with the swing up problem can be categorized into two groups: non-optimal swing up and optimal swing up.

Nowadays, many approaches have been presented for the non-optimal swing up problems, including energy-based method [3], smooth controllers [4], siding mode control [5], feedforward-feedback control [1], [6], and integral back-stepping sliding mode control [7]. Unlike the non-optimal swing up problems, when it comes to the optimal swing up instances, a desired cost function must also be minimized. Linear and nonlinear programming ([8], [9]), and ant colony optimization method [10] have been reported for the optimal swing up of the inverted pendulum.

Al-Jana et al. presented performance optimizing of a double inverted pendulum (DIP) via a uniform neuro multiobjective genetic algorithm as a direct method [11]. A suboptimal nonlinear control law based on passivity analysis and dynamic programming was presented for a rotary Pendubot pendulum for which switch control law was not required [12]. Time-optimal control of single inverted pendulum has been presented by some researchers [13-16]. The energy-optimal trajectory planning for the Pendubot and Acrobot was presented by Gregory et al. [17]. Recently, Horibe *et al.* adressed the problem of optimal swing up of a single inverted pendulumn via stable manifold method [18].

The approaches followed to solve the optimal control problems are generally classified as either direct or indirect methods. Direct methods are based on the conversion of the optimal control problem into a parameter optimization problem, while the indirect ones explicitly solve the optimality condition stated in terms of the Pontryagin's maximum principle (PMP), the co-state equation and suitable boundary conditions. Compared to the indirect methods, the direct methods have been proved to be quite robust and globally convergent, but rather computationally more expensive. In contrast, the indirect methods suffer from limited domain of convergence but result in an accurate solution ([19], [20]). Most of the previous works dealing with optimal swing up have employed the direct methods [8-12], leading to solely approximate solutions.

The indirect methods lead to a two-point boundary value problem (TPBVP) which is usually sensitive to the initial condition, with its limited domain of convergence. Recently, with the development of various methods for solving the TPBVP, indirect methods have been extensively applied to come with highly accurate optimal trajectories for fully actuated robot manipulators ([17], [19], [20]), but, to the best of our knowledge, only a few works have focused on underactuated systems, possibly due to the further difficulty of the convergence problem in an underactuated system rather than a fully actuated one. In recent attempts to solve this problem, energy-optimal trajectory planning [17] and stable manifold method [18] were adopted for systems with only one underactuated degree of freedom (DOF), leaving the indirect solution of the optimal control problem for underactuated systems with more than one underactuated DOFs yet to be addressed. In this paper, a general inverted pendulum on the cart with *n*-1 links is considered.

Following the indirect method, the necessary conditions for optimality are derived from the PMP. The obtained equations are compiled into a TPBVP that is then solved by *bvp4c* command in MATLAB. In order to overcome the convergence problem, an inversion-based method is proposed to obtain a proper initial guess. Optimal swing up of a DIP is presented and the obtained results are compared with previous works. The rest of this paper is outlined as follows: the next section describes the dynamic equations and optimality conditions for a general inverted pendulum. Optimality conditions of DIP are derived in Section 3. Section 4 addresses simulation results for DIPs. Finally, conclusions are drawn in Section 5.

2 DERIVING THE NECESSARY OPTIMALITY CONDITIONS

Schematic model of a classical inverted pendulum with *n*-1 links is shown in "Fig. 1". As shown in the figure, x is the position of cart, u is the applied force to the cart and θ_i is the angular position of the link i from vertical axis. General dynamic equation of this system can be described as:

$$D(q)\ddot{q}+b(q,\dot{q})+h(q)=\tau, \tag{1}$$

Where $q=\begin{bmatrix} x & \theta_1 & \dots & \theta_{n-1} \end{bmatrix} \in R^n$ is the vector of joint positions, $\dot{q}=\begin{bmatrix} \dot{x} & \dot{\theta}_1 & \dots & \dot{\theta}_{n-1} \end{bmatrix} \in R^n$ is the vector of joint velocities, $D \in R^{n\times n}$ is the inertia matrix, $b(q,\dot{q}) \in R^n$ is the centripetal, coriolis and friction forces, $h(q) \in R^n$ describes the gravity effects and $\tau=[u\ \ 0\ \ \dots\ 0] \in R^n$ represents the force vector. By defining the state vector as:

$$x=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \tag{2}$$

"Eq. (1)" can be rewritten in state space form as:

$$\dot{x}= f(x,\tau) \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}=\begin{bmatrix} f_1(x,\tau) \\ f_2(x,\tau) \end{bmatrix}, \tag{3}$$

Where:

$$f_1=x_2,\ \ f_2=p(x_1,x_2)+K(x_1)\tau, \tag{4}$$

$$\begin{aligned} &p(x_1,x_2)=-D^{-1}(x_1)\left[b(x_1,x_2)+h(x_1)\right],\\ &K(x_1)=D^{-1}(x_1). \end{aligned} \tag{5}$$

Optimal swing up of an inverted pendulum is an optimization problem which can be stated in the form of the optimal control problem. The goal is to swing up the pendulum from the hanging down position to the standing up position in such a way to minimize the predefined objective function.

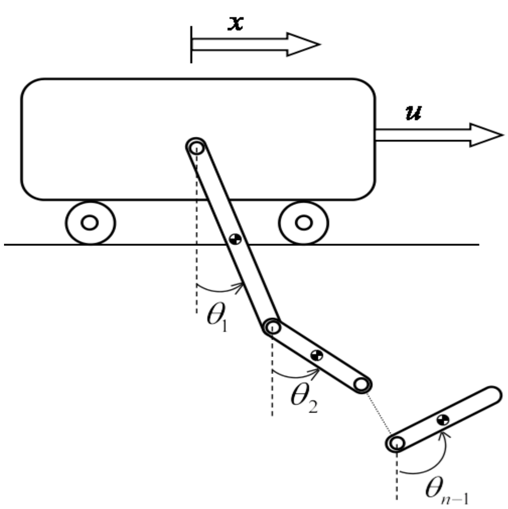

Fig. 1 Schematic view of a classical inverted pendulum with *n*-1 links.

So, the optimal control problem for a dynamic system can be stated as follows [21]: Find the continuous admissible control history $\tau:\left[t_0,t_f\right]\to\Omega\subseteq R^m$ generating the corresponding state trajectory $x:\left[t_0,t_f\right]\to R^n$ which minimizes the cost function:

$$J=\phi(x_f,t_f)+\int_{t_0}^{t_f} L(x,\tau,t)dt\,, \tag{6}$$

Subject to the system dynamics:

$$\dot{x}= f(x,u)\,, \tag{7}$$

The given initial condition:

$$x(t_0)=x_0\,, \tag{8}$$

And the prescribed final conditions:

$$x(t_f)=x_f\,. \tag{9}$$

Here, $x\in R^n$ is the state vector, $\tau\in R^m$ is the control vector, Ω is an acceptable region in R^m, t_0 and t_f are initial and final times, respectively, x_0 and x_f are predefined initial and final states, respectively, and ϕ and L are scalar continuously differentiable functions in which ϕ is the final state penalty term and L is the integrand of the cost function. By introducing the co-state vector $\lambda\in R^n$, the Hamiltonian function of the system can be defined as follows:

$$H=L+\lambda^{\mathrm{T}} f\,. \tag{10}$$

According to PMP, for the optimal trajectories $x^*(t)$ and $\tau^*(t)$, there is a non-zero co-state vector $\lambda^*(t)$ for which the following conditions along the optimal solution must be satisfied:

$$\dot{x}^*=\frac{\partial H}{\partial\lambda},\ \dot{\lambda}^*=-\frac{\partial H}{\partial x},\ \frac{\partial H}{\partial u}=0 \tag{11}$$

Where the superscript (*) denotes the extremals of $\mathrm{X}(t)$ and $\lambda(t)$. By substituting the "Eq. (4) into Eq. (10)" and defining $\lambda=\begin{bmatrix}\lambda_1^{\mathrm{T}} & \lambda_2^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$, the necessary condition (11) can be rewritten as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ p(x_1, x_2) \end{bmatrix} + \begin{bmatrix} 0 \\ K(x_1) \end{bmatrix} u, \quad (12)$$

$$\dot{\lambda}(t) = -\begin{bmatrix} \frac{\partial L}{\partial x_1} + \frac{\partial}{\partial x_1}\left[p + Ku\right]^{\mathrm{T}} \lambda_2 \\ \frac{\partial L}{\partial x_2} + \lambda_1 + \frac{\partial}{\partial x_2}\left[p\right]^{\mathrm{T}} \lambda_2 \end{bmatrix}, \quad (13)$$

$$\frac{\partial L}{\partial u} + K^{\mathrm{T}} \lambda_2 = 0. \quad (14)$$

For an inverted pendulum with n-1 links, "Eqs. (12) and (13)" represent 4n equations dealing with states and co-states, respectively, and "Eq. (14)" leads to a single equation dealing with control u. Also for the swing up problem, there are 4n fixed boundary conditions as follows:

$$\begin{aligned} q(t_0) &= \left[x(0) \quad \theta_1(0) \quad \ldots \quad \theta_{n-1}(0)\right] \\ q(t_f) &= \left[x(t_f) \quad \theta_1(t_f) \quad \ldots \quad \theta_{n-1}(t_f)\right] \\ \dot{q}(t_0) &= \left[\dot{x}(0) \quad \dot{\theta}_1(0) \quad \ldots \quad \dot{\theta}_{n-1}(0)\right] \\ \dot{q}(t_f) &= \left[\dot{x}(t_f) \quad \dot{\theta}_1(t_f) \quad \ldots \quad \dot{\theta}_{n-1}(t_f)\right]. \end{aligned} \quad (15)$$

Where:

$$x(0) = x(t_f) = \dot{x}(0) = \dot{x}(t_f) = 0,$$

And:

$$\theta_i(0) = \dot{\theta}_i(0) = \dot{\theta}_i(t_f) = 0, \theta_i(t_f) = \pi, \; i = 1 \ldots n-1$$

By substituting the control value u obtained from "Eqs. (14) into Eq. (12) and (13)" a set of 4n ordinary differential equations is obtained which in combination with the 4n boundary conditions (15) form a TPBVP. Finally, the derived TPBVP is solved to obtain 2n states and 2n co-states. In the next section, the optimality conditions for a single inverted pendulum and a DIP are derived in detail.

3 DERIVING THE EQUATIONS FOR DIP

A schematic illustration of a DIP on a cart is shown in "Fig. 2". The position of the cart is x and the angle of rotation of the link i from the vertical axis is $\theta_i, i = 1, 2$. The generalized forces associated with the generalized coordinates x, θ_1 and θ_2 are defined as follows:

$$\begin{aligned} &Q_1 = u, \; Q_2 = -c_1\dot{\theta}_1 + c_2(\dot{\theta}_2 - \dot{\theta}_1), \\ &Q_3 = -c_2(\dot{\theta}_2 - \dot{\theta}_1), \end{aligned} \quad (16)$$

Where u is the force applied to the cart, and c_1 and c_2 are the damping coefficients of the first and second joints, respectively.

Using the Euler-Lagrange equations, one can write dynamic equations for DIP as follows:

$$\begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (17)$$

Where:

$$\begin{aligned} &D_{11} = M + m_1 + m_2, D_{12} = \left(m_1 a_1 + m_2 l_1\right)\cos\theta_1, \\ &D_{13} = m_2 a_2 \cos\theta_2, \; D_{22} = m_1 a_1^2 + m_2 l_1^2 + I_1, \\ &D_{23} = m_2 l_1 a_2 \cos\left(\theta_1 - \theta_2\right), \; D_{33} = m_2 a_2^2 + I_2, \\ &b_1 = -\left(m_1 a_1 + m_2 l_1\right)\dot{\theta}_1^2 \sin\theta_1 - m_2 a_2 \dot{\theta}_2^2 \sin\theta_2, \\ &b_2 = m_2 l_1 a_2 \dot{\theta}_2^2 \sin\left(\theta_1 - \theta_2\right) + c_1\dot{\theta}_1 - c_2\left(\dot{\theta}_2 - \dot{\theta}_1\right), \\ &b_3 = -m_2 l_1 a_2 \dot{\theta}_1^2 \sin\left(\theta_1 - \theta_2\right) + c_2\left(\dot{\theta}_2 - \dot{\theta}_1\right), \\ &h_1 = 0, \; h_2 = \left(m_1 a_1 + m_2 l_1\right) g \sin\theta_1, \\ &h_3 = m_2 a_2 g \sin\theta_2, u_1 = u, \; u_2 = 0, \; u_3 = 0. \end{aligned} \quad (18)$$

In the above relationships, M is the cart mass, m_i is the mass of link i, L_i is the length of link i, I_i is the moment of inertia of link i about its center of mass, and a_i is the center of mass of link i, i = 1, 2. Now, by defining the state vector as follows:

$$\begin{aligned} &x = \left[x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6\right]^{\mathrm{T}} = \\ &\left[x \quad \theta_1 \quad \theta_2 \quad \dot{x} \quad \dot{\theta}_1 \quad \dot{\theta}_2\right]^{\mathrm{T}}, \end{aligned} \quad (19)$$

One can write the equation of motion in the state space form as below:

$$\begin{aligned} &\dot{x}_1 = f_1 = x_4, \; \dot{x}_2 = f_2 = x_5, \dot{x}_3 = f_3 = x_6, \\ &\dot{x}_4 = f_4(x,u), \dot{x}_5 = f_5(x,u), \dot{x}_6 = f_6(x,u), \end{aligned} \quad (20)$$

Where:

$$\begin{bmatrix} f_3 \\ f_4 \\ f_5 \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{bmatrix}^{-1} \left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \right). \quad (21)$$

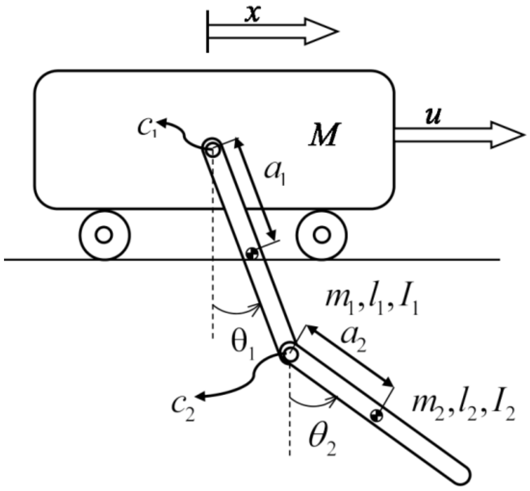

Fig. 2 A double inverted pendulum on a cart.

Here, minimum-effort optimal control problem is considered. So, by defining the co-state vector as $\lambda = [\lambda_1 \quad \dots \quad \lambda_6]^{\mathrm{T}}$ and dynamic equations in the state apace form as $f = [f_1 \quad \dots \quad f_6]^{\mathrm{T}}$, the Hamiltonian function can be obtained using Eqs. (10), (20) and (21), as follow:

$$H = 0.5u^2 + \sum_{i=1}^{6} \lambda_i f_i \tag{22}$$

Then, the co-state equations are obtained by differentiating the Hamiltonian with respect to the states, as follows;

$$\dot{\lambda}_i = -\frac{\partial H}{\partial x_i}, \quad i = 1, \dots, 6. \tag{23}$$

Next, using "Eq. (11)", the optimal control law is obtained by differentiating the Hamiltonian function with respect to control *u* as:

$$u = \frac{-\begin{bmatrix} \lambda_4 \left(D_{22}D_{33} - D_{23}^2 \right) - \lambda_5 \left(D_{12}D_{33} - D_{13}D_{23} \right) \\ -\lambda_6 \left(D_{13}D_{22} - D_{12}D_{23} \right) \end{bmatrix}}{\begin{pmatrix} D_{11}D_{22}D_{33} - D_{11}D_{23}^2 - D_{12}^2 D_{33} + \\ 2D_{12}D_{13}D_{23} - D_{13}^2 D_{22} \end{pmatrix}}. \tag{24}$$

Finally, "Eq. (24)" is substituted into "Eqs. (20) and (23)" to obtain twelve nonlinear ordinary differential equations in terms of states and co-states. These twelve equations along with the following boundary conditions:

$$\begin{aligned} &x_1(0) = 0,\ x_1(t_f) = 0,\ x_2(0) = 0,\ x_2(t_f) = \pi, \\ &x_3(0) = 0,\ x_3(t_f) = \pi, x_4(0) = 0,\ x_4(t_f) = 0, \\ &x_5(0) = 0,\ x_5(t_f) = 0, x_6(0) = 0,\ x_6(t_f) = 0. \end{aligned} \tag{25}$$

construct a TPBVP.

4 SIMULATION RESULTS FOR DIP

Here, using the equations derived in the previous section, the optimal swing up problem is solved for the DIP. However, solving the obtained boundary value problem is a challenging task due to the large number of complicated nonlinear equations. Moreover, such boundary value problems have a very limited domain of convergence and are sensitive to initial guess. Here, a simple inversion-based method is proposed to solve the problem conveniently.

To this end, as a first step, the problem is solved using the inversion-based feedforward control proposed by Graichen [1]. Then, taking the solution as an initial guess, the derived TPBVP in the previous section is solved to obtain the optimal swing up. In order to demonstrate the efficiency of the proposed method, the obtained results of optimal swing up are compared with those obtained from the inversion-based method.

Considering the DIP shown in "Fig. 2" with all required parameters given in "Table 1", the swing up problem is solved according to boundary conditions (25) within the time interval $t \in [0 \ \ t_f],\ t_f = 2.2s$. All the mechanical parameters and boundary conditions are similar to those given in [1].

In order to obtain the inversion-based solution of DIP as reported in [1], by considering the cart acceleration as the input to the system, $u = \ddot{x}$, the dynamic equation (17) can be rewritten as follow:

$$\begin{aligned} &\ddot{x} = u \\ &\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = -\begin{bmatrix} D_{22} & D_{23} \\ D_{23} & D_{33} \end{bmatrix}^{-1} \left(\begin{bmatrix} C_2 \\ C_3 \end{bmatrix} + \begin{bmatrix} G_2 \\ G_3 \end{bmatrix} + \begin{bmatrix} D_{12} \\ D_{13} \end{bmatrix} u \right) \end{aligned} \tag{26}$$

Where the first equation represents the input–output dynamics, and the latter two equations represent the internal dynamics. Then the following output trajectory is considered.

$$\begin{aligned} &x = -p_1 - p_3 - \left(p_2 + p_4 \right) \cos\left(\pi t / t_f \right) + \\ &\sum_{i=1}^{4} p_i \cos\left(\left(1 + i \right) \pi t / t_f \right), \end{aligned} \tag{27}$$

Where p_1, p_2, p_3 and p_4 are free parameters. This function satisfies the four boundary conditions dealing with the cart trajectory in "Eq. (25)". By substituting "Eq. (27) into Eq. (26)", the internal dynamics with the 8 remaining boundary conditions dealing with the link trajectories constructs a TPBVP which can be solved to obtain 8 unknown variables including the internal dynamic trajectories $\theta_1(t)$ and $\theta_2(t)$ and the free parameters p_1, p_2, p_3 and p_4. Accordingly, the free parameters are obtained as follows:

$$p_1 = -0.1014, p_2 = -0.1865, p_3 = 0.0924, p_4 = 0.1355$$

For optimal swing up, the TPBVP consists of "Eqs. (20), (23) and (24)", which are solved to obtain the cart and link trajectory and also the optimal control applied to the cart.

Table 1 Mechanical parameters of the DIP studied in this research.

Parameters	Values	Unit
Mass, cart	M = 0.8	Kg
Mass, link	m_1= 0.853, m_2 = 0.510	Kg
Center of mass,	a_1 = 0.215, a_2 = 0.223	m
Moment of inertia,	I_1 = 0.013, I_2 = 0.019	kg.m^2
Friction constant,	c_1 = c_2 = 0.005	N.m.sec/rad

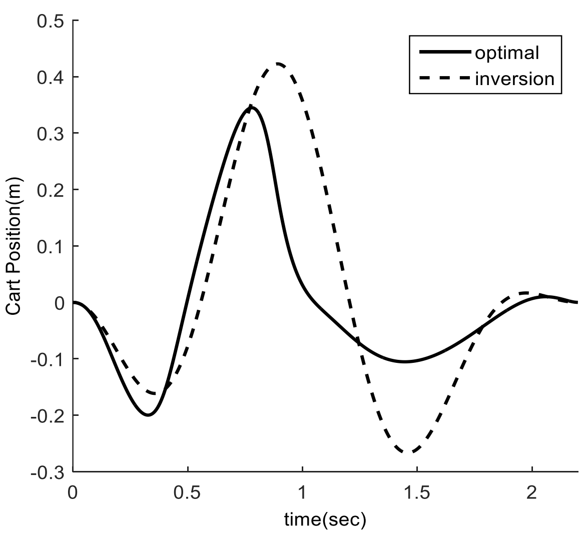

Fig. 3 Linear position of cart.

Figures 3 and 4 show the linear position and velocity of cart for both optimal and inversion-based swing ups, respectively. As shown in "Fig. 3", the cart displacement obtained from the inversion method is 0.688 m, while the corresponding value to optimal swing up is 0.545 m, indicating a reduction of about 20% in the cart displacement for optimal swing up, as compared to the inversion-based method.

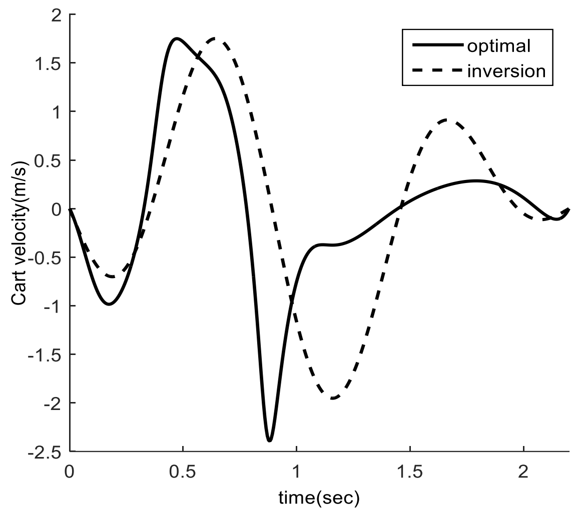

Fig. 4 Linear velocity of cart.

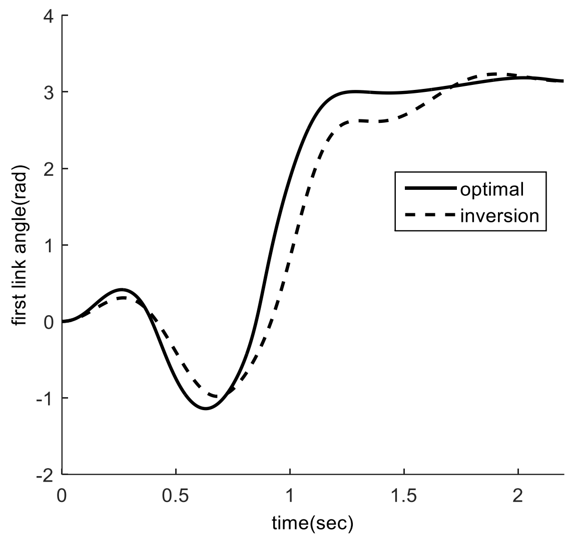

Fig. 5 Angular position of the first link.

Angular position and velocity of the first and second links are shown in "Figs. 5 to 8". As it can be seen from "Figs. 5 and 7", angular positions of the links reached from zero to π within 2.2 s. Figure 9 shows the applied force to the cart. As it can be seen, maximum torque for the inversion-based method reported in [1] is 50 N.m and the corresponding value to the optimal method is 30 N.m. Performance indices are also calculated as follows:

$$J_{inversion} = \int_0^{2.2} 0.5u^2 dt = 188.54,$$
$$J_{optimal} = \int_0^{2.2} 0.5u^2 dt = 70.15. \quad (28)$$

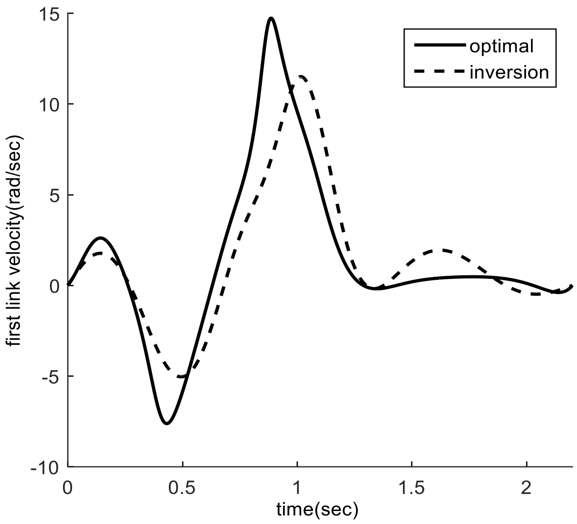

Fig. 6 Angular velocity of the first link.

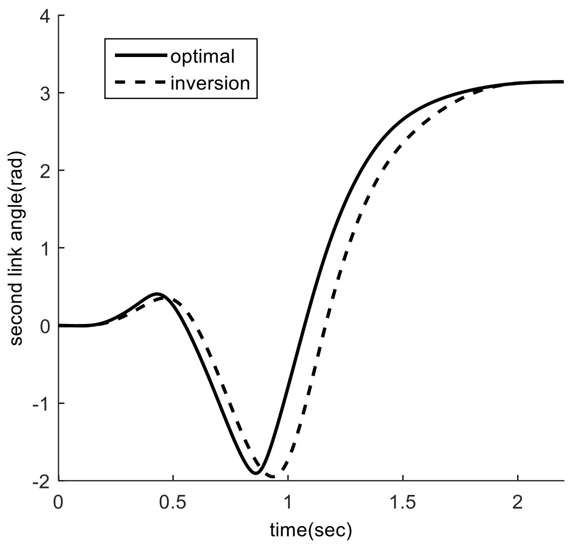

Fig. 7 Angular position of the second link.

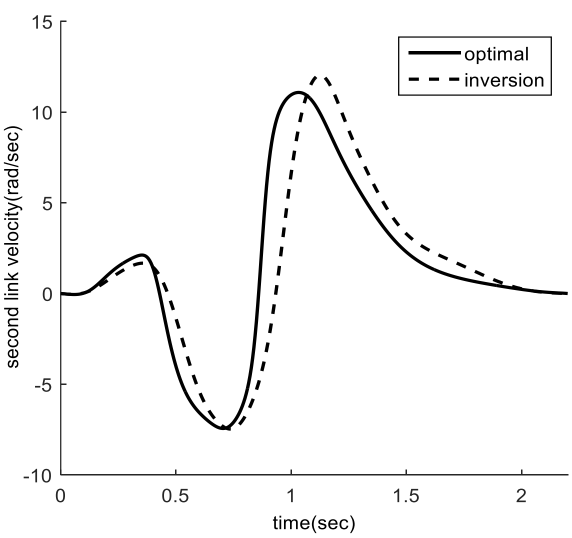

Fig. 8 Angular velocity of the second link.

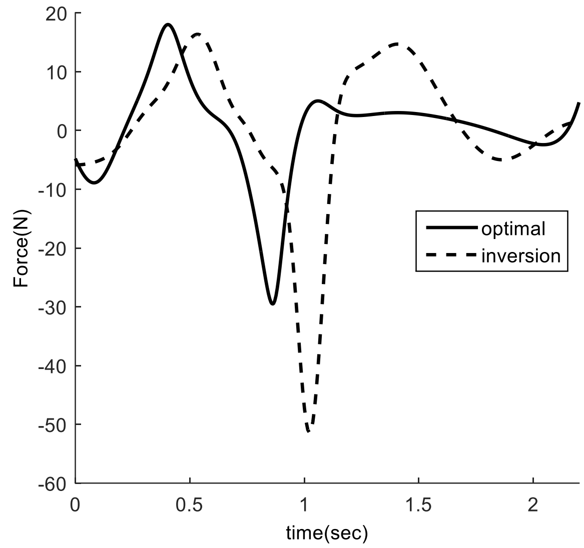

Fig. 9 Applied force to the cart.

So, the performance index for optimal swing up was found to decrease by approximately 60% in comparison with the inversion-based method.

5 CONCLUSION

In this paper, swing up of DIP with minimum effort was considered. To this end, optimal swing up was formulated in the form of an optimal control problem and then solved using a PMP-based indirect approach leading to a two-point boundary value problem (TPBVP). The TPBVP was then numerically solved using the *bvp4c* function in MATLAB. In order to achieve a faster convergence rate, the inversion-based method was also used to obtain the initial guess required for solving the TPBVP. The obtained results were compared with those of the inversion-based method. From the simulation results for DIP, it was seen that, the performance index for the proposed optimal swing up was approximately 60% lower than that of the inversion-based method reported by Graichen [1].

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