Thermal Optimization of an Array of Needle-Shaped using Constructal Theory

Maryam Hoseinzadeh
Department of Mechanical Engineering,
Shahrekord University, Iran
E-mail: m72.9843@gmail.com

Afshin Ahmadi Nadooshan*
Department of Mechanical Engineering,
Shahrekord University, Iran
E-mail: ahmadi@eng.sku.ac.ir
*Corresponding author

Morteza Bayareh
Department of Mechanical Engineering,
Shahrekord University, Iran
E-mail: m.bayareh@eng.sku.ac.ir

Received: 27 April 2018, Revised: 11 July 2018, Accepted: 20 August 2018

Abstract: In the present paper, the constructal theory is employed to determine the optimal configuration of three rows of needle-shaped fins. The heat transfer across the fins is due to laminar forced convection. Second order upwind scheme is used for discretization of the diffusion terms of governing equations. The pressure–velocity coupling is performed using the SIMPLE algorithm. The heat transfer is optimized subject to constant fin volume. The effect of Reynolds number and thermal conductivity on the optimal configuration is investigated. The results obtained from the present simulations are in good agreement with the numerical results. The results show that pin–fins flow structure leads to the best performance when the pin–fin diameters and heights are non-uniform. At Re = 100 and 200, the optimal value of $H_3/H_1$is 1.3. It is revealed that at Re = 50, the optimal value for D_3/D_1 is approximately 1.1. The results demonstrate that heat transfer rate is an increasing function of the Reynolds number.

Keywords: Constructal Theory, Forced Convection, Needle-Shaped Fins, Optimization, Reynolds Number

Reference: Hoseinzadeh, M., Ahmadi Nadooshan, A., and Bayareh, M., “Thermal Optimization of an Array of Needle-Shaped using Constructal Theory”, Int J of Advanced Design and Manufacturing Technology, Vol. 11/No. 3, 2018, pp. 55–61.

Biographical notes: M. Hoseinzadeh received her MSc in Mechanical Engineering from Shahrekord University, Iran. **A. Ahmadi Nadooshan** received his PhD in Mechanical Engineering from Isfahan University of technology. He is currently Associated Professor at the Department of Mechanical Engineering at Shahrekord University, Iran. His current research interest includes heat transfer, thermodynamics and multi-phase flows. **M. Bayareh** is Assistant Professor of Mechanical engineering at Shahrekord University, Iran. He received his PhD in Mechanical engineering from Isfahan University of technology. His current research focuses on Turbulent flows, multi-phase flows and thermodynamics.

1 INTRODUCTION

The heat transfer requires a difference in temperature between the two media [1]. Complex fins are an important part of heat transfer devices in electronic cooling and other applications [2]. One of the techniques for optimizing the amount of heat transfer is constructal theory [3]. The Constructal theory states that geometry is not an input variable, but the achievement of the best structure is an important result. Almogbel and Bejan [2] employed constructal law to minimize the thermal resistance of a system of cylindrical trees of pin fins.
Yang et al. [4] considered an array of cylindrical fins in order to achieve optimal heat transfer. They concluded that when the shape of heat-sink is fixed, there is an optimal diameter of the fins that maximizes the heat transfer. They also revealed that the pressure drop decreases with the diameter of the fins. Bello-Ochende and Bejan [5] and Page et al. [6] studied a cross flow over an array of cylinder fins and obtained the optimal geometry in which the heat transfer rate was maximum. Olakoyejo and Meyer [7] investigated a three dimensional geometry of a set of square cross-sections fins using constructal law. They assumed that the volume of the Fins and their base temperature are constant. They achieved a geometry in which heat transfer from solid to fluid reaches the lowest value.
Goshayeshi and VafaToroghi [8] studied the effect of natural convective heat transfer on a plate with a triangular blade mounted vertically on the plate. Their results showed that the heat transfer decreases at first, then increases and attains an optimum value by increasing the gap between the blades. Goodarzian et al. [9] studied the efficiency of a series of blades with different structures when heat transfer and mass transfer occur simultaneously. Rubio-Jimenez et al. [10] studied the heat transfer from an array of micro-pin fin heat sink with variable density using constructal law.
Salimpour et al. [11] used the constructal theory to optimize the thermal performance of micro-channel heat sinks with noncircular cross sections. They optimized the geometry to minimize the temperature of the micro-channel using three cross sections: ellipsoid, triangle and rectangular ones. Jadhav et al. [12] examined fluid flow and heat transfer on a pair of parallel vertical pin-fins to cool the plates. The main objective of the present study is to determine the optimal configuration of three needled-shape fins. The optimization corresponds to an electronic configuration using the constructal law to achieve the maximum heat transfer subject to constant fin volume.

2 PROBLEM SETUP AND GOVERNING EQUATIONS

Three rows of a multi-scale cylindrical pin–fin are shown in “Fig. 1”.

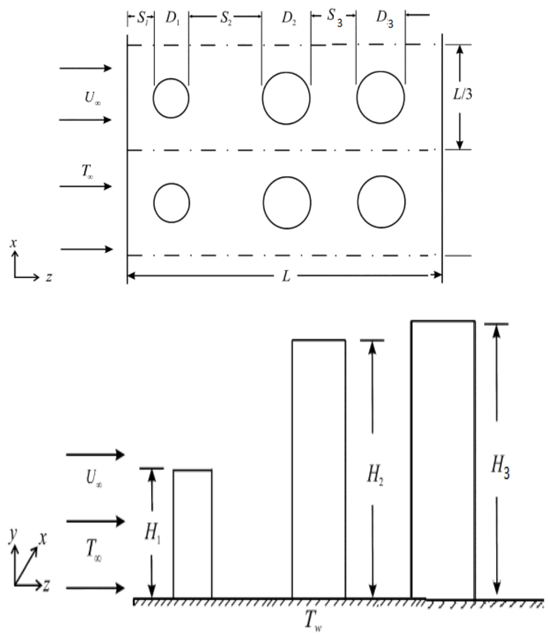


Fig. 1 Three rows unequal pin–fins. left: up view. right: side view.

S_1 is the distance between the first row of fins and the leading edge, s_2 is the distance between the first fin and the second one and s_3 is the distance between the second fin and the third one. The heights of fin 1, fin 2, and fin 3 is H_1 and H_2 and H_3, respectively, while their respective diameters are D_1 and D_2 and D_3. The swept length is L that is assumed to be constant. The fluid flow is uniform and isothermal. Because of the symmetry, a computational domain includes only three fins on a rectangular wall is considered. The objective of the present work is to explore the optimal configuration in which the rate of heat transfer from the solid to the fluid is maximum subject to constant volume of fin V:

$$D_1^2H_1 + D_2^2H_2 + D_3^2H_3 = \frac{4}{\pi}V = \text{Constant.} \quad (1)$$

This constraint is dictated by the weight and the material cost of the fins structure which is a limiting parameter in the design of electronic devices and modern vehicles. In addition, there is an overall size limitation:

$$S_1 + D_1 + S_2 + D_2 + S_3 + D_3 + S_4 = L \quad (2)$$

The configuration has nine variables:
$S_1, S_2, S_3, H_1, H_2, H_3, D_1, D_2$ and D_3. S_1 is assumed to be constant such that the first fin is close to the leading edge. Also, S_2, $\frac{H_2}{H_1}$ and $\frac{D_2}{D_1}$ are constant according to Bejan's investigation [10]. Hence, based on the two

constraints (1) and (2), the number of degrees of freedom is three. The flow is assumed to be steady, laminar, incompressible and two dimensional. All the thermo-physical properties are considered constant. The conservation equations for mass, momentum and energy are [14]:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}}+\frac{\partial \tilde{v}}{\partial \tilde{y}}+\frac{\partial \tilde{w}}{\partial \tilde{z}}=o \quad (3)$$

$$\mathrm{Re}\left(\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}}+\tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}}+\tilde{w}\frac{\partial \tilde{u}}{\partial \tilde{z}}\right)=-\frac{\partial \tilde{P}}{\partial \tilde{x}}+\nabla^2\tilde{u} \quad (4)$$

$$\mathrm{Re}\left(\tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}}+\tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}}+\tilde{w}\frac{\partial \tilde{v}}{\partial \tilde{z}}\right)=-\frac{\partial \tilde{P}}{\partial \tilde{y}}+\nabla^2\tilde{v} \quad (5)$$

$$\mathrm{Re}\left(\tilde{u}\frac{\partial \tilde{w}}{\partial \tilde{x}}+\tilde{v}\frac{\partial \tilde{w}}{\partial \tilde{y}}+\tilde{w}\frac{\partial \tilde{w}}{\partial \tilde{z}}\right)=-\frac{\partial \tilde{P}}{\partial \tilde{z}}+\nabla^2\tilde{w} \quad (6)$$

$$\mathrm{RePr}\left(\tilde{u}\frac{\partial \tilde{T}}{\partial \tilde{x}}+\tilde{v}\frac{\partial \tilde{T}}{\partial \tilde{y}}+\tilde{w}\frac{\partial \tilde{T}}{\partial \tilde{z}}\right)=\nabla^2\tilde{T} \quad (7)$$

For the volume occupied by the cylindrical fins, the energy equation reduces to:

$$\nabla^2\tilde{T}=0 \quad (8)$$

Where, $\nabla^2=\left(\frac{\partial}{\partial \tilde{x}^2}+\frac{\partial}{\partial \tilde{y}^2}+\frac{\partial}{\partial \tilde{z}^2}\right)$. The coordinate system $(\tilde{x},\tilde{y},\tilde{z})$ and velocity components $(\tilde{u},\tilde{v},\tilde{w})$ are defined in Fig. 1. "Eqs. (3-7)" are non-dimensionalized by the following parameters:

$$(\tilde{u},\tilde{v},\tilde{w})=\frac{(u,v,w)}{U_\infty} \text{ و } (\tilde{x},\tilde{y},\tilde{z})=\frac{(x,y,z)}{L} \quad (9)$$

$$\tilde{T}=\frac{T-T_\infty}{T_w-T_\infty} \text{ و } \tilde{P}=\frac{P}{\mu U_\infty/_L} \quad (10)$$

Where, Pr is the Prandtl number $\frac{v}{\alpha}$, and Re is the Reynolds number:

$$\mathrm{Re}=\frac{U_\infty L\rho}{\mu} \quad (11)$$

The boundary conditions are as follows: no-slip and no-penetration boundary conditions are imposed on the fins and the wall surfaces. Also, $\tilde{w}=1, \frac{\partial \tilde{u}}{\partial \tilde{z}}=\frac{\partial \tilde{v}}{\partial \tilde{z}}=0$ are used for the inlet of the flow and $\partial(\tilde{u},\tilde{v},\tilde{w})/\partial \tilde{x}=0$ is employed for the exit. $\partial(\tilde{u},\tilde{v},\tilde{w})/\partial \tilde{y}=0$ is used at the top of the computational domain. In addition, the thermal boundary conditions are: $\tilde{T}=1$ for the walls and $\tilde{T}=0$ for the inlet plane of the computational domain. The symmetry surface is adiabatic ($\frac{\partial T}{\partial n_\Omega}$=0). The continuity of the temperature and flux at the interface of the solid and fluid surfaces requires:

$$-\tilde{k}\,\widetilde{\frac{\partial T_S}{\partial n}}_{\Omega}=-\frac{\partial \tilde{T}}{\partial n_\Omega} \quad (12)$$

Where, $\tilde{k}$ is thermal conductivity ratio k_s /k. The distance between the fins varies. The geometric configuration ($\tilde{D}_2$, $\tilde{D}_3$, $\tilde{H}_2$, $\tilde{H}_3$, $\tilde{S}_3$) should be determined in which the overall heat transfer between the fins and their surrounding flow is maximum. The dimensionless measure of the overall heat transfer is [2]:

$$\tilde{q}=\frac{q/L}{K(T_w-T_\infty)} \quad (13)$$

Where q is the total heat transfer rate integrated over the surface of the fins and the elemental surfaces.

3 NUMERICAL METHOD

Eqs. (3-8) are solved using a finite-volume method. The domain is discretized using polyhedral elements and the governing equations are integrated over each control volume. Second order upwind scheme is used for the diffusion terms. The pressure–velocity coupling is performed with the SIMPLE algorithm. Convergence is obtained when the normalized residuals for the mass and momentum and energy equations are smaller than 10^{-6}. The length of the computational domain is chosen long enough to ensure that the boundaries do not affect the results. It is found that the doubling of the length results in 1% difference in the total heat transfer rate (based on Eq. 13).
Grid independence test is carried out for all the fin arrangements. The grid study shows that for a control volume with a mesh size of 0.009 per unit length in the x-direction, 0.003 per unit length in the y-direction, and 0.003 per unit length in the z-direction assured a grid independent solution in which the maximum changes in total heat transfer rate are less than 1% when the mesh per unit length is doubled sequentially. The meshes in the vicinity of the optimal configurations are more refined to capture the fluid flow in this region.

4 RESULTS

The fin flow structure has three degrees of freedom which are designated as $\frac{H_3}{H_1}$, $\frac{D_3}{D_1}$ and $\tilde{S}_3$. The simulations are started by fixing the distance between the first fin and the leading edge, $\tilde{S}_1=0.05$, the distance between the second fin and the third one, $\tilde{S}_2=0.01$, $\frac{H_2}{H_1}=0.9$, $\frac{D_2}{D_1}=1.1$. The total volume of the fins is $\tilde{v}=0.01$. The degrees of freedom are the ratios of the fin's height, $\frac{H_3}{H_1}$, the diameter ratio $\frac{D_3}{D_1}$, and the spacing between the cylinders, $\tilde{S}_3$. The dimensions of the flow structure are set as follows: the non-dimensional length $\tilde{L}$ is equal to 1, the flow width (the distance between two symmetry planes in the x-direction) is $\tilde{L}/3$ (Fig. 1). For the first simulations, Re = 50, $\tilde{k}$ = 100, $\frac{D_3}{D_1}$ = 1, and $\tilde{S}_3$ = 0.05. The fin height ratio varies. An optimal height ratio is found for this configuration. This procedure is repeated for other $\tilde{S}_3$ values in the range $0.05 \leq \tilde{S}_3 \leq 0.2$, as shown in Fig. 2, once an overall optimum is found for this configuration, i.e. the optimal height ratio $\frac{H_3}{H_1}$ that corresponds with the maximum total heat transfer rate.

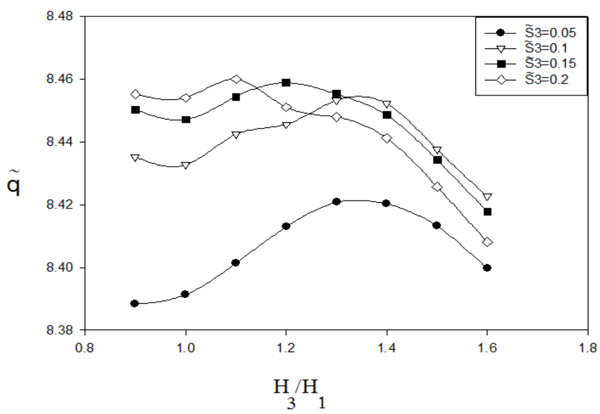


Fig. 2 Total heat transfer rate as a function of the distance between the two-second diameter fins and the height ratio for the diameter ratio of 1.

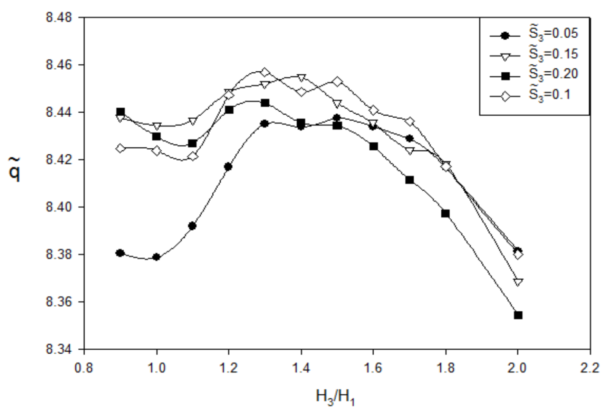


Fig. 3 Total heat transfer rate versus the distance between the two-second diameter fins and the height ratio for the diameter ratio of 1.1.

Now, for the diameter ratio, $\frac{D_3}{D_1}$ = 1.1, the simulations are performed in the range $0.05 \leq \tilde{S}_3 \leq 0.2$. The results are shown in Fig. 3. A new optimal configuration is found with the corresponding maximum total heat transfer rate. Fig. 4 shows the behavior of the optimal configuration for $\frac{D_3}{D_1}$ = 1.2. Figs. 2, 3 and 4 show that as the $\frac{D_3}{D_1}$ increases, the optimal value of $\frac{H_3}{H_1}$ decreases. The optimal pin–fin configuration for Re = 50 and $\tilde{k}$ = 100 lies between $0.05 \leq \tilde{S}_3 \leq 0.2$, $1 \leq \frac{D_3}{D_1} \leq 1.2$ and $0.85 \leq \frac{H_3}{H_1} \leq 2$.

Fig. 5 summarizes the optimal results and shows the effect of the diameter ratio on maximum total heat transfer rate for the range of parameters given. The optimal value of $\frac{D_3}{D_1}$ is approximately 1.1, but at this optimal diameter ratio, the effect of $\frac{D_3}{D_1}$ is negligible.

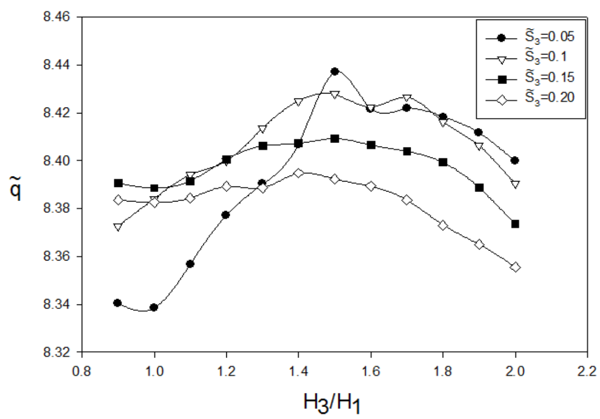


Fig. 4 Total heat transfer rate versus the distance between two-second fins and the height ratio for the diameter ratio of 1.2.

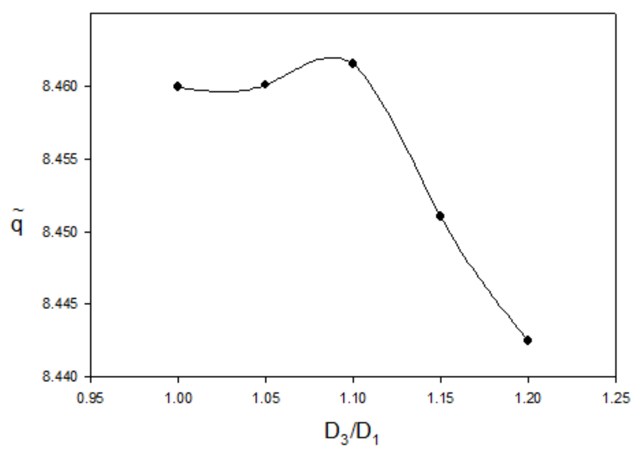


Fig. 5 Total heat transfer rate versus the distance between the two-second diameter fins, the height ratio and the diameter ratio.

The optimization procedure is performed for higher Reynolds numbers, Re=100 and Re = 200 (Fig. 6). In this range of Reynolds numbers, $(\frac{H_3}{H_1})_{opt}$ is equal to 1.3 and indicates that the height of the third row of fins should be slightly higher than that of the first one. This is a worth noting result that establishes that the diameters of the fins should not be uniform. The results demonstrate that maximum total heat transfer rate increases with the Reynolds number.

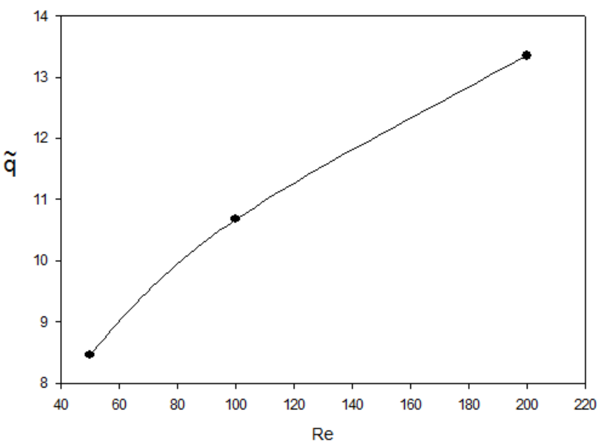


Fig. 6 The effect of Reynolds number on the optimal fin configuration.

The effect of thermal conductivity ratio $\tilde{k}$ is also investigated. The results are presented in Fig. 7 for the range of $30 \leq \tilde{K} \leq 300$ at Re = 100. The figure shows that heat transfer increases with increasing heat transfer coefficient.

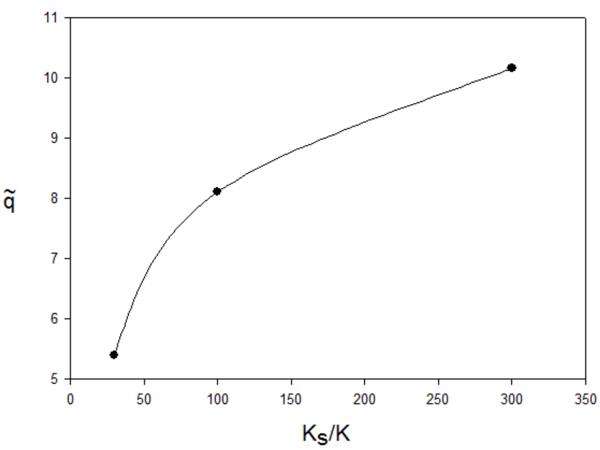
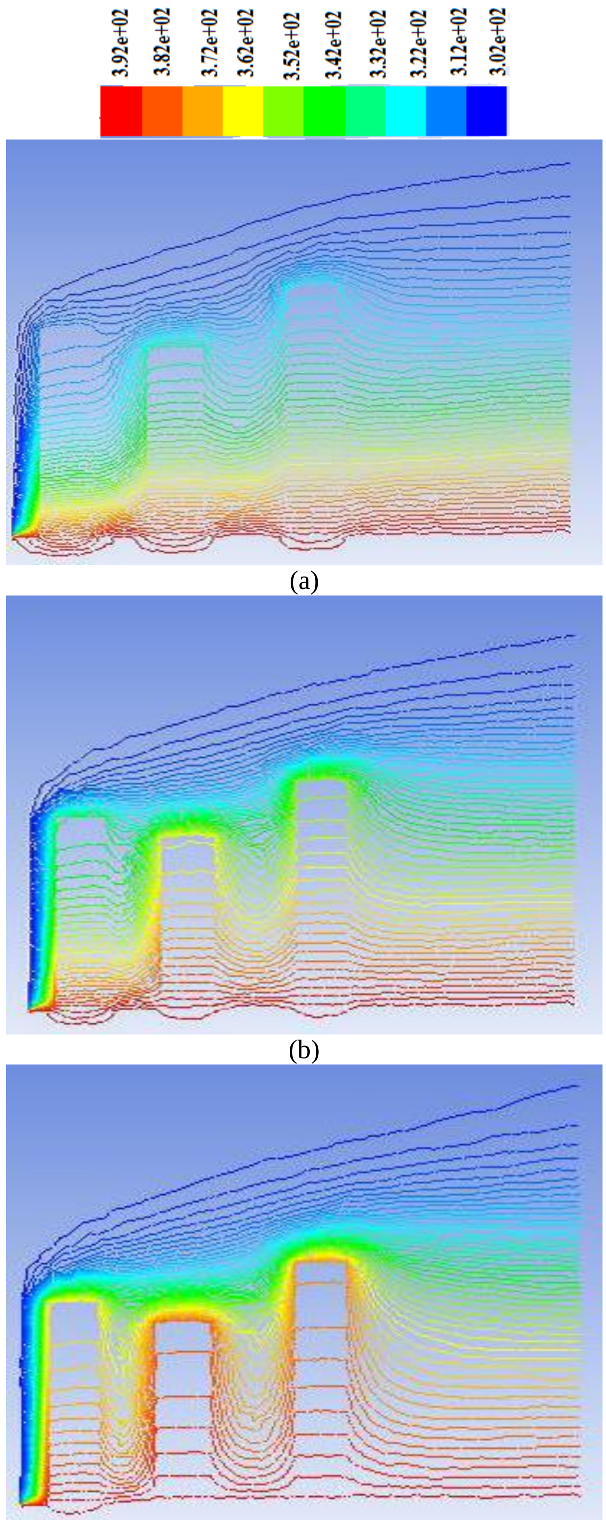


Fig. 7 Heat transfer versus thermal conductivity coefficient.

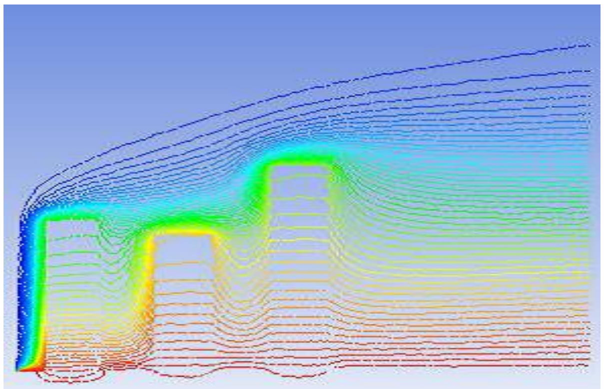
The effect of the thermal conductivity ratio is displayed in Fig. 8. As the dimensionless thermal conductivity ratio increases, the blue color of the temperatures profiles penetrates between the fins. The isothermal contours become denser at lower thermal conductivity ratios indicating the higher rate of heat transfer.



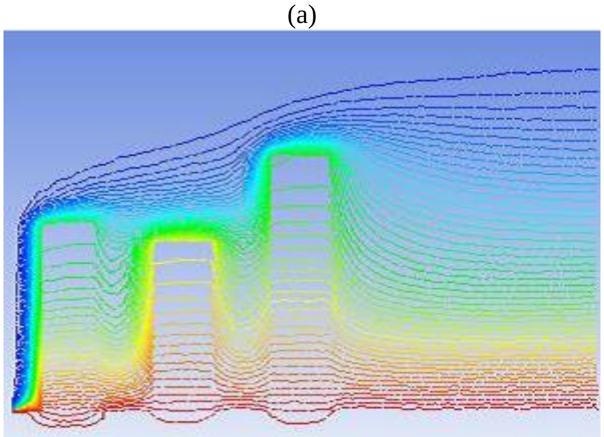
(c)

Fig. 8 Effects of dimensionless conductivity ratio on the temperature fields for the optimal configuration: (a): $\tilde{k} = 30$, (b): $\tilde{k} = 100$ and (c): $\tilde{k} = 300$.

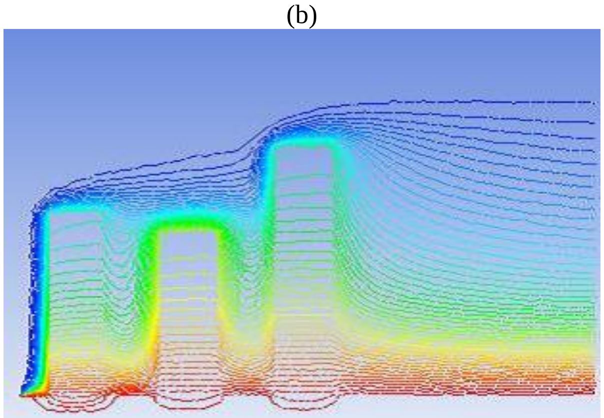
Fig. 9 shows the temperature distribution in the middle cross-section of the pin–fin for different Reynolds numbers and different dimensionless thermal conductivity ratios. All the temperature profiles are presented for Pr = 0.71. The temperature profiles change with the Reynolds number. As the Reynolds number increases, the blue color of the temperatures profiles penetrates the fin structure, and this can be attributed to the increase in the flow strength and the flattening of the thermal boundary layers.



(a)



(b)



(c)

Fig. 9 Effects of Reynolds number on the temperature fields for the optimal configuration, (a) Re = 50, (b) Re = 100, (c) Re = 200.

5 CONCLUSIONS

In the present paper, the constructal theory is employed to introduce a new type of needle-shaped fins arranged in three rows. The pin–fins are cooled by laminar forced convection flow. The heat transfer is optimized subject to constant fin volume. Numerical optimization was performed to determine the optimal configuration (relative diameter, height and spacing between fins). The results demonstrated that maximum total heat transfer rate increases with the Reynolds number. The heat transfer increases with the ratio of thermal conductivity. The results showed that pin–fins flow structure leads to the best performance when the pin–fin diameters and heights are non-uniform.

6 NOMENCLATURE

D_1	The diameter of the first fin, m
D_2	The diameter of the second fin, m
D_3	The diameter of the third fin, m
h	Heat transfer coefficient, m
H_1	The height of the first fin, m
H_2	The height of the second fin, m
H_3	The height of the third fin, m
K	Thermal conductivity, W/mK
L	The swept axial length, m
P	Pressure, Pa
Pr	Prandtl number
q	Total heat transfer rate, W
$\tilde{q}$	Dimensionless heat transfer density, Eq. (13)
Re	Reynolds number
S_1	The spacing between the first cylinder and the trailing edge, m
S_2	The spacing between the D1 and D2 cylinders, m
S_3	The spacing between the D2 and D3 cylinders, m
S_4	The spacing between the third cylinder and the end of plate, m
T	Temperature, K
T_W	Wall temperature, K
T_1	Free-stream temperature
u, v, w	Velocity components, m/s
U_1	Free-stream velocity, m/s
V	Total volume of the fins
x, y, z	Cartesian coordinates, m
Greek symbols	
α	Thermal diffusivity, m^2/s
μ	Dynamic viscosity, Pa.s
v	kinematic viscosity, m^2/s
ρ	Density, kg/m^3

Subscripts

1	Leading row
2	Trailing row
max	Maximum
Opt	Optimum
S	Solid
w	Wall

Superscripts

~	Dimensionless variables

REFERENCES

[1] Incropera, F. P., DeWitt D. P. Introduction to Heat Transfer. J. Wiley & Sons, 1990.

[2] Almogbel, M., Bejan, A., Cylindrical Trees of Pin Fins, International Journal of Heat and Mass Transfer, Vol. 43, No. 23, 2000, pp. 4285-4297.

[3] Bello-Ochende, T., Meyer, J. P., and Bejan, A., Constructal Multi-Scale Pin–Fins, International Journal of Heat and Mass Transfer, Vol. 53, No. 13, 2010, pp. 2773-2779.

[4] Yang, A., Chen, L., Xie, Z., Feng, H., and Sun, F., Constructal Heat Transfer Rate Maximization for Cylindrical Pin-Fin Heat Sinks, Applied Thermal Engineering, Vol. 108, 2016, pp. 427-425.

[5] Bello-Ochende, T., Bejan, A., Constructal Multi-Scale Cylinders in Cross-Flow, International Journal of Heat and Mass Transfer, Vol. 48, No. 7, 2005, pp. 1373-1383.

[6] Page, L. G., Bello-Ochende, T., and Meyer, J. P., Constructal Multi Scale Cylinders with Rotation Cooled by Natural Convection, International Journal of Heat and Mass Transfer, Vol. 57, No. 1, 2013, pp. 345-355.

[7] Olakoyejo, O. T., Meyer, J. P., Numerical Optimization of Square Pin-Fins for Minimum Thermal Resistance with Non-Uniform Design Dimensions, International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics, 14–16 July, 2014.

[8] Goshayeshi, H. R., Vafa Toroghi, R., An Experimental Investigation of Heat Transfer of Free Convection on Triangular Fins in Order to Optimize the Arrangement of Fins, International Journal of Science, Technology and Society, Vol. 2, No. 5, 2014, pp. 152-160.

[9] Goodarzian, H., Sahebi, S. A., Shobi, M. O., and Safaee, J., A Collocation Solution on the Optimization of Straight Fin with Combined Heat and Mass Transfer, International Journal of Physical Sciences, Vol. 6, No. 9, 2011, pp. 2268-2275.

[10] Rubio-Jimenez, C. A., Kandlikar, S. G., and Hernandez-Guerrero, A., Numerical Analysis of Novel Micro Pin Fin Heat Sink with Variable Fin Density, IEEE Transactions on Components, Packaging and Manufacturing Technology, Vol. 2, No.5, 2012, pp. 825-833.

[11] Salimpour, M. R., Sharifhasan, M., and Shirani, E., Constructal Optimization of Microchannel Heat Sinks with Noncircular Cross Sections, Heat Transfer Engineering, Vol. 34, No. 10, 2013, pp. 863-874.

[12] Jadhav, R. S., Balaji, C., Fluid Flow and Heat Transfer Characteristics of a Vertical Channel with Detached Pin-Fin Arrays Arranged in Staggered Manner on Two Opposite Endwalls, International Journal of Thermal Sciences, Vol. 105, 2016, pp. 57-74.