# Dynamic Modelling and Control of a Dielectric Elastomer Actuator with Two Degrees of Freedom

## Yaser Hesari
Department of Mechanical Engineering,
Science and Research Branch, Islamic Azad University, Tehran, Iran.
E-mail: yhesary2006@gmail.com

## Shahram Etemadi Haghighi*
Department of Mechanical Engineering,
Science and Research Branch, Islamic Azad University, Tehran, Iran.
E-mail: setemadi@srbiau.ac.ir
*Corresponding author

**Abstract:** Dielectric elastomer actuators are capable of creating multi degrees of freedom in a single joint. In this paper, a double-cone dielectric elastomer actuator is assumed as a planar joint with two degrees of freedom. Because of theoretical complexities, mathematical formulation of dynamic equations is too complicated. To obtain the dynamic equations of motion, at first, experimental charts are used. At this stage forms of relations between displacements, voltages, forces and moments are proposed, and coefficients are optimized to keep the difference between experimental and estimated charts in minimum. Then dynamic equations of motion are derived based on Newton-Euler method, and state-space form of equations of the joint are obtained. As a second objective, joint stabilization around working point is considered. To stabilize the joint against external loads, or initial dislocations, a regulator controller is designed. The joint is over actuated. So using constraint equations, control rule is extracted and simulated. Simulations show successful performance around the working point.

**Keywords:** Dielectric elastomer actuator, Electro active cone membrane actuator, Lyapunov controller

**Biographical notes: Y. Hesari** received his BSc and MSc in Mechanical Engineering respectively from Shahid Rajaie and Islamic Azad University, Science and Research Branch, in 2017. His research interests include Manufacturing of elastomer dielectric actuator and control. **S. Etemadi Haghighi** received his PhD in Mechanical Engineering from Sharif University of Technology in 2011. He is currently Assistant Professor at the Department of Mechanical and Aerospace Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran. His current research interest includes vibration and control**.**

## 1    INTRODUCTION

In traditional robotics, each joint has one degree of freedom (DOF) but the dielectric elastomer actuators (DEA) can create multiple degrees of freedom (Multi-DOF) in a single joint. This is advantageous because mechanisms can be made in more compact form. Based on the application of dielectric elastomer (DE) in sensor design, a new self-sensing method without using any additional sensing device is introduced by Jung et. al. [1].

A 3-DOF DEA is also designed by Koo et. al. [2] with a rod in the middle. He et. al. discussed the problem of instability in DE and created an optimum electromechanical transducer mechanism [3]. He et. al. also studied the static behavior of DE membrane, using large deformations field theory [4]. Cone DEA mechanism is first introduced by Luan et. al. [5]. They studied electric field, thickness, and stress to find out the best pretension in membrane. Conn and Rossiter proposed a double cone DEA with 5-DOF, fabricated from VHB 4910 acrylic elastomer [6]. Performance of the true electric field DEAs with Numerical method has been addressed by He et. al. [7]. They determined the effect of pre stretch on actuator displacement. Conn et. al. experimentally obtained the relations between voltage, displacement and rotation of the double cone DEA [8].

Branz et. al. used the finite element method to estimate the maximum torque of the actuator in low frequency excitation [9]. They also studied the kinematics and control of a double joint four degree of freedom robotic arm [10]. Branz and Francesconi dynamically modelled and controlled a double cone DEA and checked the performance on a FEM model [11]. Wang et. al. introduced a cubic six degree of freedom DEA for stabilizing camera and similar apparatus [12]. Every membrane in this mechanism is capable of exciting two degrees of freedom. Nguyen et. al. fabricated a six legged robot using DEAs which was capable of moving with the velocity of 3cm/s [13].

DEAs can be easily fabricated as circular membranes, but in order to determine the force-displacement relations, experimental data is necessary, because analytical study in this field usually leads to very complicated problems [14]. In this paper we use experimental data as well. The data is used to propose and optimize force-displacement-voltage relations. As an innovation, based on these relations, dynamic governing equations of the double cone DEA are derived. Finally, in order to stabilize the actuator around the working point, a Lyapunov controller is designed. Simulations of the performance of closed loop system are added at the end of the paper.

## 2    MATERIAL AND METHOD

The joint has a double-conical membrane actuator. The geometric characteristics of the actuator related to the dimensions is expressed in "Table 1" as shown in "Fig. 1". The rigid components used in the actuator comprise three members, two rings, central rod and an elastomer membrane. The ring is fabricated from acrylic and formed with laser cut. The rod is made of 6.6 nylon with rounded ends used to minimize stress at the elastomer interface.
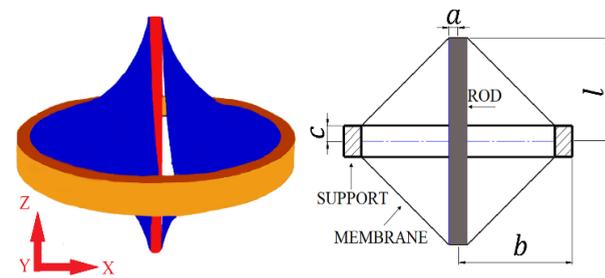


**Fig. 1**    Schematic diagrams of cone membrane actuator.

**Table 1** Geometric parameters of membrane actuator

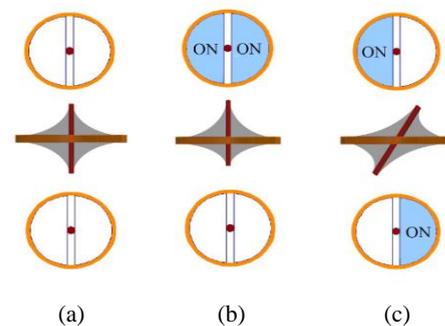| Parameters | Values | Unit |
|---|---|---|
| m=mass | $6.3 \times 10^{-3}$ | kg |
| I=polar moment of inertia | $36.6 \times 10^{-3}$ | kg/mm² |
| a=radius central rod | 4.5 | mm |
| l=half central rod length | 24.75 | mm |
| c=thick ring | 4 | mm |
| b=inner radius ring | 30 | mm |
| outer radiusring | 65 | mm |



**Fig. 2**    Schematic of the elastomer cone membrane actuator:( a): actuator is in equilibrium, (b): z-axis transition and (c): rotation about x, y-axes.

The film used in the elastomer membrane is made of DE with 3MVBH4905/4910 characteristic. There are four elastomer membrane in the joint mechanism, two are placed in the top and the other two are placed underneath, symmetrically.

Copper electrode elements by coated carbon grease are used to actuate elastomers [8].The actuation algorithms are shown in "Fig. 2". By proper actuation, axial and rotational motions of the joint can be obtained.

## 3    MODELLING OF CONE MEMBRANE ACTUATOR

The dynamical equations of elastomeric actuators with simpler geometries have already been extracted, but in the case of cone membrane actuator presented in this paper, due to the excessive complicacy in theoretical analysis, experimental diagrams need to be used. Based on the plotted results in "Fig. 3, Fig. 4 and Fig. 5", the relationships between force-voltage-displacement, as well as the moment-voltage-rotation, are extracted. "Fig. 3a" shows the relation between voltage and moment and "Fig. 3b" indicates relation between voltage and force, where zero rotation and displacement are assumed. [8]
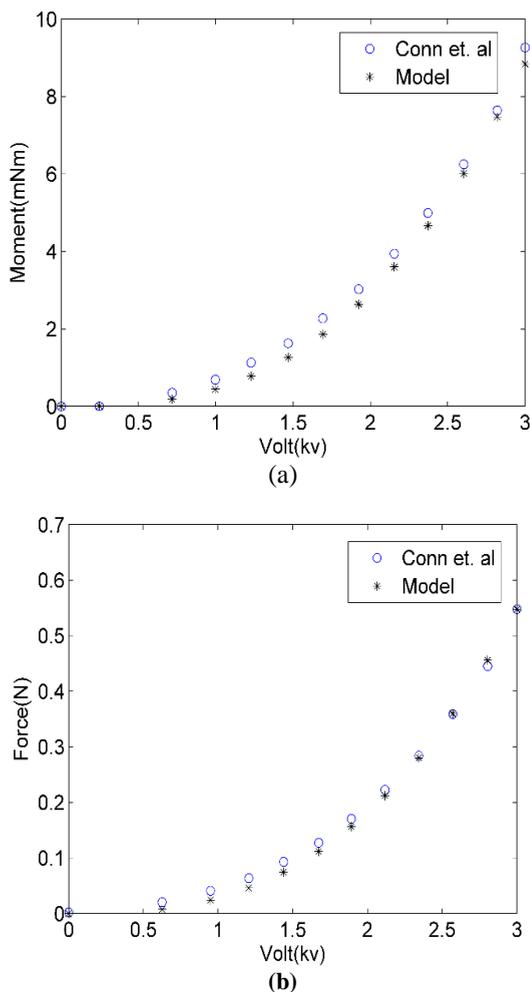
(a)

(b)

**Fig. 3**    Experimental and estimated diagrams: (a): Moment –Voltage and (b): Force-Voltage.

Analogously "Fig. 4a" describes the relation between voltage and rotation and "Fig. 4b" indicates the relation between voltage and displacement in z-axis, where moment and force are assumed zero [8]. The behavior of the experimental data can be assumed exponential.
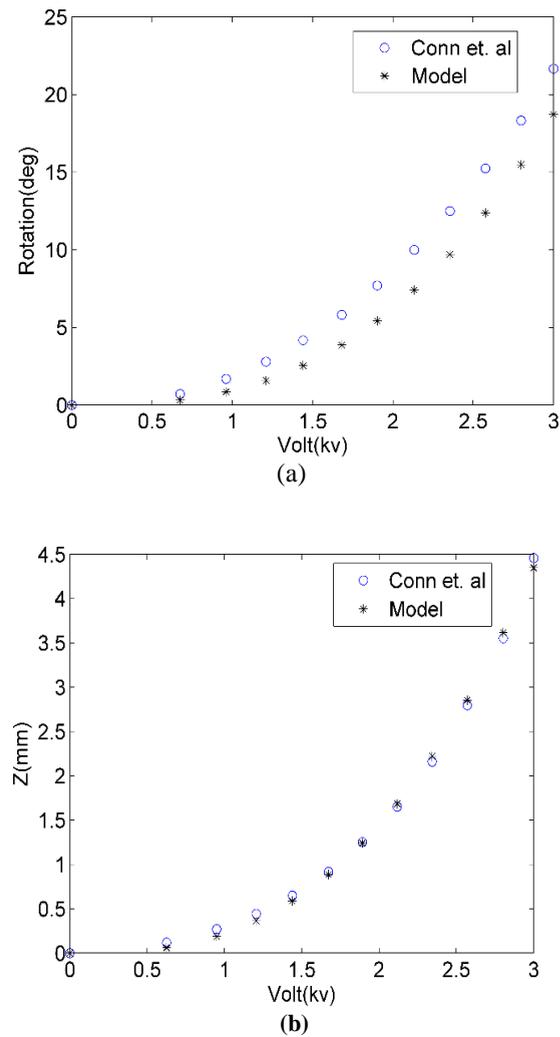
(a)

**(b)**

**Fig. 4**    Experimental and estimated diagrams: (a): Rotational Displacement-Voltage and (b): Linear Displacement-Voltage.

"Fig. 5a" describes the relation between voltage and rotation and "Fig. 4b" indicates the relation between voltage and displacement in z-axis, where moment and force are assumed zero [8]. Linear behavior in both diagrams are clearly observable. Note that "Fig. 3, Fig. 4 and Fig. 5", express the static behavior of the actuator [8]. For dynamical modeling, Newton-Euler rules can be used.
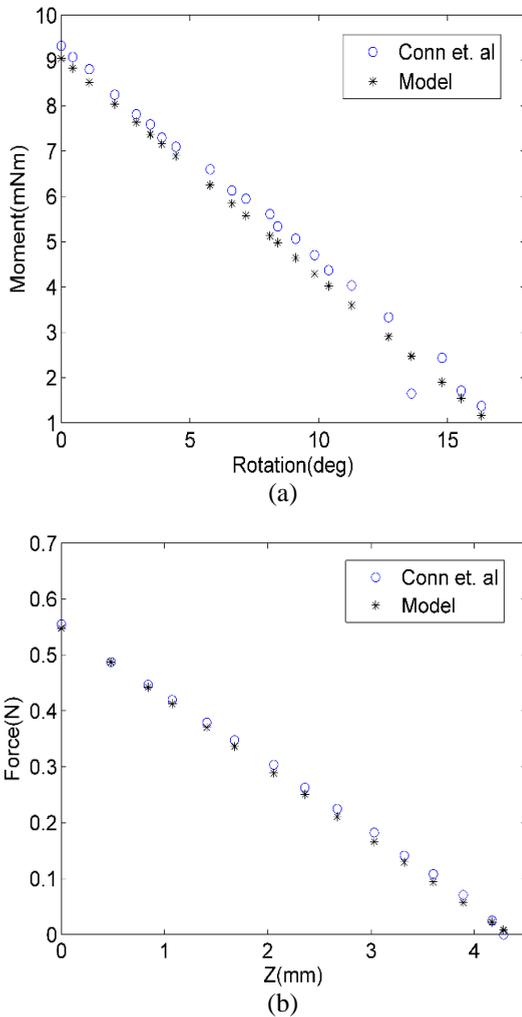
**Fig. 5** Experimental and estimated diagrams: (a): Moment-Rotational Displacement and (b): Force-Linear Displacement.

## 4    DYNAMIC EQUATION

There are four pieces of elastomer actuators that are symmetrically installed in the joint. It can be assumed that the elastomer materials used in the actuator are uniform. So, the joint has the ability to move along the linear z-axis and rotate about x, y axes. Based on the actuator free body diagram as shown in "Fig. 6", and by applying the forces $F_1$, $F_2$, $F_3$, $F_4$ and moments $M_1$, $M_2$, $M_3$ and $M_4$, respectively for displacement and rotation, Newton Euler method is used. The actuator consists of four elements and every force and moment is related to one of the elastomer elements. For example, if the joint must move linearly then forces $F_1$ and $F_2$ need to be equal and forces $F_3$ and $F_4$ can be zero. The same approach can also be used on the rotation. For positive rotation without

displacement, $M_1$ and $M_4$ must be equal and $M_2$ and $M_3$ might have zero value.
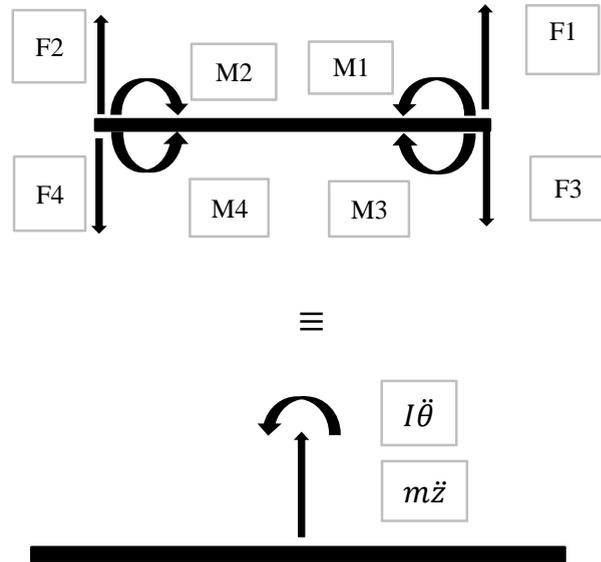


**Fig. 6**    Free body diagram of the joint with two degrees of freedom.

From "Fig.5b" for V=0: $F_Z = 0.555 - 0.126Z$ which is also noted in [8]. From the free body diagram (FBD) in "Fig. 6" by considering the geometric symmetry and employing the "Fig. 3b, Fig. 4b and Fig. 5b", the general form of actuator force is guessed as:

$$F_z = \left(\alpha V_1^n - \beta Z - c_1 \dot{Z}\right) + \left(\alpha V_2^n - \beta Z - c_1 \dot{Z}\right) + \left(-\alpha V_3^n - \beta Z - c_1 \dot{Z}\right) + \left(-\alpha V_4^n - \beta Z - c_1 \dot{Z}\right) + F \quad (1)$$

$F_Z$: Total force applied by elastomer in z direction (N)
$F$:  External excitation force (N)
$V_i$: Applied voltage to each elastomeric parts (kV) (i = 1, 2, 3, 4)
$\alpha, \beta, c_1$, n: Coefficients of the force equation units.
Assuming that $V_3 = V_4 = 0$ in "Eq. (1)", and from "Fig. 5b", the value of the coefficient $\beta$ can be obtained as: $4\beta = 0.126$.
In "Fig. 4b", $F_z$, F and $\dot{Z}$ are assumed equal to zero. The relation between voltage and displacement corresponding to "Fig. 4b" can be written as follows.

$$\begin{cases} F_z = 0 \\ \dot{Z} = 0 \\ V_3 = V_4 = 0 \\ V_1 = V_2 = V \end{cases} \implies 0 = 2\,\alpha\,V^n - 4\beta Z \implies Z = (\alpha\,/2\beta)\,V^n \quad (2)$$

Using the same method, a similar equation can be obtained which corresponds to "Fig. 3b". For Z=0, $V_3$=$V_4$=0 and $\dot{Z}$=0, we have:

$$F_z = 2\,\alpha\,V^n \tag{3}$$

From "Fig. 5a" for V=0: M=9.313– 0.483θ. From the FBD in "Fig. 6" by considering the geometric symmetry and employing the "Fig. 3a, Fig. 4a and Fig. 5a", the general form of actuator moment is guessed as:

$$M= \left(\gamma V_1^n - \mu\theta - c_2\dot{\theta}\right) + \left(\gamma V_4^n - \mu\theta - c_2\dot{\theta}\right) + \left(-\gamma V_2^n - \mu\theta - c_2\dot{\theta}\right) + \left(-\gamma V_3^n - \mu\theta - c_2\dot{\theta}\right) + Me \tag{4}$$

$V_i$: Applied voltage (kV) (i = 1, 2, 3, 4)
M: Moment applied by the joint (mNm)
$M_e$: External moment (mNm)
$c_2, \gamma, n, \mu$= Are the coefficients related to the moment equation.
According to the "Fig. 5a", and by assuming $V_3$=$V_4$=0, we have 4μ=0.483.

$$\left[ \begin{array}{l} M= 0 \\ \dot{\theta} = 0 \\ V_3 = V_4 = V \quad \Rightarrow \quad 0 = 2\gamma V^n - 4\mu\theta \quad \Rightarrow \quad \theta = (\gamma/2\mu)\,V^n \\ V_1 = V_2 = 0 \end{array} \right. \tag{5}$$

Using the same method, a similar equation can be obtained which corresponds to "Fig. 3a". For θ =0, $V_3$=$V_2$=0 and $\dot{\theta}$=0, we have:

$$M = 2\gamma V^n \tag{6}$$

The proposed "Eq. (1) and Eq. (4)", seem to be capable of producing all experimental diagrams with approvable tolerance. To obtain the coefficients γ, n and α "Fig. 3a, Fig. 3b, Fig. 4a and Fig. 4b" are used along with least squares method and values are reported in "Table 2".

**Table 2** The extracted coefficients through the power method

| Method | Power | Unit |
|--------|-------|------|
| $\alpha$ | 0.013785 | N/kV |
| $\beta$ | 0.0315 | N/mm |
| $\gamma$ | 0.2275 | mNm/kV |
| $\mu$ | 0.12075 | mNm/deg |
| $n$ | 2.721 | dimensionless |

To graphically demonstrate the results, experimental diagrams in figures (3), (4) and (5) are regenerated by the data from "Eq. (1) and Eq. (4)". The error is always less than five percent and seems an acceptable estimation.

By assuming a linear dynamic system around the equilibrium point z=0, and knowing that linear inertia of the joint equals 3.6g, and by using $F_z = 0.555 - 0.126z$, the natural frequency of the system $\omega_n$=5.916 (Rad/sec) is calculated. In other words, from Newton's second law, we have:

$$\sum F_z = m\ddot{Z} \tag{7}$$

$$m\ddot{Z} + 4c_1\dot{Z} + 4\beta Z = \alpha(V_1^n + V_2^n - V_3^n - V_4^n) + F \tag{8}$$

By substituting values as $\alpha(V_1^n + V_2^n) = 0.555$, $V_3$=$V_4$=0 and F=0 we have:

$$(3.6 \times 10^{-3})\ddot{Z} + 4c_1\dot{Z} + 0.126Z = 0.555 \tag{9}$$

Referring to conn et al, the amount of damping force is $4c_1$=0.640 [8].
By the same method and from the "Eq. (4) and Fig. 6", the joint rotational dynamic can be obtained as follows:

$$\sum M = I\ddot{\theta} \tag{10}$$

$$I\ddot{\theta} + 4c_2\dot{\theta} + 4\mu\theta = \gamma(V_1^n + V_4^n - V_2^n - V_3^n) + Me \tag{11}$$

By substituting values as $\gamma(V_1^n + V_4^n) = 9.313$, $V_2^n = V_3^n = 0$ and Me=0 we have:

$$(36.6 \times 10^{-3})\ddot{\theta} + 4c_2\dot{\theta} + 0.483\theta = 9.313 \tag{12}$$

Referring to the Branz [11], the amount of damping moment equation in the range of 0.1 to 10 hertz is calculated as $4c_2$=6.49.
In the space state form, dynamic equations of motion can be expressed as follows. Note that the state variables are defined as:

$$
\begin{aligned}
z &= x_1 \\
\dot{z} &= \dot{x}_1 = x_2 \\
\theta &= x_3 \\
\dot{\theta} &= \dot{x}_3 = x_4 \\
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \alpha/m\,(V_1^n + V_2^n - V_3^n - V_4^n) - ((4 \times \beta)/m)(x_1) \\
&\quad + F/m \\
&\quad - ((4 \times c_1)/m)(x_2) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \gamma/I\,(V_1^n + V_2^n - V_3^n - V_4^n) - \\
&\quad ((4 \times \mu)/I)(x_3) + Me/I - \\
&\quad ((4 \times c_2)/I)(x_4)
\end{aligned}
\tag{13}
$$

## 5   REGULATOR DESIGN

In order to stabilize the system against minor external incitements about the equilibrium point, a regulator

controller must be designed. For V=0, actuators produce no displacement. The equilibrium point must be located in the middle of actuator course. To do so, a bias voltage of V = 2.35 (kV) must be applied and control output must be added to this value.

Lyapunov approach is used here and the Lyapunov candidate function and its derivative are chosen according to "Eq. (14) and Eq. (15)":

$$S = 1/2 \, (x_{1+}^2 x_2^2 + x_{3+}^2 x_4^2) \tag{14}$$

$$\dot{S} = (x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 + x_4\dot{x}_4) = -\eta_1 x_2^2 - \eta_2 x_4^2 \tag{15}$$

$S$ is a scalar positive definite function and $\dot{S}$ is negative semi definite. Note that according to the definition, $\dot{S} < 0$ can only happen near the system equilibrium point in the origin, which means local asymptotic stability of the origin in the work space. Assuming $u_i = v_i^n$ and by employing the "Eq. (13) and Eq. (15)" we have:

$$\begin{aligned}
\dot{S} = x_1 x_2 &+ x_2((\alpha/m)(u_1 + u_2 - u_3 - u_4)) \\
&- ((4 \times \beta)/m)\, x_1 + F/m \\
&- ((4 \times c_1)/m)\, x_2) + x_3 x_4 \\
&+ x_4(\gamma/I\,(u_1 + u_4 - u_2 - u_3) \\
&- ((4 \times \mu)/I)\, x_3 + Me/I \\
&- ((4 \times c_2)/I)\, x_4)
\end{aligned} \tag{16}$$

$$\begin{aligned}
x_2 &\binom{\alpha/m\,(u_1 + u_2 - u_3 - u_4) - ((4 \times c_1)/m)\, x_2}{+x_1(1 - (4 \times \beta)/m) + F/m} \\
+x_4 &\binom{\gamma/I\,(u_1 - u_2 - u_3 + u_4)}{-((4 \times c_2)/I)x_4 + x_3(1 - (4 \times \mu)/I) + Me/I} \\
&= -\eta_1 x_2^2 - \eta_2 x_4^2
\end{aligned} \tag{17}$$

The above relation can be rewritten as:

$$\begin{aligned}
x_2 &\binom{-\eta_1 x_2 - ((4 \times c_1)/m)\, x_2 +}{x_1(1 - (4 \times \beta)/m) + F/m + \alpha/m\,(u_1 + u_2 - u_3 - u_4)} \\
+x_4 &\binom{-\eta_2 x_4 - ((4 \times c_2)/I)x_4 +}{x_3(1 - (4 \times \mu)/I) + Me/I + \gamma/I\,(u_1 - u_2 - u_3 + u_4)} \\
&= 0
\end{aligned} \tag{18}$$

As a solution, it can be assumed that both parentheses are zero so that:

$$\begin{cases} u_1 + u_2 - u_3 - u_4 = A \\ u_1 + u_4 - u_2 - u_3 = B \end{cases} \tag{19}$$

Where:

$$\begin{cases}
A = ((-\eta_1 + (4 \times c_1)/m)x_2 - x_1(1 - (4 \times \beta)/m) \\
\qquad - F/m)\, m/\alpha \\
B = ((-\eta_2 + (4 \times c_2)/I)x_4 - x_3(1 - (4 \times \mu)/I) \\
\qquad - Me/I)\, I/\gamma
\end{cases} \tag{20}$$

Solving algebraic system as:

$$\begin{cases} u_1 = -u_3 = 1/4\,(A + B) \\ u_2 = -u_4 = 1/4\,(A - B) \end{cases} \tag{21}$$

Assuming that $u_1 = -u_3$ and $u_2 = -u_4$, the control rules for each of four pieces of elastomer around the midpoint of the actuator length is obtained as follows:

$$\begin{aligned}
&u_1 = -u_3 = 1/4\,(A + B) \tag{22} \\
&= 1/4\,((-\eta_1 + (4 \times c_1)/m)x_2\,(m/\alpha) + (-\eta_2 \\
&+ (4 \times c_2)/I)x_4\,(I/\gamma) - (x_1(1 - (4 \times \beta)/m)(m/\alpha)) \\
&- (x_3(1 - (4 \times \mu)/I)\,(I/\gamma)) - F/\alpha - Me/\gamma) \\
&u_2 = -u_4 = 1/4\,(A - B) = 1/4\,((-\eta_1 + \\
&(4 \times c_1)/m)x_2\,(m/\alpha) - (-\eta_2 + \\
&(4 \times c_2)/I)x_4\,(I/\gamma) - (x_1(1 - (4 \times \beta)/m)(m/\alpha)) + \\
&(x_3(1 - (4 \times \mu)/I)\,(I/\gamma)) - F/\alpha + Me/\gamma)
\end{aligned}$$

By this control law, the origin is the only stable point in the workspace, and convergence to the origin is guaranteed. The speed of convergence is controllable by adjusting the coefficients $\eta_1$ and $\eta_2$.

## 6   SIMULATION

System performance against external excitation is evaluated by simulation. External excitations are in the form of force and torque. At first, the system is introduced to a 10 N force and an external moment with value of 1 mNm. "Fig. 7 and Fig. 8" show the reactions of the system. S in "Fig. 7" is always decreasing as we expected from the definition of the Lyapunov function. In "Fig. 8" by increasing the amount of controller coefficient, the system returns to equilibrium faster and shows less oscillations which is a consequence of increasing closed loop damping.
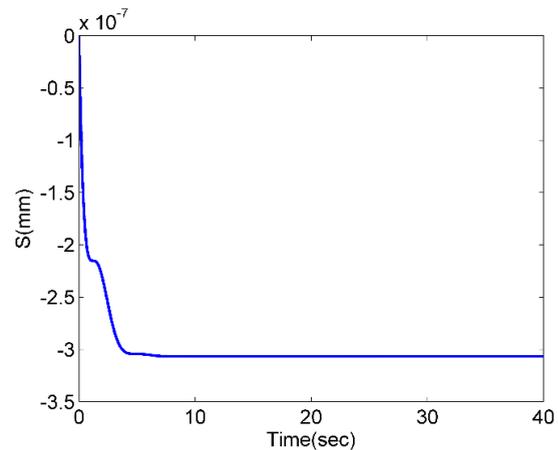


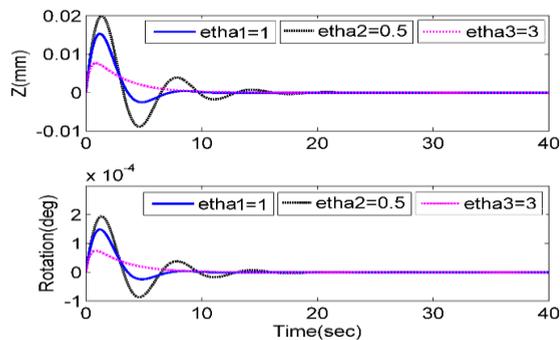**Fig. 7**    Changes of Lyapunov candidate function versus time.

**Fig. 8**     Response of the closed circuit system against external force and moment.
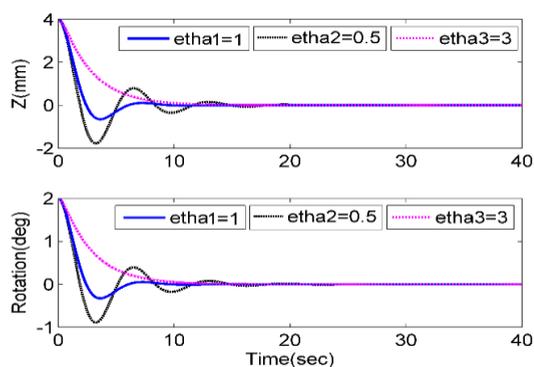


**Fig. 9**     The response of the closed circuit system against the initial non-zero displacement by applying different control coefficients.

## 7     CONCLUSION

Estimation of static responses from experimental data was the first objective in this research. This is important because theoretical analysis is too complicated in the case of the mechanism discussed in this paper. Since the estimating formulations have generated accurate, they are being used in dynamic modeling of the joint. As the second objective of the paper, stabilization of the joint against external motivations and initial dislocations are discussed. A regulator controller based on Lyapunov method is designed and applied to the joint, and simulations show approvable performance.

As future plan, authors intend to add stiffness control to the control algorithm of the joint.

## REFERENCES

[1]   Jung, K., Kim, K. J. and Choi, H. R., A Self-Sensing Dielectric Elastomer Actuator, Sensors and Actuators A: Physical, Vol. 143, No. 2, pp. 343-351.

[2]   Koo, J. C., Choi, H. R., Jung, M. Y., Jung, K. M., Nam, J. D. and Lee, Y. K., Design and Control of Three-DOF Dielectric Polymer Actuator, Key Engineering Materials, Vol. 297, 2005, pp. 665-670.

[3]   He, T., Zhao, X. and Suo, Z., Equilibrium and Stability of Dielectric Elastomer Membranes Undergoing Inhomogeneous Deformation, Journal of Applied Physics, Vol. 106, No. 8, 083522, pp. 1-28.

[4]   He, T., Cui, L., Chen, C. and Suo, Z., Nonlinear Deformation Analysis of a Dielectric Elastomer Membrane–Spring System, Smart Materials and Structures, Vol. 19, No. 8, 085017, pp. 1-7.

[5]   Luan, Y., Wang, H. and Zhu, Y., Design and Implementation of Cone Dielectric Elastomer Actuator with Double-Slider Mechanism, Journal of Bionic Engineering, Vol. 7, 2010, pp. S212-S217.

[6]   Conn, A. T., Rossiter, J., Antagonistic Dielectric Elastomer Actuator for Biologically-Inspired Robotics, SPIE Smart Structures and Materials+ Nondestructive Evaluation and Health Monitoring, Vol. 7976, 2011, pp. 79761Z1-79761Z10.

[7]   He, T., Miao, G. and Chen, C., The Effect of Pre Stretch on the Performance of a Dielectric Elastomer Membrane, Remote Sensing, Environment and Transportation Engineering (RSETE), Jun. 2011, pp. 6463-6467.

[8]   Conn, A. T., Rossiter, J., Towards Homonymic Electro-Elastomer Actuators with Six Degrees of Freedom, Smart Materials and Structures, Vol. 21, No. 3, 035012, pp. 1-9.

[9]   Branz, F., Sansone, F. and Francesconi, A., Design of an Innovative Dielectric Elastomer Actuator for Space Applications, SPIE Smart Structures and Materials+ Nondestructive Evaluation and Health Monitoring, Vol. 9056, 2014, pp. 90560Z1-90560Z10.

[10]  Branz, F., Antonello, A., Carron, A., Carli, R. and Francesconi, A., Kinematics and Control of Redundant Robotic Arm Based on Dielectric Elastomer Actuators, SPIE Smart Structures and Materials+ Nondestructive Evaluation and Health Monitoring, Vol. 9430, 2015, 943023, pp. 1-13.

[11]  Branz, F., Francesconi, A., Modelling and Control of Double-Cone Dielectric Elastomer Actuator, Smart Materials and Structures, Vol. 25, No. 9, pp. 095040.

[12]  Wang, P., Conn, A. T., Elastic Cube Actuator with Six degrees of Freedom Output, Actuators Vol. 4, No. 3, pp. 203-216.

[13]  Nguyen, C. T., Phung, H., Nguyen, T. D., Jung, H. and Choi, H. R., Multiple-Degrees-of- Freedom Dielectric Elastomer Actuators for Soft Printable Hexapod Robot, Sensors and Actuators A: Physical, Vol. 267, 2017, pp. 505-516.

[14] Hau, S., York, A. and Seelecke, S., Performanc Eprediction of Circular Dielectric Electro-Active Polymers Membrane Actuators with Various Geometries, Electroactive Polymer Actuators and Devices (EAPAD), Vol. 9430, 2015, pp. 94300C1-94300C8.