Available online at http://ijim.srbiau.ac.ir
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 4, No. 4, Year 2012 Article ID IJIM-00201, 9 pages
Research Article

Target setting in additive models with preferences and interval data

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Abstract

In this paper, the focus is on additive models with interval data.An additive model can be converted to a multi-objective linear problem if information about preferences of the consumption of inputs and the production of outputs are taken into account. Here in this study, data are not exact and are of interval kind. Moreover, the most preferred solutions with available information by interval additive models are sought. It has also been shown that if additional information is available, an axial solution can be applied. Also, the most preferred target settings will be computed too. In this study twenty bank branches in Iran are evaluated, and target settings and efficiency are compared with the original case and significant decisions are made. .

Keywords: Data envelopment analysis; Additive model; Axial solution; Partial information; Interval data.

1 Introduction

Data envelopment analysis is a powerful tool to measure the relative efficiency of each decision making unit. Unlike the original DEA model, we assume further that the levels of inputs and outputs are not known exactly. Hinojosa and M?rmol (2011) introduced an axial solution for MOLP , in which partial information is available. Whenever a group of DMUs has the same objective functions with different weights, we can apply the axial solutions, and then the most preferred solutions are sought among all solutions. Reduction of the number of optimal solutions is the result of this performance. Determination of the target setting becomes difficult in problems with multiple inputs and outputs. In this case, a set of weights has to be determined in order to aggregate the outputs and inputs. The goal is the computation of optimal solutions in an additive model that could be set as target values for a DMU with partial information about preferences.

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Determination of the target setting becomes difficult in problems with multiple inputs and multiple outputs. In this case, a set of weights has to be determined to aggregate the outputs and inputs. The goal is the computation of optimal solutions in additive model that could be as set target values for a DMU with partial information about preferences.

2 MOLP WITH PARTIAL INFORMATION

A multi-objective linear problem in general form is as follows:

$$\begin{aligned} &\max f(\lambda) = [f_1(\lambda), ..., f_r(\lambda), ..., f_s(\lambda)] \\ &s.t \qquad \lambda \in \Omega \end{aligned} \tag{2.1}$$

Ω is the feasible set in decision space, and $f_r : R^n \to R^1; r = 1, ..., s$, are the linear continuous differentiable real value function. The information about preferences is the set of information $\Lambda \subseteq \Delta^{s-1}$ that $\Delta^{s-1} = \{\alpha \in R^s_+; \sum_{r=1}^{s} \alpha_r = 1\}$.$\Lambda$ is the set of admissible weights for $DMUs$. Axial solutions are all $\lambda' s$ in relation $(2-2)$.

$$t^* = \max\{t \in R; \exists\lambda \in \Omega, f(\lambda) \succeq_\Lambda t^* p \tag{2.2}$$

P is an improvement axis. Hinojosa and Mrmol (2011) proved that MOLP (2-1) can be transformed to a linear problem, as follows:

$$\begin{aligned} &\max t \\ &s.t \qquad \alpha^h f(\lambda) \geq t\alpha^h P, \qquad h = 1, 2, ..., k \\ &\quad \lambda \in \Omega \end{aligned} \tag{2.3}$$

3 AXIAL SOLUTIONS OF TARGET SETTINGS IN AN ADDITIVE MODEL

Assume we have N decision making units ($DMUs$), and each one produces n outputs denoted by y_{rj}, and consumes m inputs denoted by x_{ij}.

3.1 AN Additive Model

The problem of slack may arise in any optimal solution of a radial DEA model. The aim of a radial model is to expand all outputs or contract all inputs by the same proportion, but non-radial models allow the individual outputs to increase or the inputs to decrease at different rates. The non-radial approach is called the additive variant of the DEA model. The objective of an output-oriented additive DEA model is to maximize the total slacks in all outputs existing in the observed input-output bundles. Similarly, in an input-oriented model, one would maximize the total slacks in inputs (Ray. Subhash C (2004)). The output-oriented additive DEA model for constant return scale technology is

$$\begin{array}{l} \max s = \sum_{r=1}^{m} s_r^+ \\ s.t. \quad \sum_{j=1}^{N} \lambda_j y_{rj} - s_r^+ = y_{ro}; (r = 1, ..., m); \\ \sum_{j=1}^{N} \lambda_j x_{ij} \leq x_{io}; (i = 1, ..., n); \\ \lambda_j \geq 0; (j = 1, ..., N); s_r^+ \geq 0; (r = 1, ..., m) \end{array} \tag{3.4}$$

An additive model doesn't measure the efficiency value whose job is introducing efficient and inefficient DMUs and determining the target settings.

3.2 An interval additive model

Now if data belong to interval, model (3-4) should be changed, that is should be written as two models to compute the lower and upper bound of objective functions and target settings. Models (3-5) and (3-6) are formulated, we consider both input- and output-oriented models.

$$\begin{array}{l} \max \underline{s} = \sum_{r=1}^{m} s_r^+ + \sum_{i=1}^{n} s_i^- \\ s.t. \quad \sum_{j=1, j\neq o}^{N} \lambda_j \underline{y}_{rj} + \lambda_o \overline{y}_{ro} - s_r^+ = \overline{y}_{ro} \quad ; (r = 1, ..., m); \\ \sum_{j=1, j\neq o}^{N} \lambda_j \overline{x}_{ij} + \lambda_o \underline{x}_{io} + s_i^- = \underline{x}_{io}; (i = 1, 2, ..., n); \\ \lambda_j \geq 0; (j = 1, ..., N); s_r^+ \geq 0; (r = 1, ..., m); s_i^- \geq 0; (i = 1, ..., n); \end{array} \tag{3.5}$$

$$\begin{array}{l} \max \overline{s} = \sum_{r=1}^{m} s_r^+ + \sum_{i=1}^{n} s_i^- \\ s.t. \quad \sum_{j=1, j\neq o}^{N} \lambda_j \overline{y}_{rj} + \lambda_o \underline{y}_{ro} - s_r^+ = \underline{y}_{ro} \quad ; (r = 1, ..., m); \\ \sum_{j=1, j\neq o}^{N} \lambda_j \underline{x}_{ij} + \lambda_o \overline{x}_{io} + s_i^- = \overline{x}_{io}; (i = 1, 2, ..., n); \\ \lambda_j \geq 0; (j = 1, ..., N); s_r^+ \geq 0; (r = 1, ..., m); s_i^- \geq 0; (i = 1, ..., n); \end{array} \tag{3.6}$$

3.3 An additive model is converted to MOLP

The objective function of model (3-4) is the equivalent of (3-7):

$$(\sum_{j=1}^{N} \lambda_j y_{1j} - y_{1o}, \sum_{j=1}^{N} \lambda_j y_{2j} - y_{2o}, ..., \sum_{j=1}^{N} \lambda_j y_{nj} - y_{no}) \tag{3.7}$$

Therefore model (3-4) is reformulated again to model (3-8).

$$\begin{array}{l} \max(\sum_{j=1}^{N} \lambda_j y_{1j} - y_{1o}, \sum_{j=1}^{N} \lambda_j y_{2j} - y_{2o}, ..., \sum_{j=1}^{N} \lambda_j y_{nj} - y_{no}) \\ s.t. \quad \sum_{j=1}^{N} \lambda_j x_{ij} \leq x_{io}; (i = 1, 2, ..., m); \\ \quad \sum_{j=1}^{N} \lambda_j y_{rj} - y_{ro} \geq 0; (r = 1, 2, ..., n); \\ \quad \lambda_j \geq 0; (j = 1, 2, ..., N); s_r^+ \geq 0; (r = 1, 2, ..., n); \end{array} \tag{3.8}$$

In DEA models, the improvement axis can be the output vector, and weights of the objective functions are denoted by Model (3-6) is converted to model (3-7), according to (2-3).

$$\begin{array}{l} \max t \\ s.t. \quad \sum_{r=1}^{n} \alpha_r^h \sum_{j=1}^{N} \lambda_j (y_{rj} - y_{ro}) \geq t \sum_{r=1}^{n} \alpha_r^h y_{ro}; (h = 1, 2, ..., k); \\ \quad \sum_{j=1}^{N} \lambda_j x_{ij} \leq x_{io}; (i = 1, 2, ..., m); \\ \quad \sum_{j=1}^{N} \lambda_j y_{rj} - y_{ro} \geq 0; (r = 1, 2, ..., n); \\ \quad \lambda_j \geq 0; (j = 1, 2, ..., N) \end{array} \tag{3.9}$$

We consider both input- and output- oriented models. Now, improvement axis is $P = (x_{1o}, ..., x_{no}, y_{1o}, ..., y_{no})$,for reduction of inputs and increase of outputs. Also the weight vector of the objective function is $(\alpha_1^h, ..., \alpha_n^h, \beta_1^h, ..., \beta_n^h)$.

$$\begin{array}{l} \max t \\ s.t. \quad \sum_{r=1}^{n} \alpha_r^h (\sum_{j=1}^{N} \lambda_j y_{rj} - y_{ro}) + \sum_{i=1}^{m} \beta_i^h (\sum_{j=1}^{N} -\lambda_j x_{ij} + x_{io}) \geq \\ t(\sum_{i=1}^{m} \beta_i^h x_{io} + \sum_{r=1}^{n} \alpha_r^h y_{ro}); (h = 1, 2, ..., k); \\ \quad \sum_{j=1}^{N} \lambda_j x_{ij} \leq x_{io}; (i = 1, 2, ..., m); \\ \quad \sum_{j=1}^{N} \lambda_j y_{rj} \geq y_{ro}; (r = 1, 2, ..., n); \\ \quad \lambda_j \geq 0; (j = 1, 2, ..., N) \end{array} \tag{3.10}$$

4 THE MODEL WITH INTERVAL DATA

In this section, we deal with interval data. The exact values of data are not available; the only available information to us is the lower bound and upper bound values. Models (4-1) and (4-2) show the lower and upper objective functions, respectively.

$$\begin{array}{l} \underline{t} = \max t \\ s.t. \quad \sum_{r=1}^{n} \alpha_r^h (\sum_{j=1, j \neq o}^{N} \lambda_j \underline{y}_{rj} - \overline{y}_{ro}) + \sum_{r=1}^{n} \alpha_r^h (\lambda_o \overline{y}_{ro} - \overline{y}_{ro}) \\ + \sum_{i=1}^{m} \beta_i^h (\sum_{j=1, j \neq o}^{N} -\lambda_j \overline{x}_{ij} + \underline{x}_{io}) + \sum_{i=1}^{m} (-\lambda_o \underline{x}_{io} + \underline{x}_{io}) \geq \\ t(\sum_{i=1}^{m} \beta_i^h \underline{x}_{io} + \sum_{r=1}^{n} \alpha_r^h \overline{y}_{ro}); (h = 1, 2, ..., k); \\ \quad \sum_{j=1, j \neq o}^{N} \lambda_j \overline{x}_{ij} + \lambda_o \underline{x}_{io} \leq \underline{x}_{io}; (i = 1, 2, ..., m); \\ \quad \sum_{j=1, j \neq o}^{N} \lambda_j \underline{y}_{rj} + \lambda_o \overline{y}_{ro} \geq \overline{y}_{ro}; (r = 1, 2, ..., n); \\ \quad \lambda_j \geq 0; (j = 1, 2, ..., N) \end{array} \tag{4.11}$$

$$\begin{array}{l} \overline{t} = \max t \\ s.t. \quad \sum_{r=1}^{n} \alpha_r^h (\sum_{j=1, j \neq o}^{N} \lambda_j \overline{y}_{rj} - \underline{y}_{ro}) + \sum_{r=1}^{n} \alpha_r^h (\lambda_o \underline{y}_{ro} - \underline{y}_{ro}) \\ + \sum_{i=1}^{m} \beta_i^h (\sum_{j=1, j \neq o}^{N} -\lambda_j \underline{x}_{ij} + \overline{x}_{io}) + \sum_{i=1}^{m} (-\lambda_o \overline{x}_{io} + \overline{x}_{io}) \geq \\ t(\sum_{i=1}^{m} \beta_i^h \overline{x}_{io} + \sum_{r=1}^{n} \alpha_r^h \underline{y}_{ro}); (h = 1, 2, ..., k); \\ \quad \sum_{j=1, j \neq o}^{N} \lambda_j \underline{x}_{ij} + \lambda_o \overline{x}_{io} \leq \overline{x}_{io}; (i = 1, 2, ..., m); \\ \quad \sum_{j=1, j \neq o}^{N} \lambda_j \overline{y}_{rj} + \lambda_o \underline{y}_{ro} \geq \underline{y}_{ro}; (r = 1, 2, ..., n); \\ \quad \lambda_j \geq 0; (j = 1, 2, ..., N) \end{array} \tag{4.12}$$

5 THE APPLIED EXAMPLE

We present an application to target setting in an additive model with interval data involving preferences of objective of each DMU. We apply our approach to some bank branches in Iran (Jahanshahloo.G.R. et all) There are 20 bank branches, each branch produces five outputs by consuming three inputs. Payable interest, personnel and non-performing loans are inputs and the total sum of four main deposits, other deposits, loans granted, received interest and fee are outputs. At first, preferences of objective are not taken into account. Two sets of information about DMUs exist.

$\Lambda_1 = \{y_4 \geq y_1; y_3 \geq y_5; y_2 \geq \frac{2}{3} y_1; y_2 \geq 2y_3; x_2 \geq x_1\}$

$\Lambda_2 = \{3y_4 \geq 2y_1; y_5 \geq \frac{1}{2} y_3; y_2 \geq \frac{3}{4} y_1; x_3 \geq \frac{1}{2} x_2\}$ Extreme points of these sets are weights of objective function. Lower and upper target settings for both cases with preferences and without preferences are shown in tables 1, 2 and tables 3, 4, respectively.

Table 1.Output target settings of Lower bound model

Additive model				Additive model with preference			
y_1^*	y_2^*	y_3^*	y_4^*	y_1^*	y_2^*	y_3^*	y_4^*
3126798	382545	1853365	125740	3126798	382545	1853365	125740
440355	117659	390203	37837	440355	117659	390203	37837
1061260	503089	18822028	108080	1061260	503089	1822028	108080
1213541	268460	542101	39273	1213541	268460	542101	39273
395241	12136	142873	14165	395241	12136	142873	14165
1087392	111324	574355	72257	1087392	111324	574355	72257
165818	180617	323721	45847	165818	180617	323721	45847
416416	486431	1071812	73948	416416	486431	1071812	73948
410427	449336	1802942	189006	410427	449336	1802942	189006
768593	15192	2573512	791463	768593	15192	2573512	791463
696338	241081	2285079	20774	696338	241081	2285079	20774
632339	156136	4266224	45207	1048993	177325	1130247	93137
574989	129311	3662504	122124	574989	23043	431815	50256
351127	131447	3930906	18116	935651	270814	661590	45740
317186	270708	810088	111962	317186	270708	810088	111962
347848	80453	379488	165524	347848	80453	379488	165524
835839	404579	9136507	41827	835839	404579	9136507	41827
320947	78327	334208	10980	320947	6330	29173	10878
679916	684372	3985900	95330	679916	584372	3985900	95330
120208	116908	410946	27934	120208	17495	308012	27934

Table 2.Input target settings and reference units of Lower bound model

	Additive model				Additive model with Preference			
	x_1^*	x_2^*	x_3^*	$Ref.unit$	x_1^*	x_2^*	x_3^*	Ref. unit
1	5007	36	87243	1	5007	36	87243	1
2	2927	19	9945	2	2927	19	9945	2
3	8733	26	47575	3	8733	26	47575	3
4	13759	21	19292	4	13759	21	19292	4
5	588	14	3428	5	588	14	3428	5
6	13759	19	13929	6	13759	19	13929	6
7	579	27	27827	7	579	27	27827	7
8	4646	25	9070	8	4646	25	9070	8
9	1554	20	412036	9	1554	20	412036	9
10	17528	15	8638	10	17528	15	8638	10
11	2444	20	500	11	2444	20	500	11
12	7303	23	16148	10,11,17	7303	23	16148	1,4,5,6,8,10,11
13	9852	18	17163	4,10,11,17	9852	18	17163	4,13
14	4541	15	17918	8,11,17	4541	15	17918	1,4,10
15	3040	39	51582	15	3040	39	51582	15
16	6586	26	20975	16	6586	26	20975	16
17	4209	28	419960	17	4209	28	419960	17
18	1016	7	6267	4,8,17	1016	7	6267	18
19	5800	27	19500	19	5800	27	19500	19
20	1446	13	31700	4,7,8,9,17	1446	13	31700	20

Table 3.Output target settings of upper bound model

y_1^*	y_2^*	y_3^*	y_4^*	y_1^*	y_2^*	y_3^*	y_4^*
3126798	382545	1853365	125740	2696995	263643	1675519	108635
547188	292578	2187293	128806	965032	197601	2053415	163605
1414095	156845	3240017	624505	2052294	206458	2415338	513045
1213541	268460	542101	39273	1145253	229646	468520	32363
565647	53337	1635101	291453	617282	106109	2047661	348004
897456	56998	2827200	744313	988115	74133	507502	53591
165818	180617	323721	45847	144906	180530	288513	40508
416416	486431	1071812	73948	408163	405396	1044221	56260
410427	449336	1802942	189006	335070	337971	1584722	176437
768593	15192	2573512	791463	700842	14378	2290745	662725
696338	241081	2285079	20774	641680	114183	1579961	17528
772991	252033	3048406	459483	1370650	152237	913420	108153
883649	125415	3142236	711540	1199323	80188	3232993	871547
668390	358299	2821982	241318	1230024	211043	2689262	351120
1219512	260756	4114049	88103	767535	311524	2104788	267991
816638	337658	3342988	404890	1472247	200584	2956118	510780
835839	404579	9136507	41827	618105	244250	9136507	33037
400232	125834	1465252	17935	892158	152129	1088838	44706
679916	684372	3985900	95330	640890	490508	2946797	66097
730562	140078	2370462	21168	622581	229593	1240977	114145

Table 4.Input target settings and reference units of upper bound model

x_1^*	x_2^*	x_3^*	x_1^*	x_2^*	x_3^*	Ref.unit
5007	36	87243	9613	37	87243	1,4,8
12254	20	12438	5962	20	12120	4,7,8,10,17
36548	28	35544	12512	27	50013	4,8,9,17,19
13759	21	19292	1966	23	19753	4,8,10,19
17614	15	4774	8527	15	3911	4,8,10
41448	20	15236	27359	19	15657	1,6,8,10,11
579	27	27827	1205	27	29005	4,7,8,10
4646	25	9070	9560	25	9983	8
1554	20	412036	3428	22	413902	8,9,17
17528	15	8638	36298	15	10229	10,17
2444	20	500	4956	21	937	1,4,8,17
29292	23	16772	14178	23	21353	10,11,17
40852	22	15464	19743	22	17290	4,10,11,17
19185	24	17226	9312	24	17964	8,11,17
12857	40	9435	6304	40	55136	8,9
27767	27	19225	13454	27	23992	4,7,8,10
4209	28	419960	8604	28	43103	1,4,17
4149	14	4691	2038	14	19354	4,8,17
5800	27	19500	11875	27	19569	8,17
5930	24	4235	2922	29	32061	4,7,8,9,17,19

DMUs 1,2,3,4,5,6,7,8,9,10,11,15,16,17 and 19 are efficient DMUs in model (3-2) indicating that after accounting the information sets, all of them become efficient. Also all DMUs in model (4-12) are efficient; although efficient DMUs were just 1,4,7,8,9,10 and 11 without accounting preferences. We have compared target settings in both cases (without preferences and with preferences). Target settings are compared between model (4-11) and model (4-12). Also, in this example, we notice that a DMU can become efficient with accounting preferences. These target settings give important and effective information to decision makers when objective function weights are accounted.

6 Conclusion

In this study it has been shown how a manager can be informed about the target settings, so having an inefficient or efficient organization. It is clear that models should be formulated in order to have a better performance, although data may be of the interval kind. The models were performed for a group of Iranian banks. Through using it and with the computation of target settings, a manager diagnoses how to increase outputs or decrease inputs when information about preferences is available.

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