

# A New Group Data Envelopment Analysis Method for Ranking Design Requirements in Quality Function Deployment

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Received Date: 2015-11-02    Revised Date: 2016-04-09    Accepted Date: 2016-12-11

## Abstract

Data envelopment analysis (DEA) is an objective method for priority determination of decision making units (DMUs) with the same multiple inputs and outputs. DEA is an efficiency estimation technique, but it can be used for solving many problems of management such as ranking of DMUs. Many researchers have found similarity between DEA and MCDM techniques. One of the earliest techniques in MCDM is Quality Function Deployment (QFD) which is a team-based and disciplined approach to product design, engineering and production and provides in-depth evaluation of a product. The QFD team is responsible for assessing the relationships between customer requirements (CRs) and design requirements (DRs) and the interrelationships between DRs. In practice, each member demonstrates significantly different behavior from the others and generates different assessment results, leading to the QFD with uncertainty. In this paper data envelopment analysis is used to overcome this uncertainty. Each member's subjective assessment is taken into account directly and a new data envelopment analysis method in group situation is constructed which differs from multi-objective decision making models. Then, without using Charnes-Cooper transformation, the proposed model is transformed into a linear programming problem in a completely different manner. We will call the proposed model "Grouped-QFDEA".

*Keywords* : Data Envelopment Analysis (DEA); Quality function deployment (QFD); Group situation; Multi-objective decision making models; Ranking.

## 1 Introduction

DEA [1] is one of the most popular tools in production management literature for performance measurement, while QFD [2, 3] is one of the extremely powerful tools that is useful in product design and development and for benchmarking. The goal of DEA is to determine the

production efficiency of DMUs by comparing how well the DMU converts inputs into outputs, while the goal of QFD is to produce a product with high quality by translating the CRs into DRs. A comprehensive literature review of QFD and its extensive applications is provided by Chan and Wu [4]. In practice, a QFD team is set up to determine the importance levels of DRs. Traditionally, QFD rates the DRs with respect to CRs, and aggregates the ratings to get relative importance scores of DRs [5]. Decision makers (DMs) or design engineers usually do not have sufficient information about the influence of engineering responses on CRs, due to the lack of

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information from the customer. These considerations have made the applications of imprecise problems in QFD. There are several studies to deal with this vague nature of QFD [6, 7, 8]. QFD is known by its house of quality (HOQ) which has a matrix format [2]. HOQ is an important tool for QFD activities, containing information on "what", i.e., customer requirements (CRs), "how", i.e., design requirements (DRs), relationship between "CR" and "DR", and a triangular-shaped matrix placed over the design requirements corresponds to the interrelationship between them. Traditional QFD uses the weighted sum method to rank DRs [5]. There are studies to determine the priority for CRs and DRs in QFD literature. The relative importance of CRs may be obtained using simple methods such as direct rating, or more complex ones such as the swing methods [9], the analytic hierarchy process (AHP) [10], a well-known and commonly used multi-criteria decision making method, and its variants: fuzzy AHP, analytic network process (ANP) [11] and fuzzy ANP. On the other hand, traditional QFD process does not explicitly incorporate cost and environmental factors. These factors are incorporated in further analysis. Some studies have considered the level of difficulty, some others cost or ease of implementation. However in general, studies have considered only one extra factor in their analysis [12, 13]. For the first time, in 2009, Ramanathan and Yunfeng [14] applied DEA to incorporate cost and environmental factors in QFD. They proved that the relative importance values computed by data envelopment analysis (DEA) coincide with traditional QFD calculations when only the ratings of DRs with respect to customer needs are considered and only one additional factor, namely cost, is considered. They view each DR as a decision making unit (DMU). Using the input and output definition in DEA, they classify CRs and other factors as inputs and outputs. So CRs and factors like ease of implementation are considered as outputs and factors like cost and level of difficulty as inputs. If there is no inputs (like in traditional QFD) they use a dummy input with a constant value of one for all DMUs. By solving the CCR-input oriented model for each DMU (DR) they get the relative importance of each DR. They use Assurance Region (AR) [15] in order to impose the weights of CRs. If there are significant

interrelationships between DRs, before applying the model, they use Wasserman [5] suggestion to normalize the relationship between DRs and CRs (and other additional factors). Kamvysi et al. [16] discussed the combination of QFD with analytic hierarchy process-analytic network process (AHP-ANP) and DEAHP-DEANP methodologies to prioritize selection criteria in a service context. In 2013, Azadi and Farzipoor Saen [17] applied Russell measure [18] in QFD and developed this model in imprecise situation. In this regards, QFD-impresic enhanced Russell graph measure (QFD-IERGM) is proposed for incorporating the criteria such as cost of services and implementation easiness in QFD. However, in general, these studies do not take into account DM's different ideas directly in their methodologies; only the model proposed by Ramanathan and Yunfeng i.e. QFD-DEA methodology, uses arithmetic average to obtain the final overall efficiency scores in such uncertainty environment. In this paper, DM's different ideas are considered in a group environment and in order to deal with this situation, a group methodology based on DEA is suggested. In this regards, at first a new DEA model for ranking DRs is proposed and then will be extended to the group situation. Without using Charnes-Cooper translation, the proposed model is linearized. This proposed linear programming model is applied in QFD and the uncertainty problem is overcome. In this paper we do not try to describe DEA and QFD techniques in details. Interested readers are referred to some prominent studies. The rest of this paper is organized as follows: QFD and the use of DEA in estimating the relative importances of DRs in QFD are explained in section 2. The proposed ranking method is presented in section 3. Section 4 is the extension of the proposed model in group situation. Section 5 is devoted to the numerical example using proposed methodology in QFD. The final section concludes the work.

## 2 DEA-QFD methodology

### 2.1 Quality Function Deployment

QFD begins with the identification of customer requirements and their mapping into relevant engineering design requirements, as shown in Fig 1, where  $CR_1, \dots, CR_m$  are the  $m$  identified customer requirements,  $DR_1, \dots, DR_n$  are

the  $n$  relevant engineering design requirements,  $w_1, \dots, w_m$  are the relevant importances of CRs which  $w_i > 0$  for  $i = 1, \dots, m$ ,  $R_{ij}$  is the relationship between  $CR_i$  and  $DR_j$ , and  $r_{jk}$  is the interrelationship between  $DR_j$  and  $DR_k$ , satisfying  $r_{jk} = r_{kj}$  for  $j, k = 1, \dots, n$ .

The relationship between CRs and DRs reflects the impact of the fulfillment of DRs on the satisfaction of CRs. These relationships should be developed by QFD team members. The relationship between CRs and DRs and the relationship between the DRs themselves are usually determined subjectively by ambiguous or vague judgments. However they are usually captured using symbols converted into crisp numbers using different measurement scales. The degree of these relationships is usually expressed on a scale system such as 0-1-3-9 or 0-1-3-5, representing linguistic expressions such as "no relationship", "weak/possible relationship", "medium/moderate relationship", and "strong relationship". In this paper, rating scale 0-9 is defined to characterize different strengths of the relationships between CRs and DRs as shown in Table 1. Other rating scales can also be defined. It is not our purpose to explore which rating scale is the best or more appropriate for a specific situation, which is beyond the scope of this paper. As the shape of this figure looks similar to a house, so it referred to as the house of quality (HOQ). Usual procedure for

respect to CRs is to use weighted arithmetic aggregation rule. Note that, when there is significant interrelationships between the DRs, in many studies the following normalization procedure suggested by Wasserman [5] is usually employed for this purpose:

$$R_{ij}^{norm} = \frac{\sum_{l=1}^n R_{il} \cdot r_{lj}}{\sum_{j=1}^n \sum_{l=1}^n R_{il} \cdot r_{lj}}, \quad \forall i, j \quad (2.1)$$

where  $R_{ij}$  denotes the relationship level in terms of score between the  $CR_i$  and  $DR_j$ , and  $r_{ij}$  is the interrelationship score between  $DR_i$  and  $DR_j$ .  $R_{ij}^{norm}$  is the normalized relationship value between  $CR_i$  and  $DR_j$ ,  $m$  is the number of CRs,  $n$  is the number of DRs and  $\sum_j R_{ij}^{norm} = 1$  for each  $i$ . Thus, if there is significant interrelationships between the DRs,  $R_{ij}^{norm}$  must be used in estimating the relative importances of DRs with respect to CRs, otherwise  $R_{ij}$  can be directly used.

Tables 6-9 represent, respectively a deterministic interrelationship matrix between DRs provided by DMs.

## 2.2 Incorporating additional factors in QFD using DEA

As mentioned in subsection 2.1, traditional QFD involves a simple procedure for estimating the relative importances of DRs with respect to CRs. However this procedure is disable to estimate the relative importances of DRs with respect to not only CRs but also other additional factors such as cost, ease of implementation, environmental factors, etc have to be considered. To cope with this weakness, Ramanathan and Yunfeng [14] employed DEA to compute the relative importances of DRs when several additional factors need to be considered. They proved that the relative importance scores computed by DEA coincide with traditional QFD calculations when only the ratings of DRs with respect to CRs are considered, and when only one additional factor (such as cost) is considered. They showed that DEA provides a simple and general framework facilitating QFD computations when more factors need to be considered, and has the flexibility to treat QFD ratings as qualitative factors. In this methodology each DR is considered as a DMU and the efficiency score of DR is considered as a measure of its relative importance. In order to classify the CRs and other additional factors as inputs and outputs, they use the suggestion of Golany and

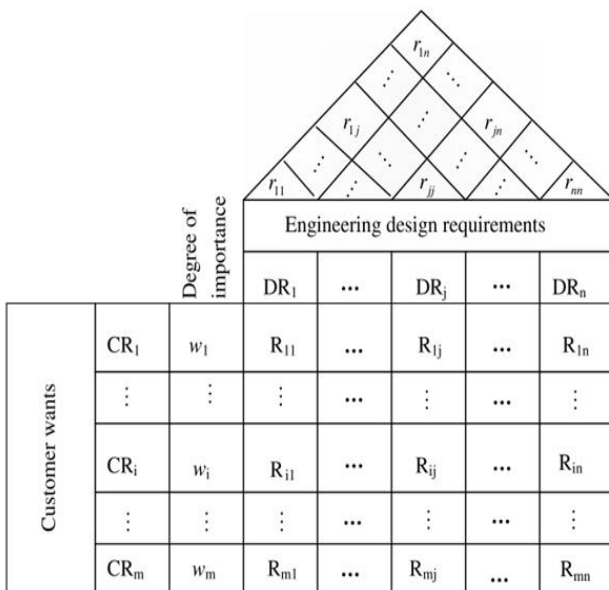


Figure 1: The house of quality in QFD

estimating the relative importances of DRs with

**Table 1:** Rating scales for the relationships between CRs and DRs, and interrelationships between DRs themselves

Rating	Definition for Relationship matrix	interrelationship matrix
9	very strong relationship	very strong interrelationship
7	strong relationship	strong interrelationship
5	moderate relationship	moderate interrelationship
3	weak relationship	weakly interrelationship
1	very weak relationship	very weakly interrelationship
0	no relationship	no intrrelationship
2,4,6,8	the relationships between these intervals	the interrelationships between these intervals

**Table 2:** Classification of CRs and additional factors as inputs and outputs

	Output 1 (CR1)	Output 2 (CR2)	⋮	Output t (CRt)	Additional output factors	Additional input factors
					Output t+1 ⋮ Output s	Input 1 ⋮ Input m
DMU 1(DR1)	$y_{11}$	$y_{21}$	⋮	$y_{t1}$	$y_{t+1,1}$ ⋮ $y_{s1}$	$x_{11}$ ⋮ $x_{m1}$
DMU 2(DR2)	$y_{12}$	$y_{22}$	⋮	$y_{t2}$	$y_{t+1,2}$ ⋮ $y_{s2}$	$x_{12}$ ⋮ $x_{m2}$
⋮	⋮	⋮	⋮	⋮	⋮ ⋮ ⋮	⋮⋮⋮
DMU n(DRn)	$y_{1n}$	$y_{2n}$	⋮	$y_{tn}$	$y_{t+1,n}$ ⋮ $y_{sn}$	$x_{1n}$ ⋮ $x_{mn}$

**Table 3:** The relative importances of CRs

CRs	CR1	CR2	CR3	CR4	CR5	CR6
The relative importance of CRs	7	10	6	5	6	6

Roll [19]. Using this logic, CRs and factors such as ease of implementation are considered as outputs, while factors such as cost and level of difficulty are considered as inputs. The corresponding output-input matrices is shown in Table 2.

In order to impose the relative importances of CRs, the method of Assurance region (AR) is employed. Thus additional constraints that specify the relationships among the multipliers, are appended to the DEA model. Hence, each DMU has m inputs and s outputs (which t of them are the CRs ( $t < s$ )), based on which the following restricted input-oriented CCR model is built to assess the efficiency score (relative importance) of DRs:

$$\begin{aligned}
 \max \quad & E_o = \sum_{r=1}^s u_r y_{ro} \\
 s.t \quad & \sum_{i=1}^m v_i x_{io} = 1 \\
 & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \forall j \quad (2.2) \\
 & u_r \geq 0, \forall r \\
 & v_i \geq 0, \forall i
 \end{aligned}$$

The importance of CRs is imposed, using additional constraints to form

$$\begin{aligned}
 u_r &= d_r u_1; \quad \forall r = 1, 2, \dots, t, \\
 d_1 &= 1
 \end{aligned}$$

For example, if CR2 and CR3, respectively, are half and thrice as important as CR1, then

**Table 4:** Assessment on the relationships between the 6 CRs and 4 DRs

CRs	DMs	DRs			
		DR1	DR2	DR3	DR4
CR1	DM1	9	0	0	0
	DM2	9	0	0	1
	DM3	8	0	0	1
	DM4	7	1	0	1
CR2	DM1	9	1	0	0
	DM2	8	1	0	0
	DM3	9	1	0	0
	DM4	9	1	1	0
CR3	DM1	3	9	9	0
	DM2	2	9	8	0
	DM3	1	9	8	0
	DM4	3	7	8	0
CR4	DM1	0	9	9	0
	DM2	0	6	9	1
	DM3	0	7	9	0
	DM4	1	7	9	0
CR5	DM1	9	3	9	0
	DM2	9	1	7	0
	DM3	9	2	9	1
	DM4	9	2	9	0
CR6	DM1	0	0	3	9
	DM2	1	1	3	9
	DM3	1	1	3	9
	DM4	0	1	3	9

**Table 5:** Assessment on the relationships between the additional factors and DRs

additional factors	DMs	DRs			
		DR1	DR2	DR3	DR4
easiness	DM1	1	4	5	1
(output factor) (9 Easy) (1 Tough)	DM2	3	3	4	1
	DM3	1	4	4	1
	DM4	4	4	5	1
cost	DM1	4	1	2	5
	DM2	2	1	2	5
	DM3	3	1	2	5
(in cost units) (input factor)	DM4	3	1	3	4
	DM1	5	1	1	2
adversity	DM2	6	2	1	2
	DM3	3	1	0	2
	DM4	5	1	1	4

**Table 6:** The interrelationships between DRs provided by DM1

DRs	DR1	DR2	DR3	DR4
DR1	0	1	3	0
DR2	1	0	9	0
DR3	3	9	0	1
DR4	0	0	1	0

**Table 7:** The interrelationships between DRs provided by DM2

DRs	DR1	DR2	DR3	DR4
DR1	0	2	2	0
DR2	2	0	8	0
DR3	2	8	0	1
DR4	0	0	1	0

**Table 8:** The interrelationships between DRs provided by DM3

DRs	DR1	DR2	DR3	DR4
DR1	0	1	3	0
DR2	1	0	7	0
DR3	3	7	0	1
DR4	0	0	1	0

**Table 9:** The interrelationships between DRs provided by DM4

DRs	DR1	DR2	DR3	DR4
DR1	0	1	3	2
DR2	1	0	9	0
DR3	3	9	0	1
DR4	2	0	1	0

**Table 10:** The efficiency scores of the four design requirements and their ranking order using Grouped QFDEA

	DR1	DR2	DR3	DR4
E.S.	0.1475	0.0407	0.0449	0.1513
R. O.	3	1	2	4

$d_2 = 0.5$  and  $d_3 = 3$ . So, the model (2.2) can be rewritten as follows:

$$\begin{aligned}
 \max \quad & E_o = \sum_{r=1}^s u_r y_{ro} \\
 s.t \quad & \sum_{i=1}^m v_i x_{io} = 1 \\
 & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \forall j \\
 & u_r = d_r u_1, r = 1, \dots, t \\
 & u_r \geq 0, \forall r \\
 & v_i \geq 0, \forall i
 \end{aligned} \tag{2.3}$$

The linear programming model (2.3) is solved for all the DMUs to estimate their relative scores.

### 3 New DEA Methodology

Let there be  $n$  decision making units as  $DMU_j$  ( $j = 1, 2, \dots, n$ ), that convert  $m$  inputs  $x_{ij}$  ( $i = 1, 2, \dots, m$ ) into  $s$  outputs  $y_{rj}$  ( $r = 1, 2, \dots, s$ ) and let  $DMU_0$  be a DMU under evaluation. Consider the fractional CCR model under the assumption that the sum of efficiency values of all DMUs equals unity i.e.  $\sum_{j=1}^n \theta_j = 1$  [20].

$$\begin{aligned}
 \max \quad & \theta_0 = \frac{\sum_{r=1}^s u_r y_{r0}}{\sum_{i=1}^m v_i x_{i0}} \\
 \text{s.t.} \quad & \theta_j = \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}}, \quad \forall j \\
 & \sum_{j=1}^n \theta_j = 1 \\
 & u_r \geq \varepsilon, \quad \forall r \\
 & v_i \geq \varepsilon, \quad \forall i \\
 & \theta_j \geq \varepsilon, \quad \forall j
 \end{aligned} \tag{3.4}$$

Where  $u_r (r = 1, 2, \dots, s)$  are the weights of outputs,  $v_i (i = 1, 2, \dots, m)$  are the weights of inputs and  $\theta_j (j = 1, 2, \dots, n)$  are the efficiency score of  $DMU_j$ . Here  $\varepsilon$  is a small amount of positive. So the last three constraints are caused that all variables  $u_r (r = 1, 2, \dots, s)$ ,  $v_i (i = 1, 2, \dots, m)$  and  $\theta_j (j = 1, 2, \dots, n)$  are always positive values.

**Theorem 3.1** *The nonlinear programming model (3.4) can be transformed to the following linear programming model:*

$$\begin{aligned}
 \min \quad & \sum_{i=1}^m v_i x_{i0} - \sum_{r=1}^s u_r y_{r0} \\
 \text{s.t.} \quad & \sum_{i=1}^m w_{ij} x_{ij} - \sum_{r=1}^s u_r y_{rj} = 0, \quad \forall j \\
 & \sum_{j=1}^n w_{ij} = v_i, \quad \forall i \\
 & u_r \geq \varepsilon, \quad \forall r \\
 & v_i \geq \varepsilon, \quad \forall i \\
 & w_{ij} \geq \varepsilon, \quad \forall i, j
 \end{aligned} \tag{3.5}$$

**Proof.** From the constraints of model (3.4), it is obvious that the value of objective function is between zero and one, i.e.  $0 \leq \theta_0 \leq 1$ , so  $0 \leq \frac{\sum_{r=1}^s u_r y_{r0}}{\sum_{i=1}^m v_i x_{i0}} \leq 1$ . Multiplying each part of the inequality with  $-\sum_{i=1}^m v_i x_{i0}$ , and then adding the term  $\sum_{i=1}^m v_i x_{i0}$  to each part of the inequality, we have

$$0 \leq \sum_{i=1}^m v_i x_{i0} - \sum_{r=1}^s u_r y_{r0} \leq \sum_{i=1}^m v_i x_{i0}.$$

So, the fractional objective function can be transformed to the linear objective function. Also by using the constraints of model (3.4), we can rearrange the constraints as

$$\begin{aligned}
 \sum_{i=1}^m x_{ij}(v_i \theta_j) - \sum_{r=1}^s u_r y_{rj} &= 0, \quad \forall j \\
 \sum_{j=1}^n v_i \theta_j &= v_i \quad \forall i.
 \end{aligned}$$

So by using the transformation  $w_{ij} = v_i \theta_j$  we can give the model (3.5).

### 4 Grouped-DEA model

Let  $\theta_0^{(k)}$  be the relative efficiency score of  $DMU_0$  obtained from the model (3.5), that provided by the  $k$ th decision maker ( $DM_k$ ) ( $k = 1, 2, \dots, K$ ) and  $h_k > 0$  be its relative importance weight satisfying  $\sum_{k=1}^K h_k = 1$ . Then, we have

$$\begin{aligned}
 \max \quad & \sum_{k=1}^K h_k \theta_0^{(k)} \\
 \text{s.t.} \quad & \theta_j^{(k)} = \frac{\sum_{r=1}^s u_r y_{rj}^{(k)}}{\sum_{i=1}^m v_i x_{ij}^{(k)}}, \quad \forall j \\
 & \sum_{k=1}^K \sum_{j=1}^n h_k \theta_j^{(k)} = 1 \\
 & u_r \geq \varepsilon, \quad \forall r \\
 & v_i \geq \varepsilon, \quad \forall i \\
 & \theta_j \geq \varepsilon, \quad \forall j.
 \end{aligned} \tag{4.6}$$

The model (4.6) is equivalent with the following multi-objective programming model:

$$\begin{aligned}
 \max \quad & h_1 \frac{\sum_{r=1}^s u_r y_{r0}^{(1)}}{\sum_{i=1}^m v_i x_{i0}^{(1)}} \\
 & \vdots \\
 \max \quad & h_K \frac{\sum_{r=1}^s u_r y_{r0}^{(K)}}{\sum_{i=1}^m v_i x_{i0}^{(K)}} \\
 \text{s.t.} \quad & \sum_{i=1}^m w_{ij}^{(k)} x_{ij}^{(k)} - \sum_{r=1}^s u_r y_{rj}^{(k)} = 0, \quad \forall j \\
 & \sum_{k=1}^K \sum_{j=1}^n h_k w_{ij}^{(k)} = v_i, \quad \forall i \\
 & u_r \geq \varepsilon, \quad \forall r \\
 & v_i \geq \varepsilon, \quad \forall i \\
 & w_{ij} \geq \varepsilon, \quad \forall i, j
 \end{aligned} \tag{4.7}$$

**Theorem 4.1** *The nonlinear multi-objective programming model (4.7) can be transformed to*

the following linear programming model:

$$\begin{aligned}
 \min \quad & \{ \sum_{i=1}^m v_i x_{i0}^{(k)} - \sum_{r=1}^s h_k u_r y_{r0}^{(k)}, \forall k \} \\
 \text{s.t.} \quad & \sum_{i=1}^m w_{ij}^{(k)} x_{ij}^{(k)} - \sum_{r=1}^s u_r y_{rj}^{(k)} = 0, \forall j \\
 & \sum_{k=1}^k \sum_{j=1}^n h_k w_{ij}^{(k)} = v_i, \forall i \\
 & u_r \geq \varepsilon, \forall r \\
 & v_i \geq \varepsilon, \forall i \\
 & w_{ij} \geq \varepsilon, \forall i, j
 \end{aligned}
 \tag{4.8}$$

**Proof.** From the constraints of model (4.7), it is obvious that the value of each objective function is between zero and one, i.e.

$$0 \leq h_k \frac{\sum_{r=1}^s u_r y_{r0}^{(k)}}{\sum_{i=1}^m v_i x_{i0}^{(k)}} \leq 1.$$

Similar the proof of Theorem 3.1, multiplying each part of the inequality with  $-\sum_{i=1}^m v_i x_{i0}^{(k)}$ , and then adding the term  $\sum_{i=1}^m v_i x_{i0}^{(k)}$  to each part of the inequality, we have:

$$0 \leq \sum_{i=1}^m v_i x_{i0}^{(k)} - \sum_{r=1}^s h_k u_r y_{r0}^{(k)} \leq \sum_{i=1}^m v_i x_{i0}^{(k)}.$$

So, the multi-objective fractional function can be transformed to the linear objective function.

**Theorem 4.2** *If an optimal solution of the following single objective programming exists, then this optimal solution will be an efficient solution of model (4.8).*

$$\begin{aligned}
 \min \quad & \sum_{k=1}^K (\sum_{i=1}^m v_i x_{i0}^{(k)} - \sum_{r=1}^s h_k u_r y_{r0}^{(k)}) \\
 \text{s.t.} \quad & \sum_{i=1}^m w_{ij}^{(k)} x_{ij}^{(k)} - \sum_{r=1}^s u_r y_{rj}^{(k)} = 0, \forall j, k \\
 & \sum_{k=1}^k \sum_{j=1}^n h_k w_{ij}^{(k)} = v_i, \forall i \\
 & u_r \geq \varepsilon, \forall r \\
 & v_i \geq \varepsilon, \forall i \\
 & w_{ij} \geq \varepsilon, \forall i, j
 \end{aligned}
 \tag{4.9}$$

**Proof.** Let  $v_i^*$  and  $u_r^*$  be an optimal solution of model (4.9). Suppose that  $v_i^*$  and  $u_r^*$  is not an efficient solution of model (4.8), then there exist  $v_i'$  and  $u_r'$  such that for some k have :

$$\sum_{i=1}^m v_i^* x_{i0}^{(k)} - \sum_{r=1}^s h_k u_r^* y_{r0}^{(k)} > \sum_{i=1}^m v_i' x_{i0}^{(k)} - \sum_{r=1}^s h_k u_r' y_{r0}^{(k)}$$

and for  $l \in K \setminus k$  have

$$\sum_{i=1}^m v_i^* x_{i0}^{(l)} - \sum_{r=1}^s h_k u_r^* y_{r0}^{(l)} < \sum_{i=1}^m v_i' x_{i0}^{(l)} - \sum_{r=1}^s h_k u_r' y_{r0}^{(l)}$$

It follows that

$$\sum_{k=1}^K \sum_{i=1}^m v_i^* x_{i0}^{(k)} - \sum_{k=1}^K \sum_{r=1}^s h_k u_r^* y_{r0}^{(k)} > \sum_{k=1}^K \sum_{i=1}^m v_i' x_{i0}^{(k)} - \sum_{k=1}^K \sum_{r=1}^s h_k u_r' y_{r0}^{(k)}$$

This contradicts that  $v_i^*$  and  $u_r^*$  is an optimal solution of model (4.9).

In particular, when  $k=1$ , the LP model (4.9) is reduced to the LP model (3.5). So, model (3.5) is a special case of the LP model (4.9) for group decision making. Solving the LP model (4.9) for each DMU, we can obtain the relative efficiency of each DMU under group decision making.

## 5 Numerical example

In this section, we apply our approach to the information existed in the Tables 3-9. Suppose there are six CRs and four DMs. The relative importances of CRs are presented in Table 3.

Table 4 represents the assessment information provided by four DMs on the relationships between six CRs and four DRs. The relationships between three additional factors (cost, ease of implementation (easiness) and adverse environmental impact (adversity)) and DRs are shown in Table 5.

By using Wasserman suggestion and applying model (4.9), the efficiency scores (relative importances) of DRs and their ranking order are obtained. The relative importances of four DRs are (0.1475, 0.0407, 0.0449, 0.1513) (see Table 10).

In the Table 10 the efficiency scores obtained by using of model (4.9) and Ranking Order are show by S.E. and R.O. Respectively. From model (4.9), between two DRs the DR with the relative importance close to zero is more important, i.e., the DR with the efficiency scores close to zero is ranked above. So, the procedure indicates that DR2 is the most important DR, followed by DR3, while DR4 is considered the least important of the all the four DRs.



## 6 Conclusion

The goal of QFD is producing a product with high quality. In this way ranking the DRs is so important in QFD, specially, when each member of QFD-team demonstrates different assessment from the others, leading to QFD with vague nature. QFD-DEA, the methodology proposed by Ramanathan, uses CCR-input oriented model to rank the DRs. This methodology uses arithmetic average to obtain the final overall efficiency scores in such uncertainty environment. In this paper, we proposed a group DEA model which generates efficiency scores for each DR while considering the subjective assessments of DMs in one model. It is expected that the new QFD-DEA methodology can play an important role in the studies and applications of the QFD and even, in the all team-based managements approaches. Our future research work is to extend the proposed methodology in Fuzzy environment. This will be researched in near future.

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