Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 7, No. 3, 2015 Article ID IJIM-00628, 7 pages
Research Article

A general approach to linguistic approximation and its application in frame of fuzzy logic deduction

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Abstract

This paper deals with one problem that needs to be addressed in the emerging field known under the name computing with perceptions. It is the problem of describing, approximately, a given fuzzy set in natural language. This problem has lately been referred to as the problem of retranslation. An approaches to dealing with the retranslation problem is discussed in the paper, that is based on a pre-defined set of linguistic terms and the associated fuzzy sets. The retranslation problem is discussed in terms of two criteria validity and informativeness.

Keywords : Fuzzy sets; Regular Function; Defuzzification; Informativeness; Validity.

1 Introduction

THe emergence of computer technology in the second half of the 20th century opened many new possibilities for machines. These possibilities have been discussed in various contexts quite extensively in the literature. One aspect of considerable interest has been a comparison of existing and prospective capabilities of machines with those of human beings. An overall observation at this time is that the range of machine capabilities has visibly expanded over the years, from numerical computation to symbol manipulation, processing of visual data, learning from experience, etc. Moreover, machines have become superior to humans in some specific capabilities, such as large-scale processing of numerical data, massive combinatorial searches of various kinds, complex symbol manipulation, or sophisticated graphics. Some important new areas have emerged due to these machine capabilities, such as fractal geometry, cellular automata or evolutionary computing. In spite of the impressive advances of machines, they are still not able to match some capabilities of human beings. Perhaps the most exemplary of them are the remarkable and very complex perceptual abilities of the human mind, which allow humans to use perceptions in purposeful ways to perform complex tasks. Although current machines are not capable of reasoning and acting on the basis of perceptions, a feasible research program for developing this capability was recently proposed by Zadeh [17]. The crux of this program is to approximate perceptions by statements in natural language and, then, to use fuzzy logic to represent these statements and deal with them as needed. This approach to developing perception-based machines is referred to in the literature as computing with words, which is a name suggested also by Zadeh [16].
Approximating statements in natural language by propositions in fuzzy logic may be viewed

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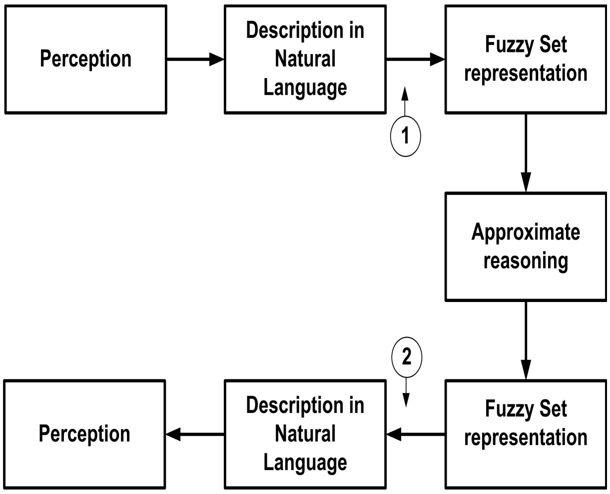
as a translation from natural language to a formalized language. Alternatively, it may be viewed as a linguistic approximation of the first kind. As is well known, this translation (or approximation) is strongly context dependent. Once it is accomplished in the context of a given application, all available resources of fuzzy logic in the broad sense can be utilized to emulate the ordinary (commonsense) human reasoning that pertains to the application (Yager et al. [15], Bezdek et al. [3]).
It is quite obvious that any relevant background knowledge should also be utilized in the reasoning Regardless of the nature of the reasoning process, its consequents are fuzzy propositions, each of which involves one or more fuzzy sets. In order to connect these fuzzy propositions to perceptions (i.e. to convey appropriate perceptions), we need to approximate them by statements in natural language. This means, in turn, that we need to express each of the fuzzy sets involved by a linguistic expression in natural language that has an understandable meaning in the given context. These issues pertain to the second kind of linguistic approximation, which may conveniently be called a retranslation. The whole process of perception-based reasoning, whose core is computing with words, is illustrated in a simplified way in Figure 1. The two kinds of linguistic approximation translation and retranslation are identified in the figure by labels 1 and 2, respectively.

While the problem of translation has been extensively studied and discussed in the literature, the problem of retranslation is far less developed. Prior to the late 1990s, this problem had been recognized only by a few authors, among them Eshragh and Mamdani [7] and Novak [10]. More recently, the problem has been addressed more substantially by Dvorak [6], Yager [14], Delgado et al. [5], Saneifard [11, 12, 13], Ezzati et al [8].
The paper is organized as follows. In Section 2, we introduce relevant concepts and notation. In this Section, we discuss the main issues involved in dealing with the problem of retranslation. Subsections *2.1* is application to fundamentally approach to retranslation. Our conclusions are covered in Section 3.



1. Linguistic approximation of the first kind: approximation of linguistic expressions by appropriate fuzzy sets.

2. Linguistic approximation of the second kind: approximation of fuzzy sets by appropriate linguistic expressions.

Figure 1: Perception-based reasoning.

2 Basic Definitions and Notations

In this paper, we assume that the reader is familiar with basics of fuzzy set theory and fuzzy logic in the broad sense. For the sake of completeness, we introduce in this section only those concepts that are relevant to our discussion of the retranslation problem. We denote all fuzzy sets in this paper by capital letters.
Classical sets are viewed as special fuzzy sets, called crisp sets, and are thus denoted by capital letters as well. For the sake of simplicity, we consider only numerical linguistic variables whose states are expressed by normal and convex fuzzy sets that are defined on some given closed interval, $X = [x_1, x_2]$, of real numbers. These fuzzy sets, usually referred to as fuzzy intervals, are viewed as concave functions from X to $[0, 1]$ whose maxima are 1.

Definition 2.1 *[10]. For any fuzzy interval $A : X \to [0, 1]$, its α-cut, A^α, is for each $\alpha \in [0, 1]$ the closed interval as follows:*

$$A^\alpha = \{x \in X \mid A(x) \geq \alpha\}.$$

Definition 2.2 *[10]. For each given fuzzy inter-*

val, A, the canonical form is as follows,

$$A(x) = \begin{cases} a_L(x) & when\ x \in [a,b), \\ 1 & when\ x \in [b,c], \\ a_R(x) & when\ x \in (c,d], \\ 0 & otherwise\ . \end{cases} \quad (2.1)$$

where $x \in X = [x_1, x_2]$ and a, b, c, d are real numbers in X such that $a \leq b \leq c \leq d$, a_L is a continuous and increasing function from $a_L(a) = 0$ to $a_L(b) = 1$, and a_R is a continuous decreasing function from $a_R(c) = 1$ to $a_R(d) = 0$.

For each value $\alpha \in [0,1]$, the α-cut of A, A^α, is a closed interval of real numbers defined by the formula

$$A^\alpha = [a_L^\alpha, a_R^\alpha], \quad (2.2)$$

where a_L^α and a_R^α are the inverse functions of a_L and a_R, respectively. The crisp sets

$$supp(A) = \{x \in X \mid A(x) > 0\},$$

$$core(A) = \{x \in X \mid A(x) = 1\},$$

are called, respectively, a support of A and a core of A. Clearly, $supp(A) = (a,d)$ and $core(A) = [b,c]$.

Definition 2.3 *[9]. A function $f : [0,1] \to [0,1]$ symmetric around $\frac{1}{2}$, i.e. $f(\frac{1}{2} - \alpha) = f(\frac{1}{2} + \alpha)$ for all $\alpha \in [0, \frac{1}{2}]$, which reaches its minimum in $\frac{1}{2}$, is called the bi-symmetrical weighted function. Moreover, the bi-symmetrical weighted function is called regular if*

(1) $f(\frac{1}{2}) = 0$,

(2) $f(0) = f(1) = 1$,

(3) $\int_0^1 f(\alpha) d\alpha = \frac{1}{2}$.

In most examples in this paper, we use trapezoidal fuzzy intervals, T, in which a_L and a_R are linear functions. That is, $a_L(x) = \frac{(x-a)}{(b-a)}$ and $a_R(x) = \frac{(d-x)}{(d-c)}$. Then, for each $\alpha \in (0,1]$,

$$T^\alpha = [a + (b-a)\alpha, d - (d-c)\alpha]. \quad (2.3)$$

Every trapezoidal fuzzy interval T is thus uniquely characterized via the quadruple

$$T = \langle a, b, c, d \rangle.$$

A special case in which $b = c$, which is called a triangular fuzzy interval, is also employed in this paper.

An important concept for dealing with the problem of retranslation is the degree of subsection, $s(A \subseteq B)$, of fuzzy set A in fuzzy set B (both defined on the same interval X), which is expressed by the formula [17],

$$s(A \subseteq B) = \frac{\int_X min\{A(x), B(x)\}dx}{\int_X A(x)dx}. \quad (2.4)$$

The minimum operator in this formula represents the standard intersection of fuzzy sets. As is well known, this is the only intersection of fuzzy sets that is cutworthy in the sense that

$$(A \cap B)^\alpha = A^\alpha \cap B^\alpha,$$

holds for all $\alpha \in [0,1]$. [2]

To deal with the retranslation problem, we also need to measure the non-specificity and fuzziness of the fuzzy sets involved.

Definition 2.4 *For any given normal and convex fuzzy set A, We define a well-justified measure of non-specificity, NS, as follows:*

$$NS(A) = \int_0^1 f(\alpha) Log_2[1 + L_m(A^\alpha)]^2 d\alpha. \quad (2.5)$$

Were $f : [0,1] \to [0,1]$ is a bi-symmetrical (regular) weighted function [9].

One can, of course, propose many regular bi-symmetrical weighted functions and hence obtain different bi-symmetrical weighted distances. Further on we will consider mainly a following function

$$f(\alpha) = \begin{cases} 1 - 2\alpha & when\ \alpha \in [0, \frac{1}{2}], \\ 2\alpha - 1 & when\ \alpha \in [\frac{1}{2}, 1]. \end{cases} \quad (2.6)$$

In Eq. (2.5), $L_m(A^\alpha)$ denotes for each $\alpha \in [0,1]$ the Lebesgue measure of A^α. In this case, $L_m(A^\alpha)$ is the length of the interval A^α for each $\alpha \in [0,1]$. The reason for choosing logarithm base 2 in this formula is to measure ambiguity in a convenient measurement unit: $NS(A) = 1$ when $L_m(A^\alpha) = 1$. Calculating ambiguity is more complicated for fuzzy sets defined on R^n when $n > 1$, but this is beyond the scope of this paper. Function NS is a special case of a more general measure of non-specificity (applicable to convex subsets of the n-dimensional Euclidean space), which is called a Hartley-like measure [14].

Table 1: Comparison results for skin friction co-efficient $(1+\frac{1}{\gamma})f''(0)$ in the case $k_0 = 0, S = 0$ and $M = 0.5$.

Sets of fuzzy numbers	Set 1		Set 2		Set 3		Set 4	
Ranking values	A	B	A	B	A	B	A	B
Cheng's method	0.58	0.70	0.58	0.58	0.58	0.58	0.46	0.58
Chu's method	0.15	0.25	0.15	0.15	0.15	0.15	0.12	0.15
Murakami's method	0.30	0.50	0.30	0.41	0.30	0.30	0.23	0.30
yager's method	0.30	0.50	0.30	0.30	0.30	0.30	0.30	0.30
The proposed method	0.44	0.48	0.42	0.44	0.44	0.47	0.35	0.44
Sets of fuzzy numbers	Set 5		Set 6		Set 7			
Ranking values	A	B	A	B	A	B	C	
Cheng's method	0.42	*	0.76	0.72	0.68	0.72	0.74	
Chu's method	0.15	*	0.28	0.26	0.22	0.26	0.27	
Murakami's method	0.41	*	0.60	0.50	0.44	0.53	0.52	
yager's method	0.30	*	0.60	0.50	0.44	0.53	0.52	
The proposed method	0.42	0.86	0.41	0.40	0.37	0.41	0.39	

Definition 2.5 *Given a convex fuzzy set A, its fuzziness, $f(A)$, can be measured by the overlap of A and its complement. Using standard operations of complementation and intersection of fuzzy sets, we have*

$$f(A) = \int_X \min\{A(x), 1 - A(x)\}dx. \tag{2.7}$$

Clearly, $f(A) = 0$ if and only if A is a crisp (classical) set, and the maximum degree of fuzziness is obtained for the unique fuzzy set in which $A(x) = 1 - A(x) = 0.5$ for all $x \in X$.
Two criteria that are considered in this paper as essential are validity and informativeness.

Definition 2.6 *The degree of validity of choosing a standard fuzzy interval F that has a linguistic interpretation to represent a given convex fuzzy set G, $v(F \mid G)$, as the degree to which G is contained in F define as follows,*

$$v(F \mid G) = \frac{\int_X \min\{G(x), F(x)\}dx}{\int_X G(x)dx}. \tag{2.8}$$

For any pair of standard fuzzy intervals, F_1 and F_2, that compete for representing a given fuzzy set G, clearly, if $v(F_1 \mid G) \geq v(F_2 \mid G)$ then F_1 is preferable to F_2 according to validity.

Definition 2.7 *The degree of informativeness of F, $i(F)$, is concerned, it is define it as the normalized reduction of non-specificity with respect to the non-specificity of X as follows:*

$$i(F) = 1 - \frac{NS(F)}{Log_2[1 + L_m(X)]}. \tag{2.9}$$

Clearly, if $i(F_1) \geq i(F_2)$ then F_1 is preferable to F_2 according to informativeness.

Example 2.1 *Let G is a fuzzy number with membership function as follows that* $X = [-10, 10]$,

$$G(x) = \begin{cases} \frac{x}{2} & when\ x \in [0,2), \\ 2.4 - 0.7x & when\ x \in [2,3), \\ 0.3 & when\ x \in [3,4), \\ 1.5 - 0.3x & when\ x \in [4,5), \\ 0 & otherwise. \end{cases} \tag{2.10}$$

Clearly

$$G^\alpha = \begin{cases} [2\alpha, (15 - 10\alpha)/3], & when\ \alpha \in (0, 0.3], \\ [2\alpha, (24 - 10\alpha)/3], & when\ \alpha \in (0.3, 1]. \end{cases} \tag{2.11}$$

There is

$$NS(G) = \int_0^{0.3} \alpha Log_2[1 + (15 - 16\alpha)/3]^2 d\alpha$$

$$+ \int_{0.3}^{1} \alpha Log_2[1 + (24 - 24\alpha)]^2 d\alpha$$

$$= 2.6317,$$

and

$$i(G) = 1 - \frac{NS(G)}{Log_2(21)} = 0.4008$$

2.1 Using Ranking Method In Fuzzy Multi-criteria Decision Making Based on An FN-IOWA Operator

Chen and Chen [4] proposed a method to handle fuzzy multi-criteria decision making problems based on fuzzy number induced ordered weighted averaging (FN-IOWA)operator and applied the algorithm to a human selection problem. In this section, we use the same example illustrated in Chen and Chen (2003) to show the efficiency of the proposed ranking method. For more detailed information about the FN-IOWA operator, (see R. R. Yager 1998, R. R. Yager and D. P. Filev 1999, Chen and Chen 2003). Here we just pay attention to the fuzzy ranking step in the final decision making process.
A new manager will be recruited among three candidates, X, Y and Z. The final scores, which can be obtained by an FN-IOWA operator, are fuzzy numbers and are listed as follows:

$$S_X = (0.2501, 0.7727, 2.2501),$$
$$S_Y = (0.0667, 0.5000, 1.8750),$$
$$S_Z = (0.1667, 0.6592, 2.2500).$$

By applying the degree of informativeness of X, Y and Z, the index regular of each alternative can be obtained as follows:

$$i(X) = 0.0988,$$
$$i(Y) = 0.0160,$$
$$i(Z) = 0.1250.$$

We can see that their ranking order $Z > X > Y$. Therefore, Candidate Z is more suitable than Candidate X, and Candidate X is more suitable than Candidate Y. The result are the same as the one presented in Chen and Chen [4].

Example 2.2 *In the following, we use seven sets of generalized trapezoidal fuzzy numbers adopted from [1, 11] to compare the proposed fuzzy ranking method with some other existing ranking methods. The eight sets of fuzzy numbers are shown in Fig. 2. A comparison of the ranking results of the proposed method with the existing methods is described as follows:*

1. From Sets I ($I = 1, \cdots, 7$), , of Fig. 2, we can see that the fuzzy numbers A and B are different fuzzy numbers, where the ranking order is $A \prec B$. From Table 1, we can see that the proposed method and the methods presented in Table 1 get the correct results.

2. In [8], the authors point out that the ranking order of Set 7, is $A \prec B \prec C$, however from Table 1, we can see that the methods presented in [4] and [18] get incorrect ranking results.

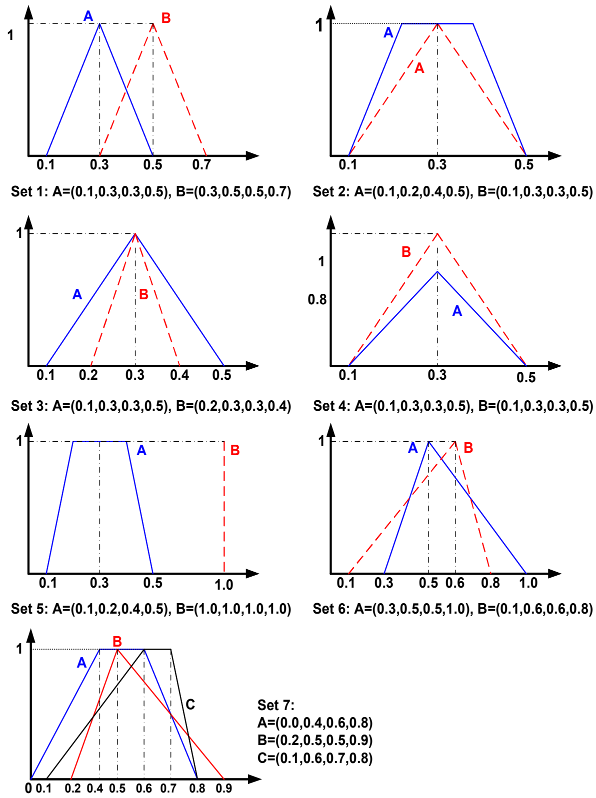


Figure 2: Seven sets of fuzzy numbers.

3 Conclusion

In this study, the researcher suggests a new approach to the problem of defuzzification using the regular weighted function of fuzzy numbers and some preliminary results on properties of such defuzzification are to be reported. Also an approaches to dealing with the retranslation problem discussed in the paper, that is based on a pre-defined set of linguistic terms and the associated fuzzy sets. In the future, we will develop a method for fuzzy set approximation based on informativeness of fuzzy numbers with any kinds of membership functions.

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