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Research Article

Developing a Data Envelopment Analysis Methodology for Supplier Selection in the Presence of Fuzzy Undesirable Factors

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Abstract
Supplier selection is a multi-criteria decision problem which includes both qualitative and quantitative factors. We present in this paper a model for supplier selection based on DEA methodology that considered both undesirable factors and fuzzy data simultaneously. The proposed method has been illustrated by a numerical example.

Keywords : Data envelopment analysis; Supplier selection; Undesirable factors; Fuzzy data.

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1 Introduction

Nowadays fierce competitive environment, characterized by thin profit margins, high consumer expectations for quality products and short lead-times, companies are forced to take advantage of any opportunity to optimize their business processes. To reach this aim, academics and practitioners have come to the same conclusion: for a company to remain competitive, it has to work with its supply chain partners to improve the chain's total performance. Thus, being the main process in the upstream chain and affecting all areas of an organization. Purchasing is one of the most important strategic activities in supply chain [1]. One of the most critical functions in purchasing is to select supplier. The objective of supplier selection process is to identify suppliers with the highest potential for meeting a manufacturer's needs consistently and acceptable overall performance. Selecting suppliers from a large number of possible suppliers with various levels of capabilities and potential is a difficult task and inherently a multi criteria decision-making

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(MCDM) problem. Supplier selection decisions are complicated because various criteria must be considered in the decision-making process [2, 5, 7]. One of the techniques for supplier selection is Data Envelopment Analysis (DEA). Data envelopment analysis measures the relative efficiency of decision making units (DMUs) with multiple performance factors which are grouped into outputs and inputs. Once DEA identifies the efficient frontier, DEA improves the performance of inefficient DMUs by either increasing the current output levels or decreasing the current input levels [8]. However, both desirable and undesirable output and input factors may be present. However, in the standard DEA model, decreases in outputs are not allowed and only inputs are allowed to decrease. (Similarly, increases in inputs are not allowed and only outputs are allowed to increase). For example, if inefficiency exists in production processes where final products are manufactured with a production of wastes and pollutants, the outputs of wastes and pollutants are undesirable and should be reduced to improve the performance[9].Traditional DEA models do not deal with imprecise data and assume that all input and output data are exactly know, but in real world, this assumption is not always true. Uncertain information or imprecise data can be expressed in interval or fuzzy number. In many real-world applications DEA (especially supplier selection problems), it is essential to take into account the existence of both undesirable factors and both qualitative and quantitative factors. This paper depicts the supplier selection process through an fuzzy data envelopment analysis (FDEA) model, while allowing for the incorporation of undesirable factors. The aim of this paper is to propose a data envelopment analysis models for selecting the best suppliers in the presence of both undesirable factors and fuzzy data. The proposed approach developed in this paper includes a number of contributions, as follows:

- This paper proposed a model capable of treating imprecise factors.
-The proposed model considers multiple criteria, this helps managers to select suppliers using a comprehensive approach that goes beyond just purchase costs.
- The proposed model does not demand weights from the decision maker.
- The proposed model can consider both fuzzy data and undesirable factors for supplier selection problems.

This paper proceeds as follows. In Section 2, Notations and definitions is presented. Section 3, introduces the model, numerical example and concluding remarks are discussed in Sections 4 and 5, respectively.

2 Notation and Definition

First, the notations which shall be used in this paper will be introduced. All fuzzy sets are fuzzy subsets of real numbers. A fuzzy number is a fuzzy set of the real line with a normal, convex and upper semicontinuous membership function of a bounded support. The family of fuzzy numbers will be denoted by E .The membership function for fuzzy number u can be expressed as

$$u(x) = \begin{cases} u_l(x), & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ u_r(x), & c \leq x \leq d, \\ 0 & otherwise. \end{cases} \tag{2.1}$$

Where $u_l : [a, b] \to [0, 1]$ and $u_r : [a, b] \to [0, 1]$ are left and right membership functions of fuzzy number . And equivalent parametric form is also given in [6] as follows:

Definition 2.1. *An arbitrary fuzzy number is presented by an ordered pair of functions* $(\underline{u}(\alpha), \overline{u}(\alpha))$ *,*$0 \leq \alpha \leq 1$ *, which satisfies the following requirements:*
1-$\underline{u}(\alpha)$ *is a bounded left continuous nondecreasing function over [0,1], with respect to any* α*.*
2-$\overline{u}(\alpha)$ *is a bounded left continuous nonincreasing function over [0,1], with respect to any* α*.*
3-$\underline{u}(\alpha) \leq \overline{u}(\alpha)$*,* $0 \leq \alpha \leq 1$*.*

The trapezoidal fuzzy number $u = (x_0, y_0, s, t)$ with two defuzzifier x_0, y_0 and left fuzziness $s > 0$ and right fuzziness $t > 0$ is a fuzzy set where the membership function is as

$$u(x) = \begin{cases} \frac{1}{s}(x - x_0 + s) & x_0 - s \leq x \leq x_0 \\ 1 & x \in [x_0, y_0] \\ \frac{1}{t}(y_0 - x + t) & y_0 \leq x \leq y_0 + t \\ 0 & otherwise. \end{cases} \tag{2.2}$$

and parametric form is

$$\underline{u}(\alpha) = x_0 - s + s\alpha, \qquad \overline{u}(\alpha) = y_0 + t - t\alpha.$$

Definition 2.2. *For arbitrary* $u = (\underline{u}(\alpha), \overline{u}(\alpha))$*,* $v = (\underline{v}(\alpha), \overline{v}(\alpha))$ *and* $k > 0$*, addition, standard substraction and standard multiplication by* k *are as follows:*

$u + v = (\underline{u}(\alpha) + \underline{v}(\alpha), \overline{u}(\alpha) + \overline{v}(\alpha))$
$u - v = (\underline{u}(\alpha) - \overline{v}(\alpha), \overline{u}(\alpha) - \underline{v}(\alpha))$

$$ku = \begin{cases} (k\underline{u}, k\overline{u}) & \text{if } k \geq 0, \\ (k\overline{u}, k\underline{u}) & \text{if } k < 0. \end{cases} \tag{2.3}$$

Definition 2.3. *[6] For arbitrary fuzzy numbers* $u = (\underline{u}(\alpha), \overline{u}(\alpha))$ *and* $v = (\underline{v}(\alpha), \overline{v}(\alpha))$*, the function*

$$d_p(u, v) = [\int_0^1 |\underline{u}(\alpha) - \underline{v}(\alpha)|^p dr + \int_0^1 |\overline{u}(\alpha) - \overline{v}(\alpha)|^p d\alpha]^{1/p} \qquad (p \geq 1) \tag{2.4}$$

is the distance between u and v.

Definition 2.4. *[3]Let* $\{A_1, A_2, \cdots, A_m\}$ *are* m *numbers, the Maximizing set* M *is fuzzy subset with membership function* $M(x)$ *given as*

$$M(x) = \begin{cases} [\frac{(x - x_{min})}{(x_{max} - x_{min})}]^k, & x_{min} \leq x \leq x_{max}, \\ 0, & otherwise, \end{cases} \tag{2.5}$$

where $x_{min} = \inf S$*,* $x_{max} = \sup S$*,* $S = \bigcup_{i=1}^n S_i$*, and* $S_i = \{x | \ A_i(x) > 0\}$*.*

Definition 2.5. *[3] Let* $\{A_1, A_2, \cdots, A_m\}$ *are* m *numbers, the Minimizing set* G *is fuzzy subset with membership function* $G(x)$ *given as*

$$G(x) = \begin{cases} [\frac{(x - x_{max})}{(x_{max} - x_{min})}]^k, & x_{min} \leq x \leq x_{max}, \\ 0, & otherwise, \end{cases} \tag{2.6}$$

where $x_{min} = \inf S$*,* $x_{max} = \sup S$*,* $S = \bigcup_{i=1}^n S_i$*, and* $S_i = \{x | \ A_i(x) > 0\}$*.*

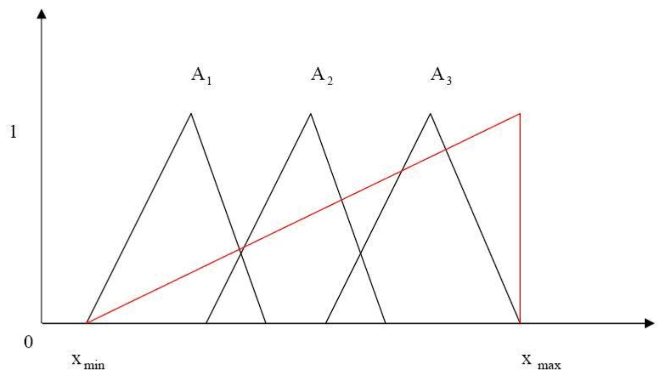

Figure 1: *The maximizing set for fuzzy numbers A_1, A_2 and A_3.*

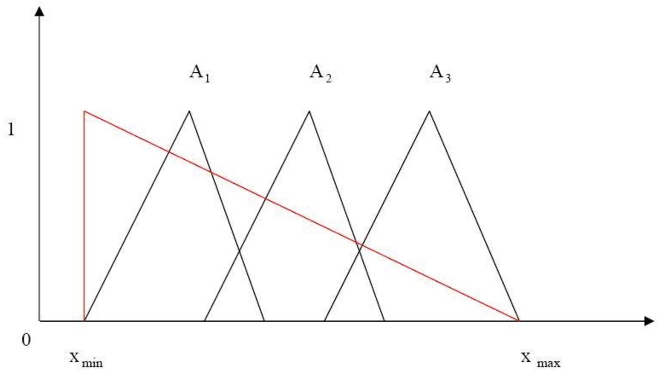

Figure 2: *The minimizing set for fuzzy numbers A_1, A_2 and A_3.*

3 Proposed approach

Consider a situation where n members of a set of n DMUs are to be evaluated in terms of s fuzzy outputs $Y_k = (y_{rk})_{r=1}^{s}$ and m fuzzy inputs $X_k = (x_{ik})_{i=1}^{m}$, where $Y_k^{(D)} = (y_{rk}^{(D)})_{r=1}^{s_1}$ and $Y_k^{(U)} = (-y_{rk}^{(U)})_{r=1}^{s_2} = (\widehat{y}_{rk}^{(U)})_{r=1}^{s_2}$ are desirable and undesirable fuzzy outputs, $X_k^{(D)} = (x_{ik}^{(D)})_{i=1}^{m_1}$ and $X_k^{(U)} = (x_{ik}^{(U)})_{i=1}^{m_2}$ are desirable and undesirable fuzzy inputs, in which $s_1 + s_2 = s$ and $m_1 + m_2 = m$.

3.1 Undesirable output

In this section, we consider the DEA efficiency analysis, when undesirable outputs are produced in production process. Let $Y_k^{(D)} = (y_{rk}^{(D)})_{r=1}^{s_1}$ and $Y_k^{(U)} = (y_{rk}^{(U)})_{r=1}^{s_2}$ are desirable and undesirable fuzzy outputs, where $s_1 + s_2 = s$. In order to improved the relative performance, we would like to increase $Y^{(D)}$ and on the contrary $Y^{(U)}$ does not allow to increase, and we would like to decrease $Y^{(U)}$.

For this purpose, we define maximizing set M_1 for desirable outputs and maximizing set M_2 for undesirable outputs .

Let $S_{rk} = supp\{y_{rk}^{(D)},\ r = 1, \cdots, s_1\}$, $S'_{rk} = supp\{-y_{rk}^{(U)},\ r = 1, \cdots, s_2\}$
$S_k = \bigcup_{r=1}^{s_1} S_{rk}$, $S'_k = \bigcup_{r=1}^{s_2} S'_{rk}$,
$x_{min} = \inf S_k$, $x_{max} = \sup S_k$, and $x'_{min} = \inf S'_k$, $x'_{max} = \sup S'_k$.
Then we define minimizing set m_1 for desirable set and minimizing set m_2 for undesirable set as follows:

$$m_1(x) = \begin{cases} \frac{(x_{max}-x)}{(x_{max}-x_{min})}, & x_{min} \leq x \leq x_{max}, \\ 0, & otherwise, \end{cases} \tag{3.7}$$

$$m_2(x) = \begin{cases} \frac{(x'_{max}-x)}{(x'_{max}-x'_{min})}, & x'_{min} \leq x \leq x'_{max}, \\ 0, & otherwise, \end{cases} \tag{3.8}$$

Also

$$m_1(\alpha) = (x_{min}, x_{max} - \alpha(x_{max} - x_{min})) \tag{3.9}$$

$$m_2(\alpha) = (x'_{min}, x'_{max} - \alpha(x_{max} - x_{min})) \tag{3.10}$$

The distance between Y^D and minimizing set m_1 is shown by $d(Y^D, m_1)$ and is defined as follows:

$$d(Y_k^D, m_1) = (\int_0^1 [(\underline{m_1(\alpha)} - \underline{Y_k^D(\alpha)})^2 + (\overline{m_1(\alpha)} - \overline{Y_k^D(\alpha)})^2] d\alpha)^{\frac{1}{2}}, \quad r = 1, \cdots, s_1. \tag{3.11}$$

Obviously, in order to improve the relative performance, we would like to increase the distance between Y_k^D and the worse case of y_{rk}^D, $r = 1, \ldots, s_1$.

The distance between Y^U and minimizing set m_2 is shown by $d(Y^U, m_2)$ and is defined as follows:

$$d(Y_k^U, m_2) = (\int_0^1 [(\underline{m_2(\alpha)} - \underline{Y_k^U(\alpha)})^2 + (\overline{m_2(\alpha)} - \overline{Y_k^U(\alpha)})^2] d\alpha)^{\frac{1}{2}}, \quad r = 1, \cdots, s_2. \tag{3.12}$$

In order to improve the relative performance, we would like to increase the distance between $Y_k^U = (-y_{rk}^U)_{r=1}^{s_2}$ and the worse case of $-y_{rk}^U$, $r = 1, \ldots, s_2$.

Now we would like to increase $d(Y^D, m_1)$ and $d(Y^U, m_2)$.
Base upon previous equations, we have the following linear program:

$$
\begin{aligned}
& Max \quad \beta && (3.13)\\
s.t. \quad & \sum_{j=1}^{n} \lambda_j X_{ij} \leq X_{ip}, \quad i = 1, \cdots, m,\\
& \sum_{j=1}^{n} \lambda_j d(y_{rj}^{D}, m_1) \geq \beta d(y_{rp}^{D}, m_1), \quad r = 1, \cdots, s_1,\\
& \sum_{j=1}^{n} \lambda_j d(y_{rj}^{U}, m_2) \geq \beta d(y_{rp}^{U}, m_2), \quad r = 1, \cdots, s_2,\\
& \sum_{j=1}^{n} \lambda_j = 1,\\
& \lambda_j \geq 0, \quad j = 1, \cdots, n,
\end{aligned}
$$

Also, if we have fuzzy inputs, the distance between fuzzy numbers X_{ij} and 0 is used. Finally we have the following models for fuzzy inputs and fuzzy desirable and undesirable outputs.

$$
\begin{aligned}
& Max \quad \beta && (3.14)\\
s.t. \quad & \sum_{j=1}^{n} \lambda_j d(X_{ij}, 0) \leq d(X_{ip}, 0), \quad i = 1, \cdots, m,\\
& \sum_{j=1}^{n} \lambda_j d(y_{rj}^{D}, m_1) \geq \beta d(y_{rp}^{D}, m_1), \quad r = 1, \cdots, s_1,\\
& \sum_{j=1}^{n} \lambda_j d(y_{rj}^{U}, m_2) \geq \beta d(y_{rp}^{U}, m_2), \quad r = 1, \cdots, s_2,\\
& \sum_{j=1}^{n} \lambda_j = 1,\\
& \lambda_j \geq 0, \quad j = 1, \cdots, n,
\end{aligned}
$$

4 Numerical example

To illustrate the proposed method, a numerical example has been presented. The data set for this example are partially taken from Farzipoor Saen [4]. The example contains specifications on 18 suppliers. As Farzipoor Saen addressed, the cardinal input is considered as total cost of shipments (TC). Supplier reputation (SR) considered as fuzzy input .The desirable output utilized number of bills received from the supplier without errors (NB)and the undesirable output is parts per million (PPM) of defective parts. NB and PPM considered as the fuzzy data output. Table 1, depicts the supplier's characters. For calculating the efficiency first, we must calculate the minimizing set for all fuzzy outputs.

Table 1: *Depicts the supplier's characters*

Supplier	Inputs		Desirable output	Undesirable output
No	TC	SR	NB	PPM
	x_{1j}	x_{2j}	y_{1j}	y_{2j}
1	253	$(0.015+\alpha, 0.229-\alpha)$	$(50+\alpha, 65-\alpha)$	$(\alpha, 2-\alpha)$
2	268	$(0.027+\alpha, 0.403-\alpha)$	$(60+\alpha, 70-\alpha)$	$(4.3+\alpha, 6.3-\alpha)$
3	259	$(0.012+\alpha, 0.182-\alpha)$	$(40+\alpha, 50-\alpha)$	$(3.6+\alpha, 5.6-\alpha)$
4	180	$(0.017+\alpha, 0.256-\alpha)$	$(100+\alpha, 160-\alpha)$	$(28+2\alpha, 32-2\alpha)$
5	257	$(0.014+\alpha, 0.204-\alpha)$	$(45+\alpha, 55-\alpha)$	$(28+2\alpha, 32-2\alpha)$
6	248	$(0.011+\alpha, 0.163-\alpha)$	$(85+\alpha, 115-\alpha)$	$(28+2\alpha, 32-2\alpha)$
7	272	$(0.022+\alpha, 0.321-\alpha)$	$(70+\alpha, 95-\alpha)$	$(28+2\alpha, 32-2\alpha)$
8	330	$(0.031+\alpha, 0.452-\alpha)$	$(100+\alpha, 180-\alpha)$	$(12.8+\alpha, 14.8-\alpha)$
9	327	$(0.024+\alpha, 0.360-\alpha)$	$(90+\alpha, 120-\alpha)$	$(2+2\alpha, 6-2\alpha)$
10	330	$(0.019+\alpha, 0.287-\alpha)$	$(50+\alpha, 80-\alpha)$	$(29+\alpha, 29-\alpha)$
11	321	$(0.054+\alpha, 0.797-\alpha)$	$(250+\alpha, 300-\alpha)$	$(25.4+\alpha, 27.4-\alpha)$
12	329	$(0.043+\alpha, 0.635-\alpha)$	$(100+\alpha, 150-\alpha)$	$(24.8+\alpha, 26.8-\alpha)$
13	281	$(0.048+\alpha, 0.711-\alpha)$	$(80+\alpha, 120-\alpha)$	$(24.8+\alpha, 26.8-\alpha)$
14	309	$(0.038+\alpha, 0.567-\alpha)$	$(200+\alpha, 350-\alpha)$	$(20.9+\alpha, 22.9-\alpha)$
15	291	$(0.034+\alpha, 0.506-\alpha)$	$(40+\alpha, 55-\alpha)$	$(8+\alpha, 10-\alpha)$
16	334	$(0.061+\alpha, 0.892-\alpha)$	$(75+\alpha, 85-\alpha)$	$(6+\alpha, 9-\alpha)$
17	249	$(0.01+\alpha, 0.145-\alpha)$	$(90+\alpha, 180-\alpha)$	$(4.3+2\alpha, 8.3-2\alpha)$
18	216	$(0.06866+\alpha, 1)$	$(90+\alpha, 150-\alpha)$	$(27.8+\alpha, 29.8-\alpha)$

For desirable outputs y_{1j} we compute minimizing set m_1, by $m_1 = (40, 350 - 310\alpha)$ and for undesirable outputs y_{2j} we have to compute minimizing set m_2 for $-y_{2j}$, in table 2, $-y_{2j}$ is computed for undesirable outputs. Then m_2 is compute by $m_2 = (-32, 32\alpha)$.

The last column of table 3, reports the results of efficiency assessments for 18 suppliers(DMUs) gained by using proposed model. Results of evaluation by using Model (3.14) show that, suppliers 5, 10, 11, 12, and 16 are efficient with a relative efficiency score of 1 and the remaining 13 suppliers with relative efficiency scores of more than 1 are considered to be inefficient.

5 Concluding remarks

Evaluation and selection of suppliers has become one of the major concerns of any corporation, and it takes multiple and conflict goals into consideration. Therefore to meet this challenge, applying multi-criteria techniques to select and evaluate the best supplier is inevitable. In this paper we have developed a new fuzzy DEA model to selection supplier in presence of fuzzy data and undesirable factors.

Table 2: *Computing $-y_{2j}$ for undesirable outputs*

Supplier No	y_{2j}	$-y_{2j}$
1	$(\alpha, 2-\alpha)$	$(-2+\alpha, -\alpha)$
2	$(4.3+\alpha, 6.3-\alpha)$	$(-6.3+\alpha, -4.3-\alpha)$
3	$(3.6+\alpha, 5.6-\alpha)$	$(-5.6+\alpha, -3.6-\alpha)$
4	$(28+2\alpha, 32-2\alpha)$	$(-32+2\alpha, -28-2\alpha)$
5	$(28+2\alpha, 32-2\alpha)$	$(-32+2\alpha, -28-2\alpha)$
6	$(28+2\alpha, 32-2\alpha)$	$(-32+2\alpha, -28-2\alpha)$
7	$(28+2\alpha, 32-2\alpha)$	$(-32+2\alpha, -28-2\alpha)$
8	$(12.8+\alpha, 14.8-\alpha)$	$(-14.8+\alpha, -12.8-\alpha)$
9	$(2+2\alpha, 6-2\alpha)$	$(-6+2\alpha, -2-2\alpha)$
10	$(29+\alpha, 29-\alpha)$	$(-29+\alpha, -29-\alpha)$
11	$(25.4+\alpha, 27.4-\alpha)$	$(-27.4+\alpha, -25.4-\alpha)$
12	$(24.8+\alpha, 26.8-\alpha)$	$(-26.8+\alpha, -24.8-\alpha)$
13	$(24.8+\alpha, 26.8-\alpha)$	$(-26.8+\alpha, -24.8-\alpha)$
14	$(20.9+\alpha, 22.9-\alpha)$	$(-22.9+\alpha, -20.9-\alpha)$
15	$(8+\alpha, 10-\alpha)$	$(-10+\alpha, -8-\alpha)$
16	$(6+\alpha, 9-\alpha)$	$(-9+\alpha, -6-\alpha)$
17	$(4.3+2\alpha, 8.3-2\alpha)$	$(-8.3+2\alpha, -4.3-2\alpha)$
18	$(27.8+\alpha, 29.8-\alpha)$	$(-29.8+\alpha, -27.8-\alpha)$

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Table 3: *Computing distance for 18 supplier and efficiency scores*

Supplier No	Inputs		Desirable output	Undesirable output	
	TC	SR	NB	PPM	
	x_{1j}	$d(x_{2j}, 0)$	$d(y_{1j}^D, m_1)$	$d(y_{2j}^U, m_2)$	Efficiency
1	253	0.24491	158.42	35.96	1.251946
2	268	0.43161	155.32	34.78	1.292175
3	259	0.19524	170.66	34.90	1.272213
4	180	0.2743	113.47	46.07	1.011721
5	257	0.21867	166.51	46.07	1
6	248	0.17432	128.48	46.07	1.011721
7	272	0.34408	137.79	46.07	1.010435
8	330	0.48341	108.89	35.53	1.246629
9	327	0.38537	127.3	34.44	1.331957
10	330	0.30722	146.32	46.61	1
11	321	0.85193	251.37	43.27	1
12	329	0.67915	116.99	42.76	1
13	281	0.76065	123.68	42.76	1.058076
14	309	0.60639	239.97	39.77	1.047506
15	291	0.54142	166.42	34.60	1.230435
16	334	0.95416	146.38	33.90	1
17	249	0.15564	103.66	34.06	1.292568
18	216	1.06866	112.14	45.39	1.000021

References

[1] N. Aissaouia, M. Haouaria, E. Hassinib, *Supplier selection and order lot sizing modeling: A review*, Computers and Operations Research 34 (2007) 3516 - 3540.

[2] C. Araz, , P. Mizrak Ozfirat, I. Ozkarahan, *An integrated multi criteria decision making methodology for outsourcing management*, Computers and Operations Research 34 (2007) 3738-3756.

[3] S. Chen, *Ranking fuzzy numbers with maximizing set and minimizing set*, Fuzzy Sets and Systems 17 (1985) 113-129.

[4] R. Farzipoor Saen, *Developing a new data envelopment analysis methodology for supplier selection in the presence of both undesirable outputs and imprecise data*, The International Journal of Advanced Manufacturing Technology 51 (2010) 1243-1250.

[5] J. Jassbi, R. Farzipoor Saen, F. Hosseinzadeh Lotfi, SH. S. Hosseininia, S. A. KHanmohammadi, *Hybrid decision-making system using data envelopment analysis and fuzzy models for supplier selection in the presence of multiple Decision makers*, International Journal Of Industrial Mathematics 3 (2011) 193-212.

[6] M. Ma, M. Friedman,A. Kandel, *A new fuzzy arithmetic*, Fuzzy Sets and Systems 108 (1999) 83-90.

[7] M. Sanei, S. Mamizadeh Chatghaye, *Evaluation of supply chain oprations using slacks-based measure of efficiency*, International Journal Of Industrial Mathematics 3 (1)(2011) 35-40.

[8] L. M. Seiford, J. Zhu, *Modeling undesirable factors in efficiency evaluation*, European Journal of Operational Research 142(2002) 16-20.

[9] Y. M. Wang, R. Greatbanks, J. B. Yang, *Interval efficiency assessment using data envelopment analysis*, Fuzzy Sets and Systems 153 (2005) 347-370.