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Research Article

Evaluation of current distribution induced on perfect electrically conducting scatterers

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Abstract

The focus of this article is on the analysis of electromagnetic scattering from conducting bodies in TE polarization. For modeling of such problems, the magnetic field integral equation is used and the current density induced on the surface of scatterer is considered as the solution of the mentioned integral equation. A numerical approach is surveyed for calculating the induced current density because it has no analytical form, in general. Finally, three scattering structures are analyzed and the current density plots are given for them.

Keywords : Magnetic field integral equation; Two-dimensional electromagnetic scattering; Integral equation modeling; Current distribution; Numerical evaluation.

1 Introduction

VAluable efforts have been spent, by researchers, on introducing novel ideas for the solution of various functional equations (for example, see [2, 3, 7, 11, 12]). The development of numerical methods for solving integral equations in Electromagnetics has attracted intensive researches for more than five decades. The use of high-speed computers allows one to make more computations than ever before. During these years, careful analysis has paved the way for the development of efficient and effective numerical methods and, of equal importance, has provided a solid foundation for a through understanding of the techniques.

Over several decades, electromagnetic scattering problems have been the subject of extensive researches (see [13, 10, 1, 8, 6, 5] and their references). Scattering from arbitrary surfaces such as square, cylindrical, circular, spherical are commonly used as test cases in computational Electromagnetics.

The key to the solution of any scattering problem is a knowledge of the physical or equivalent current density distributions on the volume or surface of the scatterer. Once these are known then the radiated or scattered fields can be found using the standard radiation integrals. A main objective then of any solution method is to be able to predict accurately the current densities over the scatterer. This can be accomplished by the integral equation (IE) method [6, 4].

In general there are many forms of integral equations. Two of the most popular for time-harmonic Electromagnetics are the electric field integral equation (EFIE) and the magnetic field integral equation (MFIE). The EFIE enforces the boundary condition on the tangential electric field and the MFIE enforces the boundary condition on the tangential components of the mag-

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netic field.
This article deals with the use of MFIE in analysis of electromagnetic scattering from two-dimensional perfect electrically conducting (PEC) structures. For this purpose, a general form of MFIE will be used for modeling of electromagnetic scattering from arbitrary structures and then the formulation will be simplified for the case of a generic two-dimensional body. Afterward, a numerical treatment will be tested to obtain the current distribution induced on some perfectly conducting scatterers from the two-dimensional MFIE model.

2 Mathematical modeling of electromagnetic scattering from perfectly conducting bodies

The MFIE is expressed in terms of the known incident magnetic field. It is based on the boundary condition that expresses the total electric current density induced at any point $r = r'$ on the surface of a conducting surface S should equal the tangential component of the total magnetic field, i.e. [4]

$$\begin{aligned}\mathbf{J}_s(r') = \mathbf{J}_s(r = r') &= \hat{n} \times \mathbf{H}^t(r = r') \\ &= \hat{n} \times [\mathbf{H}^i(r = r') \\ &+ \mathbf{H}^s(r = r')],\end{aligned} \tag{2.1}$$

where $\mathbf{J}_s$ is the electric current density on the scatterer surface; $\mathbf{H}^i$, $\mathbf{H}^s$, and $\mathbf{H}^t$ are respectively the incident, scattered, and total magnetic fields; $\hat{n}$ is unit vector perpendicular to the surface; and the unprimed and primed coordinates are respectively assigned to the observation (field) points and source points.
The scattered magnetic field can be written in terms of magnetic vector potential as [4]

$$\begin{aligned}\mathbf{H}^s(r) &= \frac{1}{\mu}\nabla \times \mathbf{A} \\ &= \nabla \times \iint_S \mathbf{J}_s(r')G(\mathbf{r},\mathbf{r}')ds',\end{aligned} \tag{2.2}$$

where $\mathbf{A}$ is the magnetic vector potential, $\nabla\times$ is the curl operator with respect to the observation coordinates, and $G(\mathbf{r},\mathbf{r}')$ is the Green's function for a three-dimensional scatterer defined by

$$G(\mathbf{r},\mathbf{r}') = \frac{e^{-j\beta R}}{4\pi R}, \tag{2.3}$$

in which j is imaginary unit, β is phase constant, and R is as follows:

$$R = |\mathbf{r} - \mathbf{r}'|, \tag{2.4}$$

where $\mathbf{r}$ and $\mathbf{r}'$ are position vectors of the observation and source points, respectively.
The differential operator $\nabla\times$ in (2.2) is in terms of the unprimed coordinates. This permits it to be moved to the front of the integral symbol, such that

$$\mathbf{H}^s(r) = \iint_S \nabla \times [\mathbf{J}_s(r')G(\mathbf{r},\mathbf{r}')]ds'. \tag{2.5}$$

Now we use the following vector identity [4]:

$$\nabla \times (\mathbf{J}_s G) = G\nabla \times \mathbf{J}_s - \mathbf{J}_s \times \nabla G, \tag{2.6}$$

where ∇ is the gradient operator with respect to the observation coordinates. $\mathbf{J}_s(r')$ is in terms of the primed (source) coordinates, therefore we have

$$\nabla \times \mathbf{J}_s(r') = 0. \tag{2.7}$$

Moreover, it can be easily observed that

$$\nabla G = -\nabla' G. \tag{2.8}$$

Considering (2.5)-(2.8) it is concluded that

$$\mathbf{H}^s(r) = \iint_S [\mathbf{J}_s(r') \times \nabla' G(\mathbf{r},\mathbf{r}')]ds', \tag{2.9}$$

and, if the observations are restricted on the surface of the scatterer ($r \to S$), then

$$\mathbf{H}^s(r = r') = \lim_{r\to S}\left\{\iint_S [\mathbf{J}_s(r') \times \nabla' G(\mathbf{r},\mathbf{r}')]ds'\right\}. \tag{2.10}$$

Now, according to boundary condition (2.1) and (2.10) we have [4]

$$\mathbf{J}_s(r') = \hat{n}\times \left[\mathbf{H}^i(r = r') + \lim_{r\to S}\left\{\iint_S [\mathbf{J}_s(r') \times \nabla' G(\mathbf{r},\mathbf{r}')]ds'\right\}\right] \tag{2.11}$$

or

$$\mathbf{J}_s(r') - \lim_{r\to S} \left\{\hat{n} \times \iint_S [\mathbf{J}_s(r') \times \nabla' G(\mathbf{r},\mathbf{r}')]ds'\right\} \tag{2.12}$$

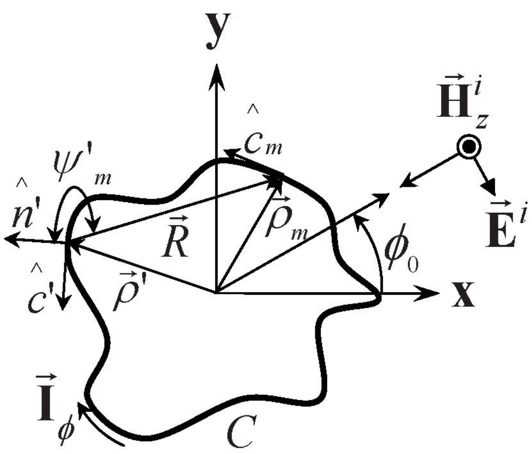

Figure 1: Two-dimensional scatterer of arbitrary cross section in TE polarization.

$$= \hat{n} \times \mathbf{H}^i(r = r').$$

Equation (2.12) is referred to as the magnetic field integral equation (MFIE). It is valid only for closed surfaces. The MFIE is the most popular for TE polarizations. However, this is a general form of MFIE for three-dimensional problems and may be simplified for two-dimensional cases. Figure 1 shows a two-dimensional scatterer of arbitrary cross section for TE polarization. Considering cylindrical coordinates system, we assume that the scatterer is very long in the $\pm z$ direction and is parallel to the z-axis. In the case of TE polarization, the two-dimensional MFIE can be concluded form (2.12), after several steps of mathematical operations, as follows [4]:

$$\frac{I_c(\rho_m)}{2} + \frac{j\beta}{4}\int_{C-\Delta C} I_c(\rho')\cos\psi'_m H_1^{(2)}\left(\beta|\rho_m - \rho'|\right)dc' = -H_z^i(\rho_m), \tag{2.13}$$

where $H_1^{(2)}$ is Hankel function of the second kind of first order, ρ_m is the position vector of any observation point on the scatterer, ρ' is the position vector of any source point on the scatterer, C is perimeter of the scatterer, I_c is the surface current distribution on the scatterer, H^i is the incident magnetic field, and ΔC is that part of the scatterer perimeter which includes ρ_m.

Equation (2.13) is the two-dimensional MFIE which is appropriate for analysis of two-dimensional perfect electrically conducting scatterers in TE polarization.

The kernel $H_1^{(2)}$ has a singular behavior when ρ_m approaches ρ'. Moreover, this kernel is complex. Therefore, the surface current distribution on the scatterer, I_c, is generally a complex function.

In general, (2.13) has no analytical solution. Hence, an appropriate numerical method is necessary to obtain an approximate solution for it. In the next section, we will do a numerical evaluation of the solution by a numerical method, and obtain values of the surface current distribution on the scatterer.

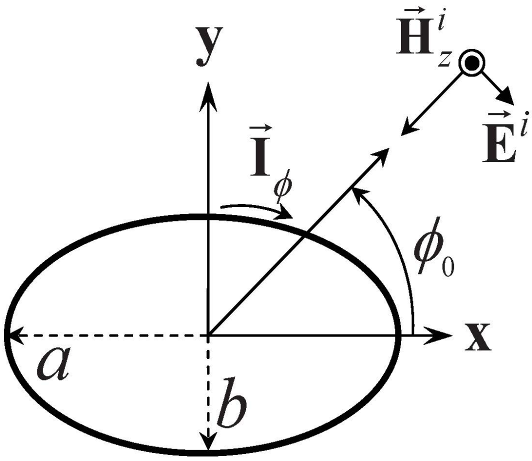

Figure 2: Conducting elliptical cylinder scatterer in TE polarization.

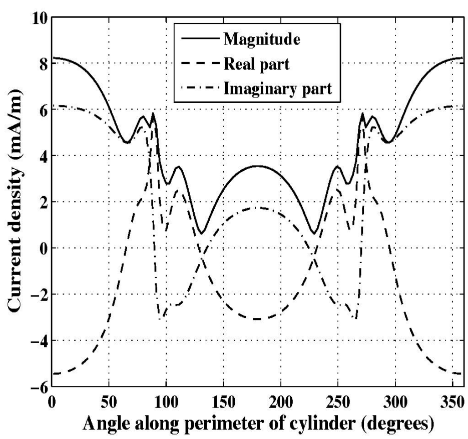

Figure 3: Current distribution along its perimeter for $\beta = 2\pi$, $\phi_0 = 0$, $a = \frac{\lambda}{3}$, and $b = 4a$.

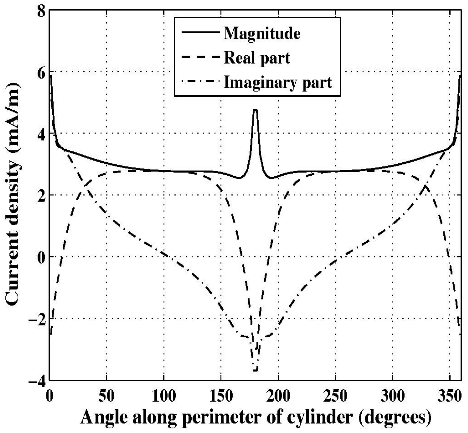

Figure 4: The results for $b = \frac{a}{4}$.

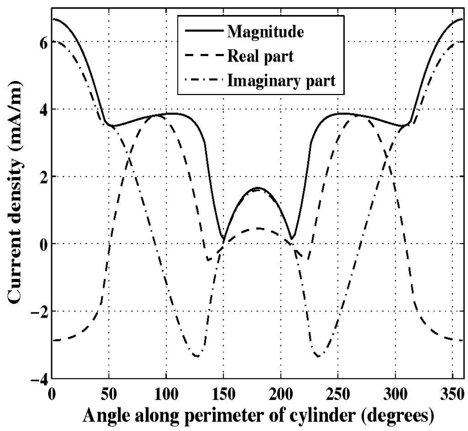

Figure 6: Current distribution along its perimeter for $\beta = 2\pi$, $\phi_0 = 0$, $a = \frac{\lambda}{3}$.

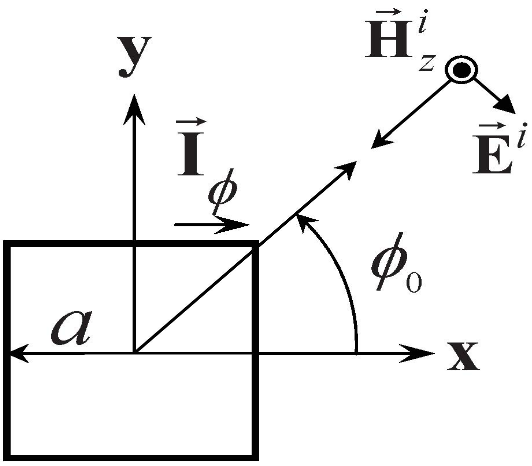

Figure 5: Conducting square cylinder scatterer in TE polarization.

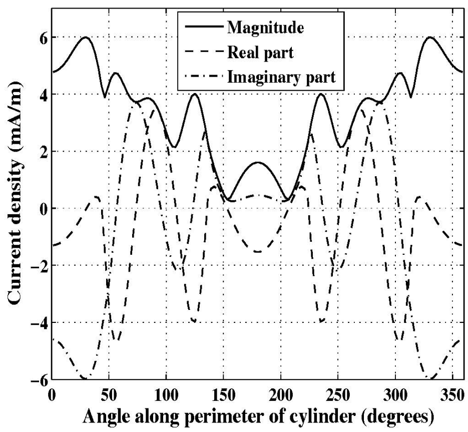

Figure 7: The results for $a = \frac{3\lambda}{4}$.

3 Numerical treatment

3.1 Brief formulation

As it was mentioned, both $H_1^{(2)}$ and I_c have complex values, and $H_1^{(2)}$ is singular too. So, we must take care of the complexity and singularity when using a numerical method for solving (2.13). An interesting method has been presented in [9] for solution of some types of functional equations. However, real and non-singular functions and kernels have been used in [9]. Here, we apply and test the method when the kernel is singular and complex, and the unknown function too has complex values. A brief formulation of the method may be considered as follows.

Let us consider Fredholm integral equation of the second kind of the form

$$x(s) + \int_0^1 k(s,t)x(t)dt = f(s), \qquad 0 \leqslant s < 1, \tag{3.14}$$

where the functions k and f are known but x is the unknown function to be determined. Also, $k \in L^2([0,1) \times [0,1))$ and $f \in L^2([0,1))$. Without loss of generality, it is supposed that the interval of integration in Eq. (3.14) is $[0,1)$, since any finite interval $[a,b)$ can be transformed to this interval by linear maps.

According to [9], an approximate solution for (3.14) may be obtained by solving the following recurrence (iterative) relation:

$$X^{(n)} = RX^{(n-1)} + F, \qquad \text{for } n = 1,2,3,\ldots, \tag{3.15}$$

where $X^{(n)}$ is the unknown vector in iteration n of the iterative process and, for any discretization size m, the m-vectors X and F are the block-pulse

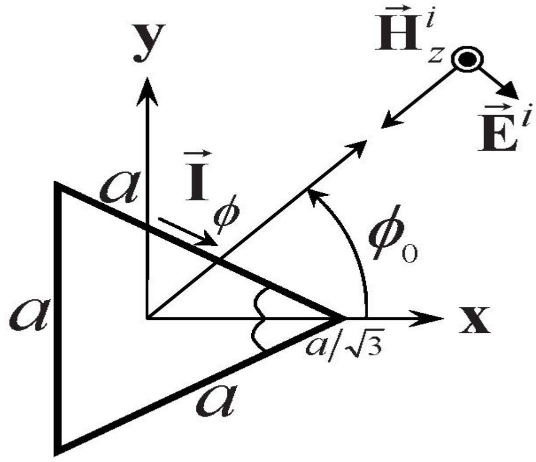

Figure 8: Conducting triangular cylinder scatterer in TE polarization.

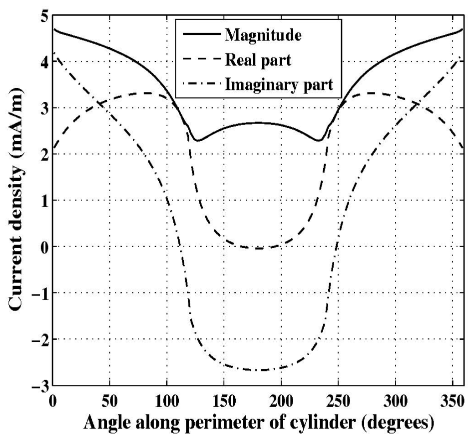

Figure 9: Current distribution along its perimeter for $\beta = 2\pi$, $\phi_0 = 0$, $a = \frac{\lambda}{3}$.

function (BPF) coefficients of functions x and f respectively, and defined as [9]

$$\begin{aligned} F &= [f_0, f_1, \ldots, f_{m-1}]^T, \\ X &= [x_0, x_1, \ldots, x_{m-1}]^T, \end{aligned} \tag{3.16}$$

in which superscript T indicates transposition and

$$\begin{aligned} f_i &= \frac{1}{h}\int_0^1 f(t)\varphi_i(t)dt, \\ x_i &= \frac{1}{h}\int_0^1 x(t)\varphi_i(t)dt, \qquad i = 0, 1, \ldots, m-1, \end{aligned} \tag{3.17}$$

where $h = 1/m$ and φ_i is ith BPF defined as

$$\varphi_i(t) = \begin{cases} 1, & \frac{i}{m} \leqslant t < \frac{(i+1)}{m}, \\ 0, & \text{otherwise}, \end{cases} \qquad i = 0, 1, \ldots, m-1. \tag{3.18}$$

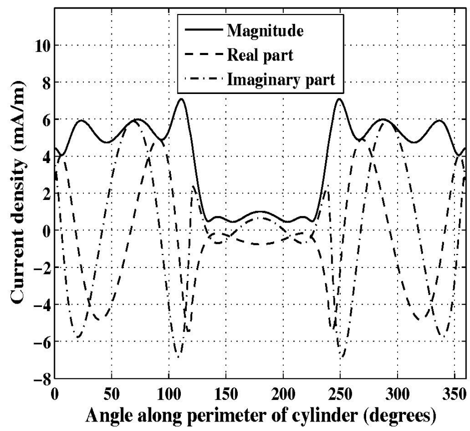

Figure 10: The results for $a = 2\lambda$.

Moreover, $R = -hK$, where K, the BPF coefficient matrix of kernel k, is an $m \times m$ matrix with elements $k_{i,j}$, $i = 0, 1, \ldots, m-1$, $j = 0, 1, \ldots, m-1$, as follows:

$$k_{i,j} = m^2 \int_0^1 \int_0^1 k(s,t)\varphi_i(s)\psi_j(t)dsdt. \tag{3.19}$$

ψ_j is jth BPF.

The expansion of functions f, X, and k over $[0, 1)$ with respect to BPFs may be compactly written as [9]

$$\begin{aligned} f(t) &\simeq \sum_{i=0}^{m-1} f_i\varphi_i(t) = F^T\Phi(t) = \Phi^T(t)F, \\ x(t) &\simeq \sum_{i=0}^{m-1} x_i\varphi_i(t) = X^T\Phi(t) = \Phi^T(t)X, \\ k(s,t) &\simeq \Phi^T(s)K\Psi(t), \end{aligned} \tag{3.20}$$

in which m-vectors Φ and Ψ are defined by

$$\begin{aligned} \Phi(t) &= [\varphi_0(t), \varphi_1(t), \ldots, \varphi_{m-1}(t)]^T, \\ \Psi(t) &= [\psi_0(t), \psi_1(t), \ldots, \psi_{m-1}(t)]^T. \end{aligned} \tag{3.21}$$

After solution of recurrence relation (3.15) and obtaining vector X, an approximate solution $x(s) \simeq X^T\Phi(s)$ can be computed for Eq. (3.14).

3.2 Error analysis and convergence evaluation

Let us set

$$\mathbf{e}^{(n)} = X^{(n)} - X, \tag{3.22}$$

where X is the exact solution for (3.15), therefore

$$X = F + RX. \tag{3.23}$$

Subtracting (3.23) from (3.15) gives

$$\mathbf{e}^{(n+1)} = R\mathbf{e}^{(n)}. \tag{3.24}$$

Using (3.24) and if $\|R\| < 1$ then we have the following error bound [9]:

$$\|\mathbf{e}^{(n)}\| \leqslant \frac{\|X^{(n+1)} - X^{(n)}\|}{1 - \|R\|}. \tag{3.25}$$

Also we can obtain two other error bounds as follows:

$$\|\mathbf{e}^{(n)}\| \leqslant \|R^n\| \|(X^{(0)} - X)\|, \tag{3.26}$$

and

$$\|\mathbf{e}^{(n)}\| \leqslant \frac{\|R\|^n}{1 - \|R\|} \|X^{(1)} - X^{(0)}\|. \tag{3.27}$$

The above three error bounds show that if $\|R\| < 1$, then $\lim_{n\to\infty} \|\mathbf{e}^{(n)}\| = 0$. This follows $\lim_{n\to\infty} X^{(n)} = X$, meaning that the sequence $\{X^{(n)}\}_{n=0}^{\infty}$ converges to X.

3.3 Numerical results

Here, we apply the mentioned approach to solve the two-dimensional MFIE for three scattering structures; a conducting elliptical cylinder; a conducting square cylinder; and a conducting triangular cylinder. Numerical results are computed for TE polarization.

Figure 2 shows the cross section of a conducting elliptical cylinder which is illuminated by a TE polarized electromagnetic plane wave and therefore an electrical current is induced on it. Solution of the two-dimensional MFIE by the mentioned method gives the numerical results for the induced current distribution. Figure 3 shows the current density along the perimeter of elliptical cylinder for $\beta = 2\pi$, $\phi_0 = 0$, $a = \frac{\lambda}{3}$, and $b = 4a$. Figure 4 gives the results for $b = \frac{a}{4}$.

Figure 5 shows the cross section of a conducting square cylinder encountered a TE polarized plane wave. Considering $\beta = 2\pi$ and $\phi_0 = 0$, the current density values are given in Figures 6 and 7 for $a = \frac{\lambda}{3}$ and $a = \frac{3\lambda}{4}$, respectively.

The cross section of a conducting triangular cylinder in TE polarization is shown in Figure 8. Considering $\beta = 2\pi$ and $\phi_0 = 0$, Figures 9 and 10 respectively give the results for $a = \frac{\lambda}{3}$ and $a = 2\lambda$.

4 Conclusion

This article dealt with the use of MFIE in analysis of electromagnetic scattering from two-dimensional PEC structures. A general form of MFIE was used for modeling of electromagnetic scattering from three-dimensional structures and then the formulation was simplified for two-dimensional conducting bodies. For solving the models, a numerical approach was surveyed and the numerical results were computed for the current distribution induced on three electrically conducting surfaces.

The concepts illustrated in this article are feasible to be used for analysis of arbitrary two-dimensional conducting scatterers.

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