Available online at http://ijim.srbiau.ac.ir

Int. J. Industrial Mathematics Vol. 1, No. 2 (2009) 163-173

Ranking L-R Fuzzy Numbers with Weighted Averaging Based on Levels

R. Saneifard *

Department of Mathematics, Islamic Azad University, Oroumieh Branch, Oroumieh, Iran.

Abstract

In this paper, the researcher proposes a modified new method to rank L-R fuzzy numbers. The modified method uses a defuzzification of parametrically represented fuzzy numbers that have been studied in [20]. This parameterized defuzzification can be used as a crisp approximation with respect to a fuzzy quantity. In this article, the researcher uses this defuzzification for ordering fuzzy numbers. The modified method can effectively rank various fuzzy numbers and their images and overcome the shortcomings of the previous techniques. This study also uses some comparative examples to illustrate the advantages of the proposed method.

Keywords : Ranking, Fuzzy number, L-R type, Defuzzification, Weighted averaging.

1 Introduction

Since Dubios and Prade [12] introduced the relevant concepts of fuzzy numbers, many researchers proposed the related methods or applications for ranking fuzzy numbers. For instance, Bortolan and Degani [4] reviewed some methods to rank fuzzy numbers in 1985, Chen and Hwang [5] proposed fuzzy multiple attribute decision making in 1992, Choobineh and Li [6] proposed an index for ordering fuzzy numbers in 1993, Dias [11] ranked alternatives by ordering fuzzy numbers in 1993, Requena et al. [21] utilized artificial neural networks for the automatic ranking of fuzzy numbers in 1994, Fortemps and Roubens [13] presented ranking and defuzzification methods based on area compensation in 1996, and Raj and Kumar [22] investigated maximizing and minimizing sets to rank fuzzy alternatives with fuzzy weights in 1999. However, Chu and Lee-Kwang [7] argued that some of the above-mentioned methods are difficult to implement on grounds of computational complexity, and others are counterintuitive or not discriminating enough. They also observed that many methods yield different outcomes on the same problem. Chu and Tsao's

*Email address: srsaneeifard@yahoo.com, Tel:+989149737077

method [10] originated from the concepts of Lee and Li [17] and Cheng [8]. In 1988, Lee and Li proposed the comparison of fuzzy numbers, for which they considered mean and standard deviation values for fuzzy numbers based on the uniform and proportional probability distributions. Then, Cheng proposed the coefficient of variance (CV index) in 1999 to improve Lee and Li's method based on two comments presented as follows.

(a) The mean and standard deviation values cannot be the sole basis to compare two fuzzy numbers.

(b) It is difficult to rank fuzzy numbers, as higher mean value is associated with higher spread or lower mean value is associated with lower spread.

Although, Cheng overcame the problems from these comments and also proposed a new distance index to improve the method proposed by Murakami et al. [8], Chu and Tsao still believed that Cheng's method contained some shortcomings. For instance, they illustrated a ranking example shown as below. For the two triangular fuzzy numbers in their example, $A = (0.9, 1, 1.1)$ and $B = (1.2, 2, 3)$, intuitively, A should be smaller than B. However, A is bigger than B on the basis of the CV index.
To overcome these above problems, Chu and Tsao proposed a method to rank fuzzy numbers with an area between their centroid and original points. The method can avoid the problems Chu and Tsao mentioned; however, the researchers found other problems in their method. But, this method is unreasonable for some fuzzy numbers.
Having reviewed the previous methods, this article proposes here a method to use the concept of weighted averaging based on levels, so as to find the order of L-R fuzzy numbers. This method can distinguish the alternatives clearly. The main point of this article is that, the weighted averaging can be used as a crisp approximation of a fuzzy number. Therefore, by means of this difuzzification, this article aims to present a new method for ranking fuzzy numbers. In addition to its ranking features, this method removes the ambiguities resulting and overcomes the shortcomings from the comparison of previous rankings. In this work, the researcher obtains a crisp approximation with respect to a fuzzy quantity, then defines a method for ordering fuzzy numbers.
The paper is organized as follows: In Section 2, we recall some fundamental results on fuzzy numbers. In Section 3, a crisp approximation of a fuzzy number is obtained. Als, n te same section, some theorems and remarks are proposed and illustrated, and a method for ranking L-R fuzzy numbers is provided. Discussion and comparison of this work and other methods are carried out in Section 4. The paper ends with conclusions in Section 5.

2 Basic Definitions and Notations

Definition 2.1. *Let X be a universe set. A fuzzy set A of X is defined by a membership function $\mu_A(x) \rightarrow [0, 1]$, where $\mu_A(x)$, $\forall x \in X$, indicates the degree of x in A.*

Definition 2.2. *A fuzzy subset A of universe set X is normal iff $\sup_{x \in X} \mu_A(x) = 1$, where X is the universe set.*

Definition 2.3. *A fuzzy subset A of universe set X is convex iff $\mu_A(\lambda x + (1 - \lambda)y) \geq (\mu_A(x) \wedge \mu_A(y))$, $\forall x, y \in X, \forall \lambda \in [0, 1]$.*

In this article symbols $\wedge$ and $\vee$ denotes the minimum and maximum operators, respectively.

Definition 2.4. *A fuzzy set A is a fuzzy number iff A is normal and convex on X.*

Definition 2.5. *For fuzzy set A the support function is defined as follows:*

$$supp(A) = \overline{\{x|\mu_A(x) > 0\}},$$

where $\overline{\{x|\mu_A(x) > 0\}}$ is the closure of set $\{x|\mu_A(x) > 0\}$.

Definition 2.6. *A L-R fuzzy number $A = (m, n, \sigma, \beta)_{LR}$, $m \le n$, is defined as follows:*

$$\mu_A(x) = \begin{cases} L(\frac{m-x}{\sigma}), & -\infty < x < m, \\ 1, & m \le x \le n, \\ R(\frac{x-n}{\beta}) & n < x < +\infty, \end{cases}$$

where σ and β are the left-hand and right-hand spreads. In the closed interval $[m, n]$, the membership function is equal to 1. $L(\frac{m-x}{\sigma})$ and $R(\frac{x-n}{\beta})$ are non-increasing functions with $L(0) = 1$ and $R(0) = 1$, respectively. Usually, for convenience, they are denoted as $\mu_{A^L}(x)$ and $\mu_{A^R}(x)$, respectively. It should be pointed out that when $L(\frac{m-x}{\sigma})$ and $R(\frac{x-n}{\beta})$ are linear functions and $m < n$, fuzzy number A denotes a trapezoidal fuzzy number. When $L(\frac{m-x}{\sigma})$ and $R(\frac{x-n}{\beta})$ are linear functions and $m = n$, fuzzy number A denotes a triangular fuzzy number.

This definition is very general and allows quantification of quite different types of information; for instance, if A is supposed to be a real crisp number for $m \in \Re$,

$$A = (m, m, 0, 0)_{LR}, \ \forall L, \ \forall R$$

If A is a crisp interval,

$$A = (a, b, 0, 0)_{LR}, \ \forall L, \ \forall R$$

and if A is a trapezoidal fuzzy number, $L(x) = R(x) = \max(0, 1 - x)$ is implied.

3 New Approach for Ranking Fuzzy Numbers

In this section, the researcher will propose the ranking of fuzzy numbers associated with defuzzification of parametrically represented fuzzy numbers.
Let F denotes the space of L-R fuzzy numbers, then, this article, it will be assumed that the fuzzy number $A \in F$ is represented by the following representation:

$$A = \bigcup_{\alpha \in [0,1]} (\alpha, A_\alpha) \tag{3.1}$$

where

$$\forall \alpha \in [0, 1] : A_\alpha = [L_A(\alpha), R_A(\alpha)] \subset (-\infty, \infty) \tag{3.2}$$

Here, $L : [0, 1] \to (-\infty, \infty)$ is a monotonically non-decreasing function and $R : [0, 1] \to (-\infty, \infty)$ is a monotonically non-increasing left-continuous function. The functions $L(.)$ and $R(.)$ express the left and right sides of a fuzzy number, respectively. In other words ,

$$L(\alpha) = \mu_{\uparrow}^{-1}(\alpha), \quad R(\alpha) = \mu_{\downarrow}^{-1}(\alpha), \tag{3.3}$$

where $L(\alpha) = \mu_{\uparrow}^{-1}(\alpha)$ and $R(\alpha) = \mu_{\downarrow}^{-1}(\alpha)$ denote quasi-inverse functions of the increasing and decreasing parts of the membership functions $\mu(t)$, respectively. As a result, the decomposition representation of the fuzzy number A, called the L-R representation, has the following form:

$$A = \bigcup_{\alpha\in(0,1]} (\alpha, [L_A(\alpha), R_A(\alpha)]).$$

Definition 3.1. *[20]. The following value constitutes the weighted averaging based on levels representative, of the fuzzy number A:*

$$I(A) = \int_0^1 (c_L L_A(\alpha) + c_R R_A(\alpha))p(\alpha)d\alpha, \tag{3.4}$$

where the parameters c_L and c_R denote the "optimism/pessimism" coefficient in conducting operations on fuzzy numbers. The function $p(\alpha)$ is the distribution function of the importance of the level sets. The latter satisfies the conditions

$$c_L \geq 0, \quad c_R \geq 0, \quad c_L + c_R = 1,$$

and

$$p : [0, 1] \to E_+, \quad \int_0^1 p(\alpha)d\alpha = 1.$$

The function $p(\alpha)$ is also called the weighted averaging parameter. In actual applications, function $p(\alpha)$ can be chosen according to the actual situation. In this article, the author assumes that

$$p(\alpha) = (k + 1)\alpha^k, \tag{3.5}$$

where $k > 0$ is a parameter.

Theorem 3.1. *[20] Suppose $A = (m, n, \sigma, \beta)_{LR}$ is a L-R trapezoidal fuzzy number with distribution of the function of the importance of the degrees having the form of relation (3.4). Then the following formula is valid for weighted averaging:*

$$I(A) = c_R\left(\beta - \frac{k+1}{k+2}(\beta - \sigma)\right) + c_R\left(m + \frac{k+1}{k+2}(n - m)\right). \tag{3.6}$$

Since this article aims to approximate a fuzzy number by a scalar value, the researcher has to use an operator $I : F \to \Re$ which transforms fuzzy numbers into family of real line. Operator I is a crisp approximation operator. Thus, since any parameterized defuzzification can be used as a crisp approximation of a fuzzy number, the resulting value is used to rank the fuzzy numbers. Thus, $I(A)$ is used to rank fuzzy numbers. The larger $I(A)$, the larger the fuzzy number.

Definition 3.2. *For any two L-R fuzzy numbers A and B, the ranking order by $I(.)$ is determined based on the following rules:*

$I(A) > I(B)$ *if and only if* $A \succ B$,

$I(A) < I(B)$ *if and only if* $A \prec B$,

$I(A) = I(B)$ *if and only if* $A \sim B$.

Then, this article formulates the order $\succeq$ and $\preceq$ as $A \succeq B$ if and only if $A \succ B$ or $A \sim B$, and $A \preceq B$ if and only if $A \prec B$ or $A \sim B$.

Remark 3.1. *If* $\inf supp(A) \geq 0$*, then* $I(A) \geq 0$.

Remark 3.2. *If* $\sup supp(A) \leq 0$*, then* $I(A) \leq 0$.

Remark 3.3. *For two arbitrary L-R fuzzy numbers, A and B, this article assume*

$$I(A + B) = I(A) + I(B).$$

This work considers the following reasonable axioms that Wang and Kerre [23] proposed for ranking fuzzy quantities.
Let I be an ordering method, S the set of fuzzy quantities for which the method I can be applied, and $\mathcal{A}$ a finite subset of S. The statement "two elements A and B in $\mathcal{A}$ satisfy that A has a higher ranking than B when I is applied to the fuzzy quantities in $\mathcal{A}$" will be written as "$A \succ B$ by I on $\mathcal{A}$" , "$A \sim B$ by I on $\mathcal{A}$", and "$A \succeq B$ by I on $\mathcal{A}$" are similarly interpreted. [23], The axioms as the reasonable properties of ordering fuzzy quantities for an ordering approach I are as follows:

A-1 For an arbitrary finite subset $\mathcal{A}$ of S and $A \in \mathcal{A}$; $A \succeq A$.

A-2 For an arbitrary finite subset $\mathcal{A}$ of S and $(A, B) \in \mathcal{A}^2$; $A \succeq B$ and $B \succeq A$ by I on $\mathcal{A}$, this method should assume $A \sim B$.

A-3 For an arbitrary finite subset $\mathcal{A}$ of S and $(A, B, C) \in \mathcal{A}^3$; $A \succeq B$ and $B \succeq C$ by I on $\mathcal{A}$, this method should assume $A \succeq C$.

A-4 For an arbitrary finite subset $\mathcal{A}$ of S and $(A, B) \in \mathcal{A}^2$; inf supp$(A)>$sup supp(B), this method should assume $A \succeq B$.

A'**-4** For an arbitrary finite subset $\mathcal{A}$ of S and $(A, B) \in \mathcal{A}^2$; inf supp$(A)>$sup supp(B), this method should assume $A \succ B$.

A-5 Let S , S' be two arbitrary finite sets of fuzzy quantities in which I can be applied and A , B are in $S \cap S'$. This method obtains the ranking order $A \succeq B$ on S' iff $A \succeq B$ on S.

A-6 Let A, B, $A + C$ and $B + C$ be elements of S. If $A \succeq B$ by I on A, B, then $A + C \succeq B + C$.

A'**-6** Let A, B, $A + C$ and $B + C$ be elements of S. If $A \succ B$ by I on A, B, then $A + C \succ B + C$.

Theorem 3.2. *The function I has the properties* $(A - 1), (A - 2), ..., (A' - 6)$.

Proof. It is easy to verify that the properties $(A - 1), (A - 2), ..., (A - 5)$ hold. For the proof of $(A - 6)$, this article considers the fuzzy numbers A, B and C. Let $A \succeq B$, then from relation (3.4), there is

$$I(A) \geq I(B),$$

by adding $I(C)$,

$$I(A) + I(C) \geq I(B) + I(C),$$

and by Remark (3 -3),

$$I(A+C) \geq I(B+C).$$

Therefore

$$A+C \geq B+C.$$

by which the proof is complete. Similarly (A'- 6) holds.

Remark 3.4. *If $A \preceq B$, then $-A \succeq -B$.*

Hence, this approach can imply the ranking order of the images of the fuzzy numbers.

4 Numerical Examples

In this section, four numerical examples are used to illustrate the proposed approach to ranking L-R fuzzy numbers. Now, the author compares the proposed method with those in [4, 6, 9, 10]. Throughout this section it is assumed that $p(\alpha) = 2\alpha$ ($k = 1$), and the "optimism/pessimism" coefficient is 0.5.

Example 4.1. *Consider the following sets (see Yao and Wu [25]).*
Set 1: $A = (0.5, 0.5, 0.1, 0.5)_{LR}$, $B = (0.7, 0.7, 0.3, 0.3)_{LR}$, $C = (0.9, 0.9, 0.5, 0.1)_{LR}$.
Set 2: $A = (0.4, 0.7, 0.4, 0.1)_{LR}$, $B = (0.5, 0.5, 0.3, 0.4)_{LR}$, $C = (0.6, 0.6, 0.5, 0.2)_{LR}$.
Set 3: $A = (0.5, 0.5, 0.2, 0.2)_{LR}$, $B = (0.5, 0.8, 0.2, 0.1)_{LR}$, $C = (0.5, 0.5, 0.2, 0.4)_{LR}$.

By the approach in this paper, the ranking index values of set 1 can be obtained as $I(A) = 0.3666$, $I(B) = 0.5$ and $I(C) = 0.6333$. Then, the ranking order of the fuzzy numbers is $A \prec B \prec C$. As for set 2, the ranking index values are $I(A) = 0.4500$, $I(B) = 0.4166$ and $I(C) = 0.5000$. The ranking order is $B \prec A \prec C$. As for set 3, the ranking index values are $I(A) = 0.3500$, $I(B) = 0.4333$ and $I(C) = 0.3833$. The ranking order is $A \prec C \prec B$. Based on the analysis results from [1, 2, 3], the computation results using our approach and other ones are given in Table 1. Our computation procedure is simpler than that of others. In set 2, by the approach proposed in [25], the ranking order is $A \prec B \sim C$. By the CV index approach, the ranking order is $B \prec C \prec A$. By Fig. 2, it is easy to see that neither of those methods is consistent with human intuition.

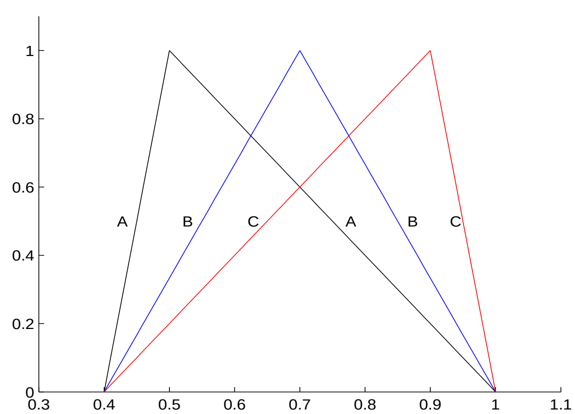

Fig. 1. Set 1.

Table 1
Comparative results of example 4.1

Method	Fuzzy number	Set1	Set2	Set3
Sign Distance method with p=1	A	1.2000	0.0950	1.0000
	B	1.4000	1.0500	1.2500
	C	1.6000	1.0500	1.1000
Results		$A \prec B \prec C$	$A \prec B \sim C$	$A \prec C \prec B$
Sign Distance method with p=2	A	0.8869	0.7853	0.7257
	B	1.0194	0.7958	0.9416
	C	1.1605	0.8386	0.8165
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$
Distance Minimization	A	0.6	0.475	0.5000
	B	0.7	0.525	0.6250
	C	0.9	0.525	0.5500
Results		$A \prec B \prec C$	$A \prec B \sim C$	$A \prec C \prec B$
Choobineh and Li	A	0.3333	0.5000	0.3330
	B	0.5000	0.5833	0.4146
	C	0.6670	0.6111	0.5417
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Chu and Tsao	A	0.2990	0.2440	0.2500
	B	0.3500	0.2624	0.3152
	C	0.3993	0.2619	0.2747
Results		$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Yao and Wu	A	0.6000	0.4750	0.5000
	B	0.7000	0.5250	0.6250
	C	0.8000	0.5250	0.5500
Results		$A \prec B \prec C$	$A \prec B \sim C$	$A \prec C \prec B$
Cheng distance	A	0.7900	0.7106	0.7071
	B	0.8602	0.7256	0.8037
	C	0.9268	0.7241	0.7458
Results		$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Cheng CV uniform distribution	A	0.0272	0.0693	0.0133
	B	0.0214	0.0385	0.0304
	C	0.0225	0.0433	0.2750
Results		$B \prec C \prec A$	$B \prec C \prec A$	$A \prec C \prec B$
Cheng CV proportional distribution	A	0.1830	0.0471	0.0080
	B	0.0128	0.0236	0.0234
	C	0.0137	0.0255	0.0173
Results		$B \prec C \prec A$	$B \prec C \prec A$	$A \prec C \prec B$

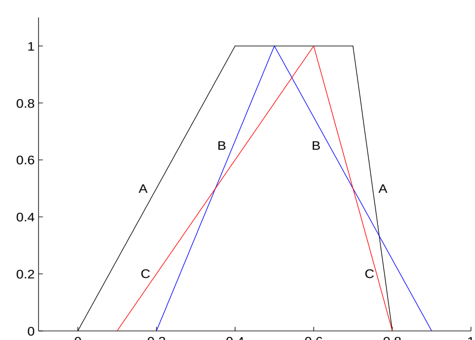

Fig. 2. Set 2.

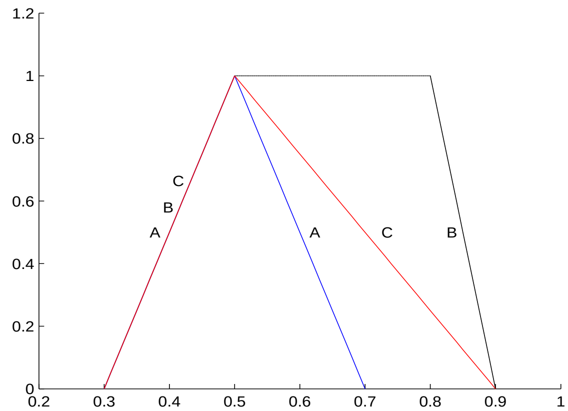

Fig. 3. Set 3.

Example 4.2. *Consider fuzzy numbers* $A = (2, 2, 1, 3)_{LR}$*, and the general number,* $B = (2, 2, 1, 2)$*, shown in Fig. (4). The membership function of* A *is defined by*

$$\mu_A(x) = \begin{cases} x - 1 & \text{when } x \in [1, 2], \\ \frac{5-x}{3} & \text{when } x \in [2, 4], \\ 0 & \text{otherwise.} \end{cases}$$

The membership function of B *is defined by*

$$\mu_B(x) = \begin{cases} \sqrt{1 - (x-2)^2} & \text{when } x \in [1, 2], \\ \sqrt{1 - \frac{1}{4}(x-2)^2} & \text{when } x \in [2, 4], \\ 0 & \text{otherwise.} \end{cases}$$

In Liou and Wang's ranking method [16], different rankings are produced for the same problem when applying different indices of optimism. In the Sign Distance method with $p = 1$, $d_p(A, A_0) = 5$, $d_p(B, A_0) = 4.78$*, and with* $p = 2$, $d_p(A, A_0) = 3.9157$, $d_p(B, A_0) = 3.8045$*, the ranking order* $A \succ B$ *is obtained. In Chu and Tsao's ranking method, there is* $S(A) = 1.2445$ *and* $S(B) = 1.1821$*, therefore,* $A \succ B$*. By using this new approach, there is* $I(A) = 1.8333$ *and* $I(B) = 1.66$*. Thus, the ranking order is* $A \succ B$*, too. Also, the result of the Distance Minimization method was similar to our method. Obviously, this method can also rank fuzzy numbers other than triangular and trapezoidal ones. Compared to Liou and Wang's method, and along with Chu and Tsao's method our method produces a simpler ranking result.*

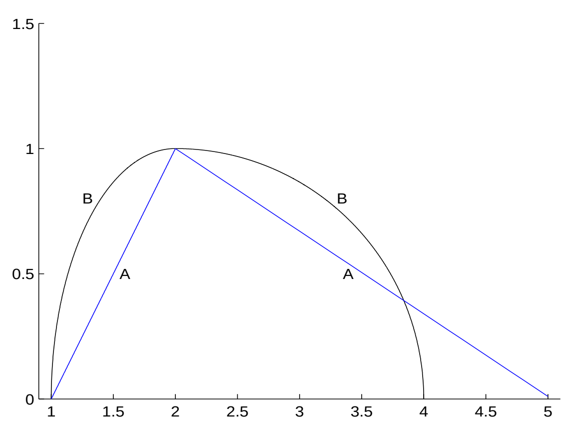

Fig. 4.

Example 4.3. *The two triangular fuzzy numbers* $A = (3,3,2,2)_{LR}$ *and* $B = (3,3,1,1)_{LR}$ *shown in Fig. 5 taken from [10].*

Through the proposed approach in this paper, the ranking index values can be obtained as $I(A) = 2.5$ *and* $I(B) = 2$*. Then, the ranking order of fuzzy numbers is* $A \succ B$*. Because fuzzy numbers* A *and* B *have the same mode and symmetric spread, most of the existing approaches fail in ranking them appropriately. For instance, in [1], different ranking orders are obtained when different index values* (p) *are taken. When* $p = 1$ *and* $p = 2$*, the ranking order of fuzzy numbers is* $A \sim B$ *and* $A \succ B$*, respectively. Meanwhile, using the approaches in [2, 10, 25, 24], the ranking order is the same, i.e.,* $A \sim B$*. Nevertheless, inconsistent results are produced when the distance index and the* CV *index of Cheng's approach [8] are respectively used. Moreover, the ranking order obtained by Wang's approach [24] is* $A \succ B$*. Additionally, by the approaches provided in [18, 19], different ranking orders are obtained when different indices of optimism are taken. However, decision makers prefer the result* $A \succ B$ *intuitionally.*

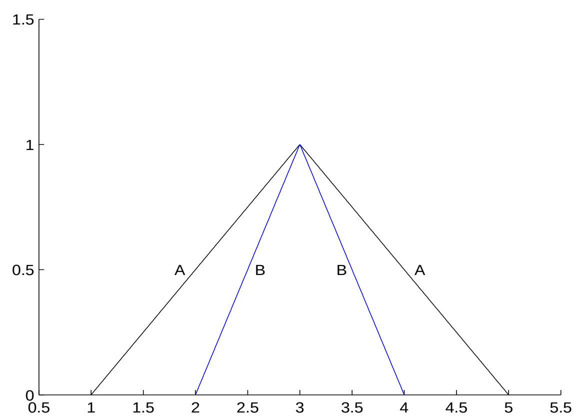

Fig. 5.

Example 4.4. *Consider the three fuzzy numbers* $A = (2,2,1,3)_{LR}$*,* $B = (3,3,3,1)_{LR}$ *and* $C = (2.5, 2.5, 0.5, 0.5)_{LR}$ *(see Fig. 6).*

By using this new approach, $I(A) = 1.8333$*,* $I(B) = 2.6666$ *and* $I(C) = 1.5$*. Hence, the ranking order is* $C \prec A \prec B$ *too. Obviously, the results obtained by "Sign distance" and "Distance Minimization" methods are unreasonable. To compare with some of the other methods in [23], the reader can refer to Table 2.*

Furthermore, in the aforesaid example $I(-A) = -1.8333$*,* $I(-B) = -2.6666$ *and* $I(-C) = -1.5$*, consequently the ranking order of the images of three fuzzy number is* $-B \prec -A \prec -C$*. Clearly, this proposed method has consistency in ranking fuzzy numbers and their images, which could not be guaranteed by the* CV *index method.*

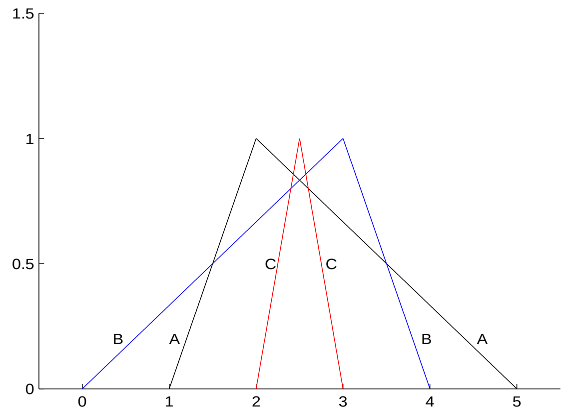

Fig. 6.

Table 2
Comparative results of example 4.4

Fuzzy number	New approach	Sign distance p=1	Sign distance p=2	Distance Minimization	Chu and Tsao
A	1.8333	5	3.9157	2.5	0.7407
B	2.6666	5	3.9157	2.5	0.7407
C	1.5000	5	3.5590	2.5	0.75
Results	$C \prec A \prec B$	$C \sim A \sim B$	$C \prec A \sim B$	$C \sim A \sim B$	$A \sim B \prec C$

All the above examples show that the results of this new method are reasonable results. This method can overcome the shortcoming of other methods.

5 Conclusion

The fuzzy number defuzzification method with weighted averaging based on levels has been proposed in [20]. This parameterized defuzzification can be used as a crisp approximation with respect to a fuzzy quantity. In this article, the researcher used this for ordering L-R fuzzy numbers. The modified method effectively ranked various fuzzy numbers and their images and overcame the shortcomings which are found in the other techniques. The examples given in this paper illustrated that the proposed approach has distinctive characteristics. Additionally, the proposed approach provides decision makers with a new alternative to rank fuzzy numbers.

References

[1] S. Abbasbandy, B. Asady, Ranking of fuzzy numbers by sign distance, Information Sciences 176 (2006) 2405 - 2416.

[2] S. Abbasbandy, T. Hajjari, A new approach for ranking of trapezoidal fuzzy numbers, Computers and Mathematics with Application 57 (2009) 413 - 419.

[3] B. Asady, A. Zendehnam, Ranking fuzzy numbers by distance minimization, Applied Mathematical Modelling 31 (2007) 2589 - 2598.

[4] G. Bortolan, R. Degani, A review of some methods for ranking fuzzy numbers, Fuzzy Sets and Systems 15 (1985) 1 - 19.

[5] S. J. Chen, C. L. Hwang, Fuzzy Multiple Attribute Decision Making, Springer- Verlag, Berlin, (1972).

[6] F. Choobineh, H. Li, An index for ordering fuzzy numbers, Fuzzy Sets and Systems 54 (1993) 287 - 294.

[7] H. Chu, H. Lee-Kwang, Ranking fuzzy values ewith satisfaction function, Fuzzy Sets and Systems 64 (1994) 295 - 311.

[8] C. H. Cheng, Ranking alternatives with fuzzy weights using maximizing set and minimizing set, Fuzzy Sets and System. 105 (1999) 365 - 375.

[9] S. Chen, Ranking fuzzy numbers with maximizing set and minimizing set, Fuzzy Sets and Systems 17 (1985) 113 - 129.

[10] T. Chu, C.Tsao, Ranking fuzzy numbers with an area between the centroid point and original points, Computers and Mathematics with Application 43 (2002) 111- 117.

[11] O. Dias, Ranking alternatives using fuzzy numbers: A computational approach, Fuzzy Sets and Systems 56 (1993) 247-252.

[12] D. Dubois, H. Prade, The mean value of a fuzzy number, Fuzzy Sets and Systems 24 (1987) 279 - 300.

[13] P. Fortemps, M. Roubens, Ranking and defuzzification methods based on area compensation, Fuzzy Sets and Systems 82 (1996) 319 - 330.

[14] S. Heilpern, The expected value of a fuzzy number, Fuzzy Sets and Systems 47 (1992) 81 - 86.

[15] A. Kauffman, M.M. Gupta, Introduction to Fuzzy Arithmetic: Theory and Application, Van Nostrand Reinhold, New York, 1991.

[16] T. S. Liou, M. J. Wang, Ranking fuzzy numbers with integral value, Fuzzy Sets and Systems 50 (1992) 247-255

[17] E. S. Lee, R.J. Li, A new approach for ranking fuzzy numbers by distance method, Fuzzy Sets and Systems 95 (1998) 307 - 317.

[18] X. Liu, Measuring the satisfaction of constraints in fuzzy linear programming, Fuzzy Sets and Systems 122 (2001) 263 - 275.

[19] B. Matarazzo, G. Munda, New approaches for the comparison of L-R fuzzy numbers, Fuzzy Sets and Systems 118 (2001) 407 - 418.

[20] E. N. Nasibov, A. Mert, On methods of defuzzification of parametrically represented fuzzy numbers, Automatic Control and Computer Science 41 (2007) 265 - 273.

[21] I. Requena, M. Delgado and J. I. Verdegay, Automatic ranking of fuzzy numbers with the criterion of decision maker learnt by an artificial neural network, Fuzzy Sets and Systems 64 (1994) 1 - 9.

[22] P. A. Raj, D. N. Kumar, Ranking alternatives with fuzzy weights using maximizing set and minimizing set, Fuzzy Sets and Systems 105 (1999) 365 - 375.

[23] X. Wang, E.E. Kerre, Reasonable properties for the ordering of fuzzy quantities (I), Fuzzy Sets and Systems 118 (2001) 378 - 405.

[24] Y. J. Wang, S. H. Lee, The revised method of ranking fuzzy numbers with an area between the centroid an original points, Computers and mathematics with Applications 55 (2008) 2033 - 2042.

[25] J. Yao, K. Wu, Ranking fuzzy numbers based on decomposition principle and signed distance, Fuzzy Sets and Systems 116 (2000) 275 - 288.