



Numerical method for analysis of radiation from thin wire dipole antenna

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Received 4 April 2011; accepted 28 May 2011.

Abstract

This paper presents a numerical method for analysis of electromagnetic radiation from thin wire dipole antenna. For modeling of such structure, an appropriate form of Fredholm integral equation of the first kind is used, in which the unknown function is the current distribution on the surface of the antenna. An efficient collocation method is formulated for the solution of this integral equation whose main advantage is the use of entire domain basis functions. Finally, the radiation patterns for the antenna are given.

Keywords : Thin wire dipole antenna; Electromagnetic radiation; First kind Fredholm integral equation; Numerical solution; Radiation pattern.

1 Introduction

Many problems in Electromagnetics can be modeled by Fredholm integral equations of the first kind [3, 4, 8, 9, 10]. These equations are in general ill-posed. That is small changes to the problem's data can make very large changes to the answers obtained [6, 11]. So, obtaining the numerical solutions is very difficult.

Several regularization methods have been proposed to overcome the ill-posedness [1, 15]. In recent years, some numerical methods based on different basis functions have been illustrated. These methods often use a projection method and transform a first kind integral equation to a linear system of algebraic equations. This system usually has large condition number, therefore a suitable approach such as Conjugate Gradient (CG) method should be considered [12, 13].

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The problem of determining the radiated fields from thin wire antennas corresponds to solving *Pocklington's integral equation* or *Hallén's integral equation*, both of the form of first kind Fredholm integral equation [2, 7, 14]. The current density on the surface of the antenna is usually considered as the unknown function to be determined and the kernels appeared in the mentioned integral equations are complex. Once the current distribution is calculated, other antenna parameters such as radiation pattern can be obtained.

This paper proposes a collocation method for solving Hallén's integral equation for the thin wire dipole antenna. The method uses Sinc cardinal functions as basis functions. The advantage of this method is the use of a set of entire domain basis functions, because by calculating only a few expansion coefficients, we can have a relatively reasonable approximation of the solution for the whole of the domain.

The paper is organized as follows. Section 2 poses Hallén's integral equation for modeling of radiation from the dipole antenna. The collocation method mentioned above is presented in section 3. Then, Hallén's integral equation is solved by the proposed method to obtain the current density on the antenna and its radiation pattern. Finally, conclusions will be in section 4.

2 Hallén's integral equation for the thin wire dipole antenna

A practical center-fed dipole antenna usually consists of a pair of tubular conductors of radius a aligned in tandem so that there is a small feeding gap at the center [7], as shown in Fig. 1. The total length is ℓ . A voltage is applied across the gap, often by means of a two-wire transmission line. The resulting current distribution on the pair of tubular conductors gives rise to radiating field.

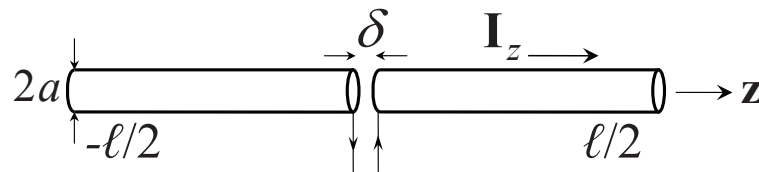


Fig. 1. Center-fed dipole antenna.

Referring to Fig. 1, let us assume that the length of the dipole is much larger than its radius ($\ell \gg a$) and its radius is much smaller than the wavelength ($a \ll \lambda$), so that the effects of the end faces of the dipole can be neglected. Therefore, the boundary conditions for a wire with infinite conductivity are those of vanishing total tangential electric field on the surface of the antenna and vanishing current at its ends [2]. For convenience, we assume that a constant voltage V_i is applied at the input terminals of the dipole, i.e., the delta-gap excitation.

Under the above conditions, an integral equation may be constructed for analysis of the dipole antenna, in which the unknown function is the current distribution on the antenna. After an extensive mathematical procedure, the final form of the integral equation is

$$\int_{-\ell/2}^{\ell/2} \frac{e^{-j\beta R}}{4\pi R} I_z(z') dz' = C \cos \beta z - \frac{j}{2\eta_0} \sin \beta |z|, \quad (2.1)$$

where

$$R = \sqrt{(z - z')^2 + a^2}. \quad (2.2)$$

In Eq. (2.1), I_z is the unknown function (current density on the antenna) to be determined, β is the phase constant, $\eta_0 = 120\pi \Omega$ is the free space intrinsic impedance, j is the imaginary unit, and C is an unknown constant coefficient which should be determined within the solution of the integral equation.

Equation (2.1) is referred to as *Hallén's integral equation* with the reduced kernel approximation. Obviously, this equation has the form of first kind Fredholm integral equation. Solution of Eq. (2.1) gives the current distribution along the dipole. Since, in this problem, the current distribution I_z is even (due to the symmetry of the structure), then Hallén's integral equation can be rewritten in the form

$$\int_0^{\ell/2} G(z, z') I_z(z') dz' = C \cos \beta z - \frac{j}{2\eta_0} \sin \beta z, \quad (2.3)$$

in which

$$G(z, z') = \frac{e^{-j\beta R}}{4\pi R} + \frac{e^{-j\beta R'}}{4\pi R'}, \quad (2.4)$$

$$R = \sqrt{(z - z')^2 + a^2}, \quad (2.5)$$

$$R' = \sqrt{(z + z')^2 + a^2}, \quad (2.6)$$

where $z \in [0, \ell/2]$.

Once the current distribution on the surface of a radiation structure is calculated one can obtain the radiation patterns for the structure. The normalized electric field pattern of a center-fed dipole antenna may be obtained in terms of the current density as follows:

$$f(\theta) = \frac{\left| \sin \theta \int_{-\ell/2}^{\ell/2} I_z(z') e^{j\beta z' \cos \theta} dz' \right|}{\max \left| \sin \theta \int_{-\ell/2}^{\ell/2} I_z(z') e^{j\beta z' \cos \theta} dz' \right|}, \quad (2.7)$$

for $0 \leq \theta \leq 180^\circ$.

3 Solution of Hallén's equation

This section presents an efficient numerical method for the solution of Hallén's integral equation to obtain the current distribution on the dipole antenna. Firstly, we introduce the basis functions required for implementation of the method.

3.1 Basis functions

The Sinc cardinal functions are mathematically defined as [5]

$$C_j(x; h) = \frac{\sin [\pi(x - jh)/h]}{\pi(x - jh)/h}, \quad (3.8)$$

where h has positive real value. The collocation points regarding these functions are evenly spaced, such that

$$x_i = ih, \quad i \in \mathbb{Z}. \quad (3.9)$$

The Sinc cardinal functions are also referred to as Whittaker cardinal functions.

An arbitrary function $f(x)$ may be approximated by a finite series of the above functions if $f(x)$ decays sufficiently fast as $|x| \rightarrow \infty$ [5]. In this case we have

$$f(x) \simeq \sum_j f_j C_j(x), \quad \text{for sufficiently small } h, \quad (3.10)$$

where $f_j = f(x_j)$ and x_j 's are defined by (3.9).

3.2 Formulation of the method

Since Hallén's equation has the form of a first kind Fredholm integral equation, therefore we propose the collocation method for the solution of a typical first kind equation as

$$\int_a^b k(s, t)x(t)dt = f(s), \quad a \leq s \leq b, \quad (3.11)$$

where the functions k and f are known but x is the unknown function to be determined. Moreover, $k \in L^2([a, b] \times [a, b])$ and $f \in L^2([a, b])$, in which L^2 is the space of square integrable functions.

Considering a finite number of the Sinc cardinal functions, we can approximate the unknown function x by (3.10) as

$$x(t) \simeq \sum_{j=n}^{n+m} x_j C_j(t), \quad (3.12)$$

in which m is the number of the basis functions such that $h = \frac{b-a}{m}$. Also, $n = \lceil \frac{a}{h} \rceil = \lceil \frac{a}{b-a} m \rceil$.

Substituting (3.12) into (3.11) gives

$$\int_a^b k(s, t) \left(\sum_{j=n}^{n+m} x_j C_j(t) \right) dt \simeq f(s). \quad (3.13)$$

By choosing $m + 1$ points $s_i = ih$ in interval $[a, b]$ we obtain

$$\int_a^b k(s_i, t) \left(\sum_{j=n}^{n+m} x_j C_j(t) \right) dt \simeq f(s_i), \quad (3.14)$$

$$i = n, n + 1, \dots, n + m,$$

or

$$\sum_{j=n}^{n+m} \left(\int_a^b k(s_i, t) C_j(t) dt \right) x_j \simeq f(s_i), \quad (3.15)$$

$$i = n, n + 1, \dots, n + m.$$

Equation (3.15) is a linear system of algebraic equations in terms of the unknown vector $\mathbf{x} = [x_n, x_{n+1}, \dots, x_{n+m}]$. Solution of this system gives the expansion coefficients for (3.12). Hence, an approximate solution $x(s) \simeq \sum_{j=n}^{n+m} x_j C_j(s)$ is obtained for Fredholm integral equation of the first kind.

3.3 Numerical results

Now, we use the proposed method for solving Hallén’s integral equation and consequently obtaining the current distribution on the dipole antenna. The numerical results are shown in figures 2 – 4 as the current density graphs for different values of ℓ and a (in terms of the wavelength). By calculating the current density we can obtain the radiation pattern for the antenna. The normalized electric field patterns of the dipole are shown in figures 5 – 7 . Also, the three-dimensional radiation patterns are given in figures 8 – 10. All the computations have been performed at frequency $f = 0.3$ GHz.

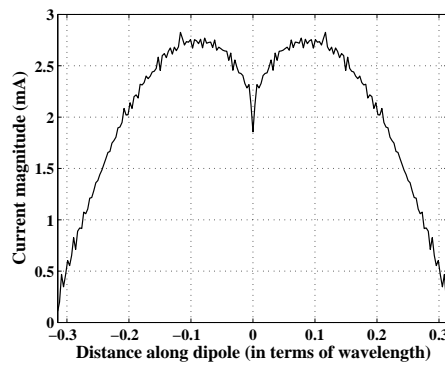


Fig. 2. Current distribution along the dipole antenna for $\ell = 0.65\lambda$ and $a = 0.001\ell$.

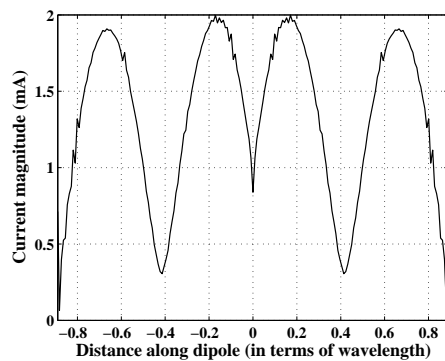


Fig. 3. Current distribution along the dipole antenna for $\ell = 1.8\lambda$ and $a = 0.001\ell$.

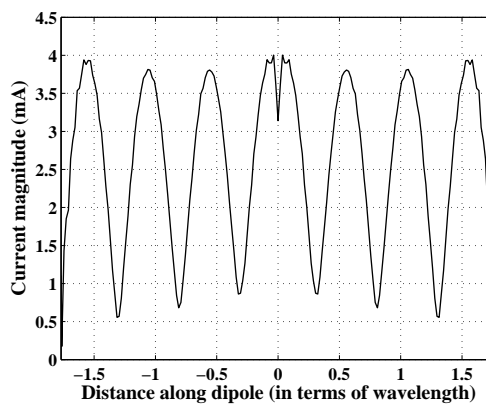


Fig. 4. Current distribution along the dipole antenna for $\ell = 3.6\lambda$ and $a = 0.0005\ell$.

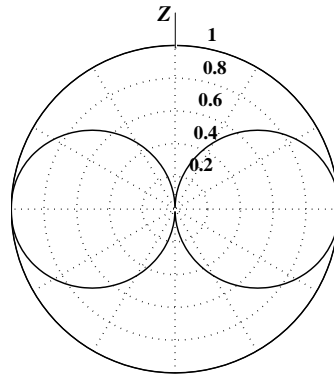


Fig. 5. Normalized electric field pattern of the dipole antenna for $\ell = 0.65\lambda$ and $a = 0.001\ell$.

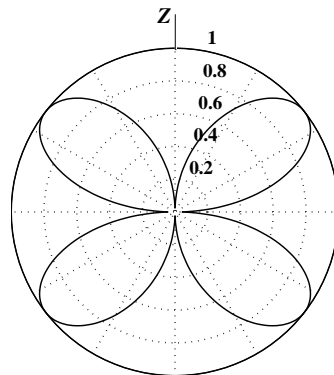


Fig. 6. Normalized electric field pattern of the dipole antenna for $\ell = 1.8\lambda$ and $a = 0.001\ell$.

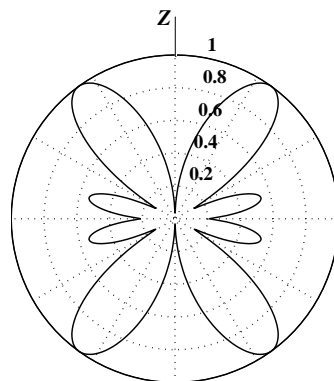


Fig. 7. Normalized electric field pattern of the dipole antenna for $\ell = 3.6\lambda$ and $a = 0.0005\ell$.

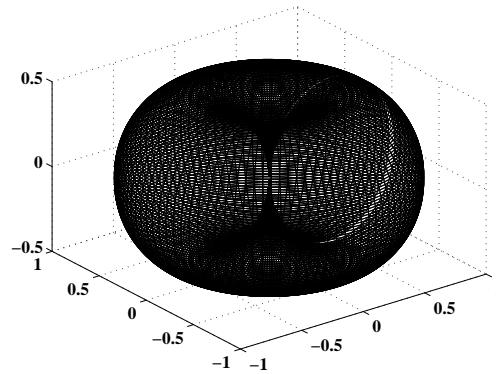


Fig. 8. Three-dimensional radiation pattern of the diploe antenna for $\ell = 0.65\lambda$ and $a = 0.001\ell$.

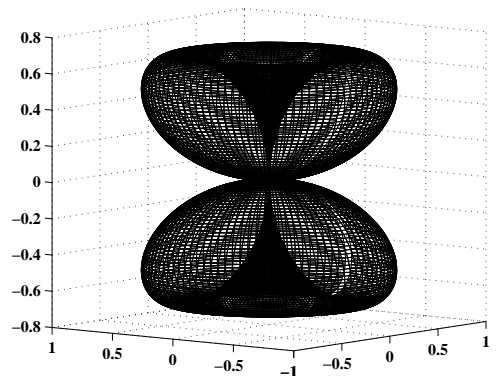


Fig. 9. Three-dimensional radiation pattern of the diploe antenna for $\ell = 1.8\lambda$ and $a = 0.001\ell$.

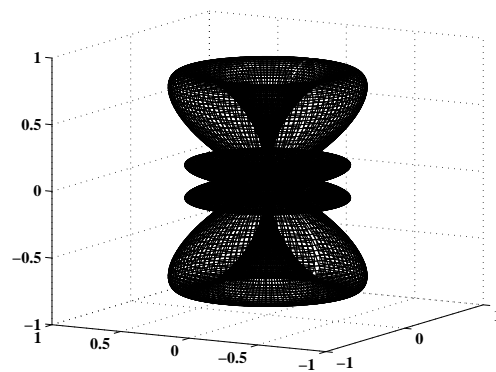


Fig. 10. Three-dimensional radiation pattern of the diploe antenna for $\ell = 3.6\lambda$ and $a = 0.0005\ell$.

4 Conclusion

An effective numerical method for analysis of thin wire dipole antenna was proposed by using a set of cardinal functions. The method was applied in the solution of Hallén's integral equation to obtain the current distribution on the surface of the antenna. The

numerical results confirmed the efficiency of the method in view of calculating the current density and radiation pattern graphs.

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