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Effect of Rotation on Thermoelastic Waves with Green-Naghdi Theory in a Homogeneous Isotropic Hollow Cylinder

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Abstract

A new model of the equations of generalized thermoelasticiy for a material of the cylinder which is supposed to be homogeneous isotropic thermally conducting is given. The formulation is applied in the context of Green and Naghdi (GN) theory of types II and III under the effect of rotation. The problem has been solved numerically using a finite element method. Numerical results for the temperature distribution, displacement, radial stress, and hoop stress are represented graphically. The results indicate that the effect of rotation was very pronounced. Comparisons are made with the results predicted by the types II and III in the presence and absence of rotation. The results obtained in this paper can be used to design various homogeneous thermoelastic elements under thermal load to meet special engineering requirements.

Keywords : Rotation, Homogeneous, Isotropic, Hollow cylinder, Finite element method, Green-Naghdi theory.

1 Introduction

During the past two decades, widespread attention has been given to thermoelasticity theories that admit a finite speed for the propagation of thermal signals. In contrast to the conventional theories based on parabolic-type heat equation, these theories are referred to as generalized theories. The problem of rotation disks or cylinders has its application in high-speed cameras, steam and gas turbines, planetary landings and in many other

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domains. First, the high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of the aircraft structure. Second, in the nuclear field, the extremely high temperature and temperature gradients originatng inside nuclear reactors influence their design and operations.
The counterparts of our problem in the contexts of the coupled thermoelasticity theory, the Green-Lindsay (GL) theory [7] and the Lord-Shulman (LS) theory [12], have been considered by Othman [13, 14] and Othman and Singh [15], respectively. At appropriate stages of our analysis, we make a comparison of our results with those obtained in these works. This comparison reveals that, on the whole, the predictions of the GN-theory (as obtained here) are qualitatively similar to those of the LS-theory. More importantly, we notice that certain physically unrealistic features inherent in the conventional coupled thermoelasticity theory and the GL-theory are not present in the GN-theory.
The classical theory of thermoelasticity as exposed, for example, in Carlson's article [3] has found generalizations and modifications in various thermoelastic models that run under the label hyperbolic thermoelasticity; see the survey of Hetnarski and Ignazack [11]. The notation hyperbolic reflects the fact that thermal waves are modeled, avoiding the physical paradox of infinite propagation speed of the classical model. In the 1990s, Green and Naghdi [8, 9, 10] proposed three new thermoelastic theories based on an entropy equality rather than the usual entropy inequality. The constitutive assumptions for the heat flux vector are different in each theory. Thus, they obtained three theories that they called thermoelasticity of type I, type II and type III. When the theory of type I is linearized, we obtain the classical system of thermoelasticity. The theory of type II (a limiting case of type III) does not admit energy dissipation. In the context of the linearized version of this theory, theorems on uniqueness of solutions have been established by Hetnarski and Ignazack [11] and Green and Naghdi [10]. Boundary-initiated waves in a half-space and in an unbounded body with cylindrical cavity have been studied by Green and Naghdi [8] and Chandrasekharaiah and Srinath [4, 5]. Also plane waves thermal shock problems have been studied by Othman et al. [16] and Othman and Song [17, 18, 19].
The exact solution of the governing equations of the generalized thermoelasticity theory for a coupled and nonlinear/linear system exists only for very special and simple initial and boundary problems. To calculate the solution of general problems, a numerical solution technique is used. For this reason the finite element method is chosen. The method of weighted residuals offers the formulation of the finite element equations and yields the best approximate solutions to linear and nonlinear ordinary and partial differential equations. Applying this method basically involves three steps. The first step is to assume the general behavior of these approximating functions in the differential equations and boundary conditions results in some errors, called the residual. This residual has to vanish in an average sense over the solution domain. The second step is the time integration. The time derivatives of the unknown variables have to be determined by former results. The third step is to solve the equations resulting from the first and the second steps by using a finite element algorithm program (see Zienkiewicz [22]). Abbas [1], Abbas and Abdalla [16] and Youssef and Abbas [21] applied the finite element method in different problems.
The aim of the present paper is to study the effect of rotation on the thermal shock problem of generalized thermoelasticity of a homogeneous isotropic hollow cylinder based on Green-Naghdi theory of type II and type III. The problem has been solved numerically using a finite element method (FEM). Numerical results for the temperature distribution, displacement, radial stress and hoop stress are represented graphically in the presence and

absence of rotation.

2 Formulation of the Problem

In the context of generalized thermoelasticity based on Green-Naghdi theory of type II and type III, the equation of motion, taking the rotation term about the $z-$axis as a body force is

$$(\mu+\lambda)u_{j,ij}+\mu u_{i,jj}-\rho\Omega^2 r-\gamma T_{,i}=\rho\ddot{u}_i \tag{2.1}$$

where Ω is the uniform angular velocity and ρ is the density of the cylinder material. The generalized energy equation can be expressed as

$$K^*T_{,ii}+K\dot{T}_{,ii}=\rho C_E\ddot{T}+\gamma T_0\ddot{u}_{i,i} \tag{2.2}$$

The constitutive equations have the form

$$\tau_{ij}=\lambda u_{i,i}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)-\gamma T\delta_{ij} \tag{2.3}$$

where λ, μ are Lame's constants, $\gamma=(3\lambda+2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion, C_E is the specific heat at constant strain, T is the temperature above reference temperature T_0, K^* and K are respectively the thermal conductivity and material constant characteristic of the theory. When $K\to 0$, Eq.(2.2) reduces to the heat conduction equation of GN type II theory.
In a cylindrical coordinate system (r,θ,z) for the axially symmetric problem, $u_r=u_r(r,z,t)$, $u_\theta=0$, $u_z=u_z(r,z,t)$. Furthermore, if only the axisymmetric plane strain problem is considered, we have $u_r=u(r,t)$ and $u_\theta=u_z=0$. The strain-displacement relations are

$$e_{rr}=\frac{\partial u}{\partial r},\qquad e_{\theta\theta}=\frac{u}{r},\qquad e_{zz}=e_{rz}=e_{r\theta}=e_{\theta z}=0 \tag{2.4}$$

The stress-strain relations are

$$\tau_{rr}=2\mu\frac{\partial u}{\partial r}+\lambda\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)-\gamma T \tag{2.5}$$

$$\tau_{\theta\theta}=2\mu\frac{u}{r}+\lambda\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)-\gamma T \tag{2.6}$$

$$\tau_{zz}=\lambda\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)-\gamma T,\quad \tau_{rz}=\tau_{r\theta}=\tau_{\theta z}=0 \tag{2.7}$$

It is assumed that there is no heat source in the medium, thus the equation of motion and energy equation have the form:

$$\frac{\partial\tau_{rr}}{\partial r}+\frac{\tau_{rr}-\tau_{\theta\theta}}{r}-\rho\Omega^2 r=\rho\frac{\partial^2 u}{\partial t^2} \tag{2.8}$$

$$K^*\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+K\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial^2 T}{\partial t\partial r}\right)=\rho C_E\frac{\partial^2 T}{\partial t^2}+\gamma T_0\frac{\partial}{\partial t^2}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right) \tag{2.9}$$

It is convenient to change the preceding equations into the dimensionless forms. To do this, the dimensionless parameters are introduced as

$$(r^0, u^0) = \frac{(r, u)}{c_1\omega_1}, \quad t^0 = \frac{t}{\omega_1}, \quad (\tau_{rr}^0, \tau_{\theta\theta}^0, \tau_{zz}^0) = \frac{1}{\mu}(\tau_{rr}, \tau_{\theta\theta}, \tau_{zz}), \quad \Theta^0 = \frac{\gamma T}{\rho c_1^2} \tag{2.10}$$

where $c_1^2 = \frac{\lambda+2\mu}{\rho}$, $\omega_1 = \frac{K}{\rho C_E c_1^2}$.
Putting (2.10) into equations (2.5)-(2.9) one may obtain (after dropping the superscript $^\circ$ for convenience)

$$\tau_{rr} = (2 + a_1)\frac{\partial u}{\partial r} + a_1\frac{u}{r} - (2 + a_1)\Theta \tag{2.11}$$

$$\tau_{\theta\theta} = a_1\frac{\partial u}{\partial r} + (2 + a_1)\frac{u}{r} - (2 + a_1)\Theta \tag{2.12}$$

$$\tau_{zz} = a_1\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) - (2 + a_1)\Theta \tag{2.13}$$

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} - \frac{\partial \Theta}{\partial r} - \Omega^2 r = \frac{\partial^2 u}{\partial t^2} \tag{2.14}$$

$$\left(\varepsilon_2 + \varepsilon_3\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\Theta}{\partial r^2} + \frac{1}{r}\frac{\partial\Theta}{\partial r}\right) - \frac{\partial^2\Theta}{\partial t^2} = \varepsilon_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) \tag{2.15}$$

$$a_1 = \frac{\lambda}{\mu}, \quad \varepsilon_1 = \frac{\gamma^2 T_0}{\rho^2 c_1^2 C_E}, \quad \varepsilon_2 = \frac{K^*}{\rho^2 c_1^2 C_E}, \quad \varepsilon_3 = \frac{K}{\rho c_1^2 C_E \omega_1}$$

From the preceding description, the initial and boundary conditions may be expressed as

$$u(r, 0) = \frac{\partial u(r, 0)}{\partial t} = 0, \quad \Theta(r, 0) = \frac{\partial\Theta(r, 0)}{\partial t} = 0 \tag{2.16}$$

$$\tau_{rr}(a, t) = 0, \quad \tau_{rr}(b, t) = 0, \quad \Theta(a, t) = H(t), \quad \frac{\partial\Theta(b, t)}{\partial r} = 0 \tag{2.17}$$

where a and b are the inner and outer radii of the hollow cylinder, respectively, and H is the heaviside unit step function.

3 Finite Element Method

In order to investigate the numerical solution of the thermal shock problem of generalized thermoelasticity of a homogeneous isotropic rotating hollow cylinder using the finite element method, the FEM (Reddy [20] and Cook et al. [6]) is adopted due to its flexibility in modeling layered structures and its capability in obtaining full field numerical solution. The governing equations (2.14) and (2.15) are coupled with initial and boundary conditions (2.16) and (2.17). The numerical values of the dependent variables like displacement and the temperature Θ are obtained at the points of interest, which are called degrees of freedom. The weak formulations of the non-dimensional governing equations are derived. The set of independent test functions to consist of the displacement δu and the temperature $\delta\Theta$ is prescribed. The governing equations are multiplied by independent weighting functions and then are integrated over the spatial domain with the boundary. Applying integration by parts and making use of the divergence theorem reduce the order of the spatial derivatives and allow for the application of the boundary conditions. The same shape functions are defined piecewise on the elements. Using the Galerkin procedure, the unknown fields u and Θ and the corresponding weighting functions are approximated by the same shape functions. The last step towards the finite element discretization is to choose the element type and the associated shape functions. Three nodes of quadrilateral elements are used. The shape function is usually denoted by the letter N and is usually the coefficient that appears in the interpolation polynomial. A shape function is written for each individual node of a finite element and has the property that its magnitude is 1 at that node and 0 for all other nodes in that element. We assume that the master element has its local coordinates in the range $[-1, 1]$. In our case, the one-dimensional quadratic elements are used, which were given by [22] as:

Linear shape functions

$$N_1 = \frac{1}{2}(1-\xi), \quad N_2 = \frac{1}{2}(1+\xi) \tag{3.18}$$

Quadratic shape functions

$$N_1 = \frac{1}{2}(\xi^2-\xi), \quad N_2 = 1-\xi^2, \quad N_3 = \frac{1}{2}(\xi^2+\xi) \tag{3.19}$$

On the other hand, the time derivatives of the unknown variables have to be determined by the Newmark time integration method (Cook et al. [6]).

4 Numerical Results

To illustrate the problem we will present some numerical results. The copper material was chosen for purposes of numerical computation, the physical data for which are given as [21]:

$$\lambda = 7.76\times10^{10}(kg)(m)^{-1}(s)^{-2}, \quad \mu = 3.86\times10^{10}(kg)(m)^{-1}(s)^{-2}, \quad T_0 = 293(K),$$
$$\rho = 8.95\times10^{3}(kg)(m)^{-3}, \quad C_E = 3.83\times10^{2}(m)^2(K)^{-1}(s)^{-2}, \quad \varepsilon_1 = 0.0168,$$
$$\alpha_t = 17.8\times10^{-6}(K)^{-1}$$

The physical quantities displacement, temperature, radial stress and hoop stress depend not only on time t and space r, but also on the characteristic parameter of the Green-Naghdi theory of type II and type III. Here, all the variables are taken in non-dimensional forms. The results for the displacement, the temperature, the radial stress and the hoop stress have been obtained by taking $t = 0.2$ based on Green-Naghdi theory of type II and type III. Fig. 1 - Fig. 4 exhibit the variation of the displacement, the temperature, the radial stress and the hoop stress with respect to r for the two types II, III of Green-Naghdi theory and three different values of $\Omega = 0, 0.8, 1.2$. Fig. 1, Fig. 3 and Fig. 4 show that the displacement, the radial stress and the hoop stress are decreasing with an increase in the rotation for $r > 0$. It is observed that the rotation has a great effect on these physical quantities. While the rotation has no effect on the temperature, as shown in Fig. 2, Fig. 5 - Fig. 8 depict the variation of the displacement, the temperature, the radial stress and the hoop stress under Green-Naghdi theory of type II (without energy dissipation) when $\varepsilon_2 = 0.2$ and $\varepsilon_3 = 0$, with respect to time t for three different values of $\Omega = 0, 0.8, 1.2$ at $r = 1.15$. It can be observed from Fig. 5, Fig. 7 and Fig. 8 that the rotation has a decreasing effect on the displacement, the radial stress and the hoop stress for $t > 0$. Fig. 6. shows that the rotation has an increasing effect on the temperature for $t > 0$. Fig. 9 - Fig. 12 demonstrate the variation of the displacement, the temperature, the radial stress and the hoop stress under Green-Naghdi theory of type III (with energy dissipation) when $\varepsilon_0.3$ and $\varepsilon_2 = 0.2$ with respect to time t for three different values of $\Omega = 0, 0.8, 1.2$ at $r = 1.15$. Fig. 9, Fig. 11 and Fig. 12 show that the rotation has a decreasing effect on the displacement, the radial stress and the hoop stress for $t > 0$. Fig. 10 shows that the rotation has no effect on the temperature with respect to Green-Naghdi theory of type III.

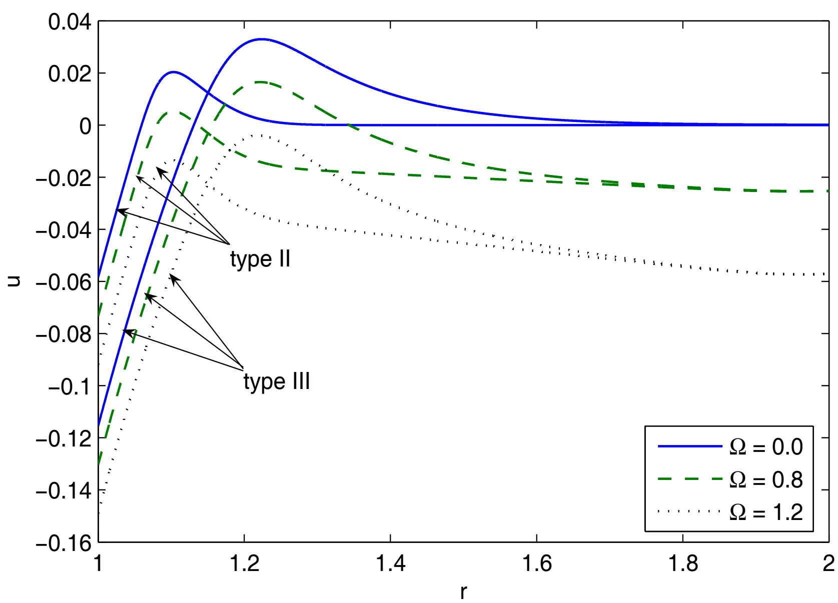

Fig. 1. The displacement distribution different values of for Ω.

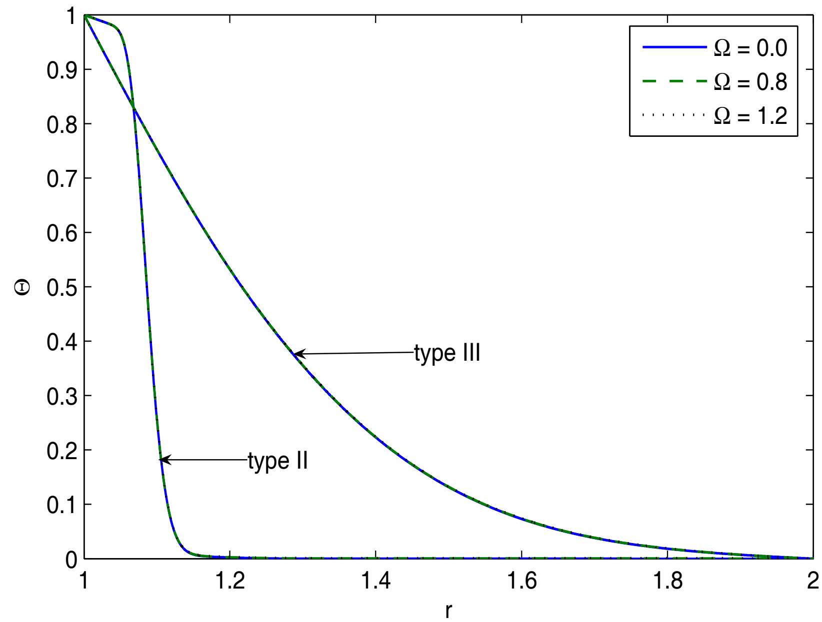

Fig. 2. The temperature distribution for different values of Ω.

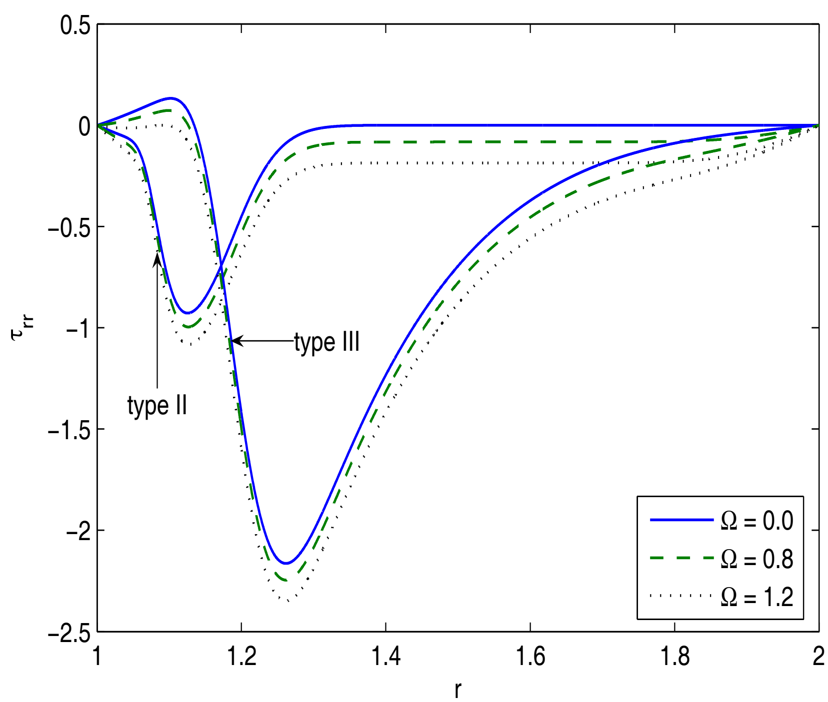

Fig. 3. The radial stress distribution for different values of Ω.

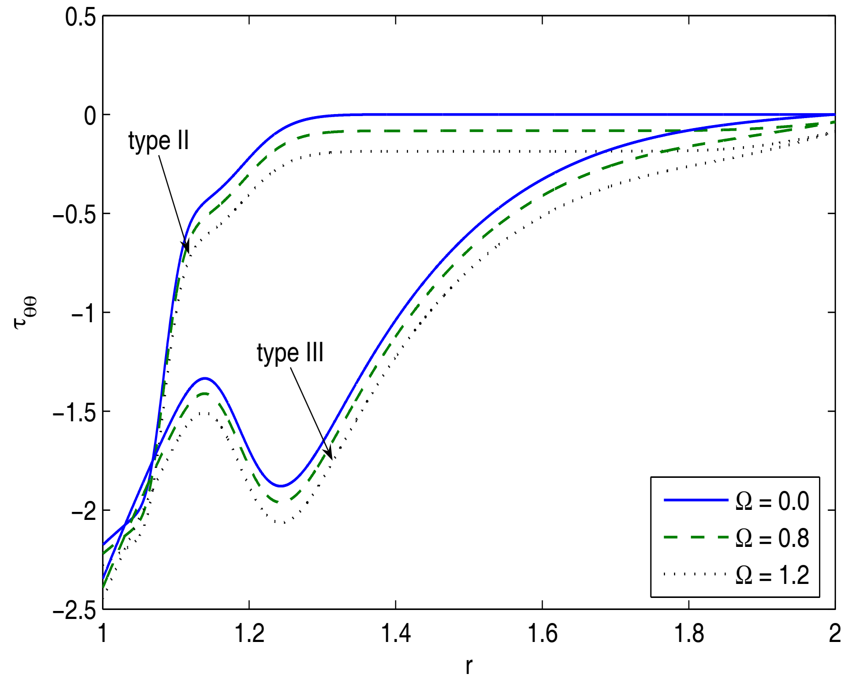

Fig. 4. The hoop stress distribution for different values of Ω.

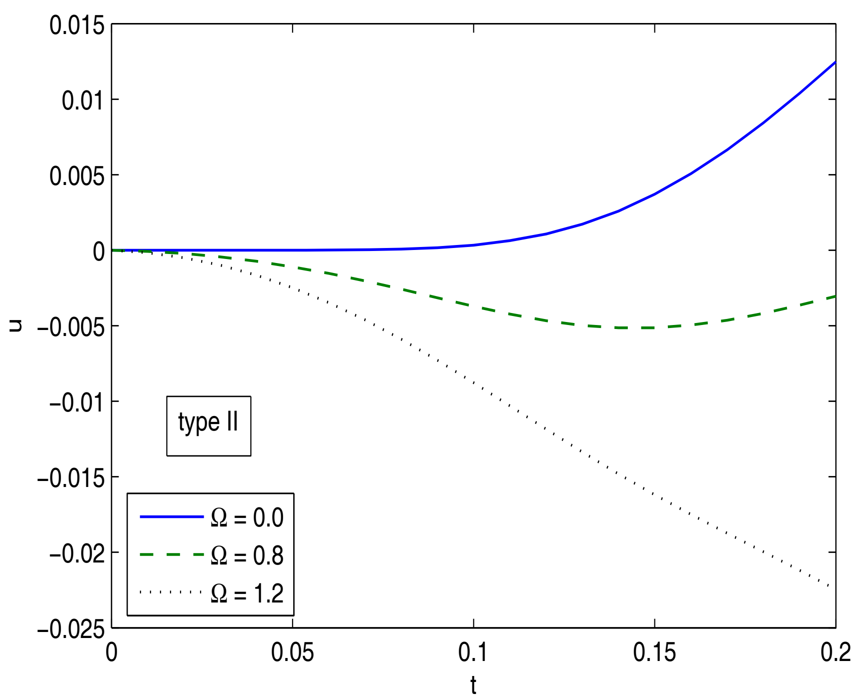

Fig. 5. The variation of displacement for type II at $r = 1.15$.

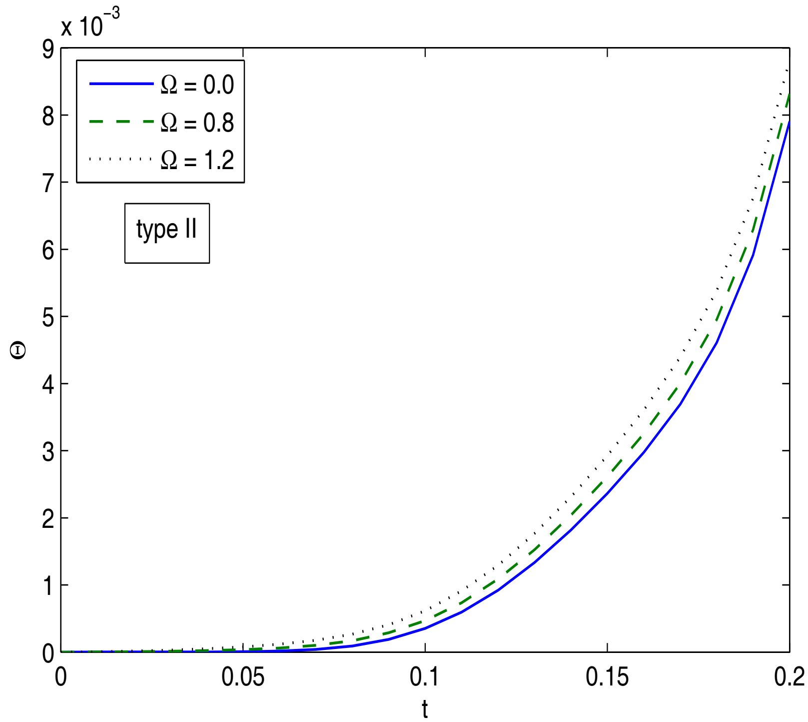

Fig. 6. The variation of temperature for type II at $r = 1.15$.

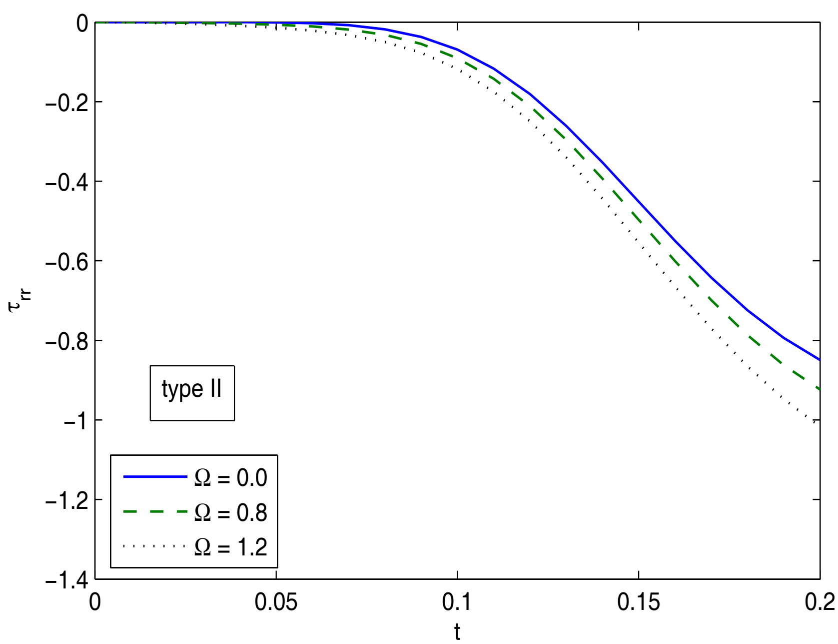

Fig. 7. The variation of radial stress for type II at $r = 1.15$.

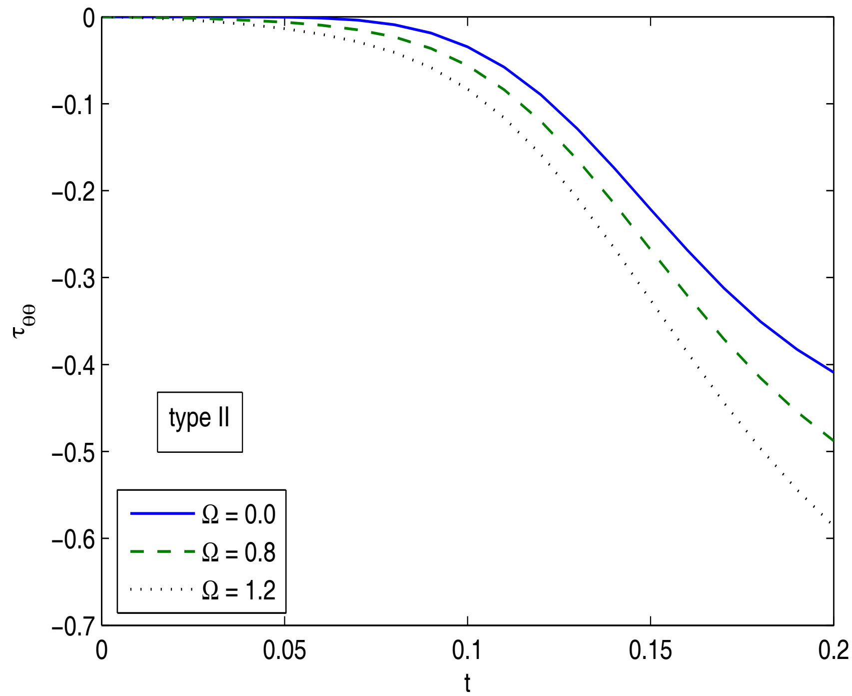

Fig. 8. The variation of hoop stress for type II at $r = 1.15$.

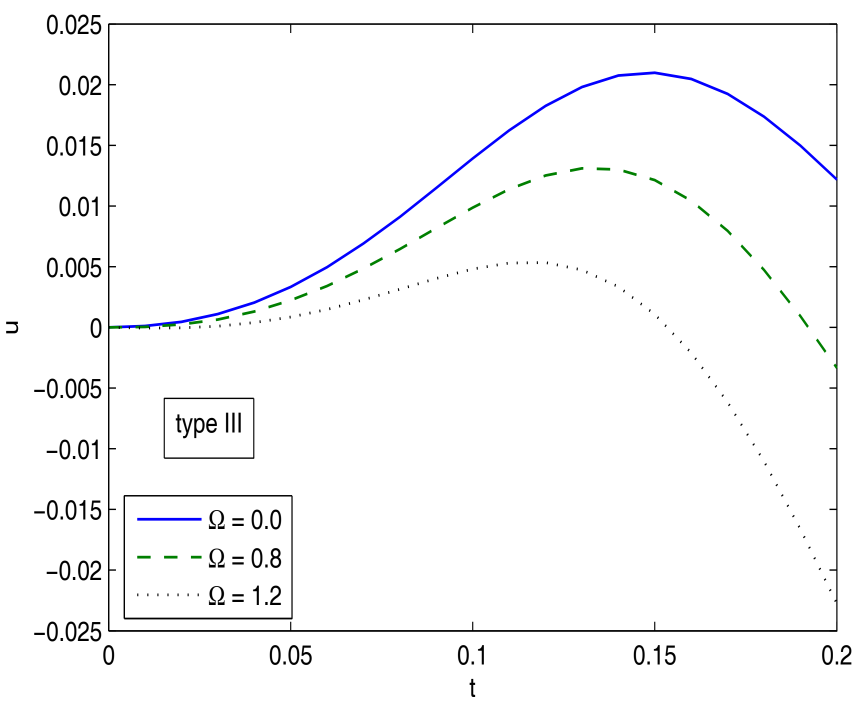

Fig. 9. The variation of displacement for type III at $r = 1.15$.

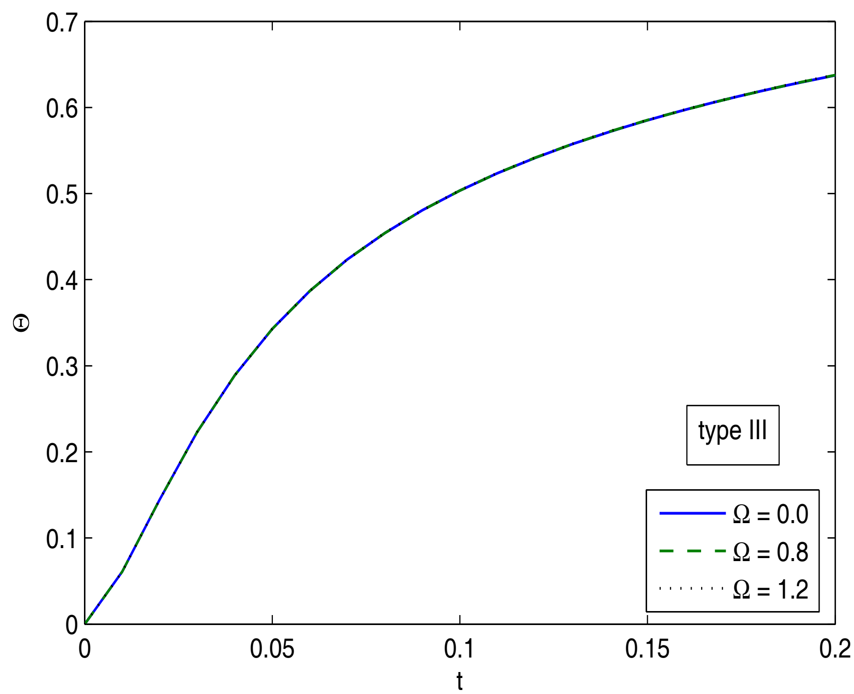

Fig. 10. The variation of temperature for type III at $r = 1.15$.

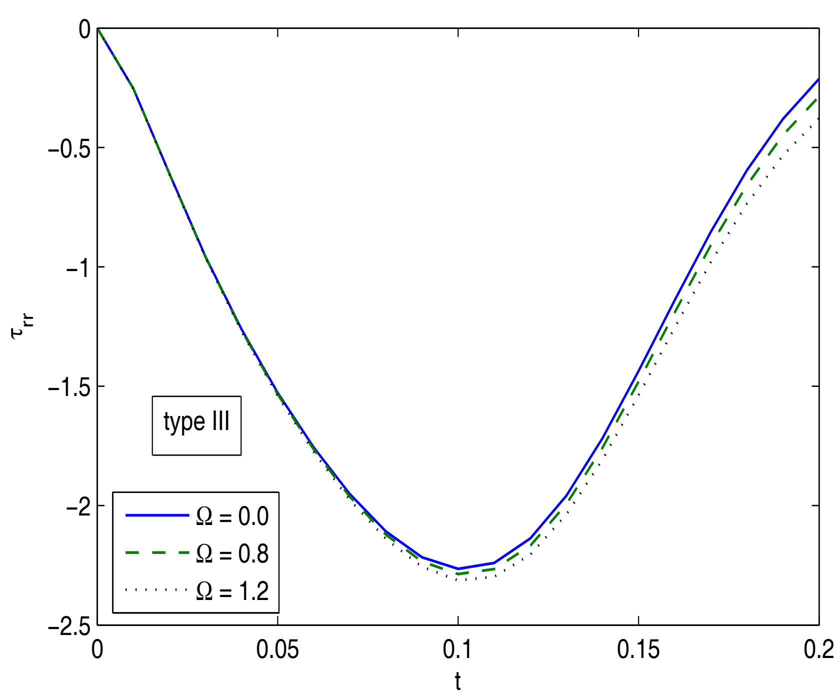

Fig. 11. The variation of radial stress for type III at $r = 1.15$.

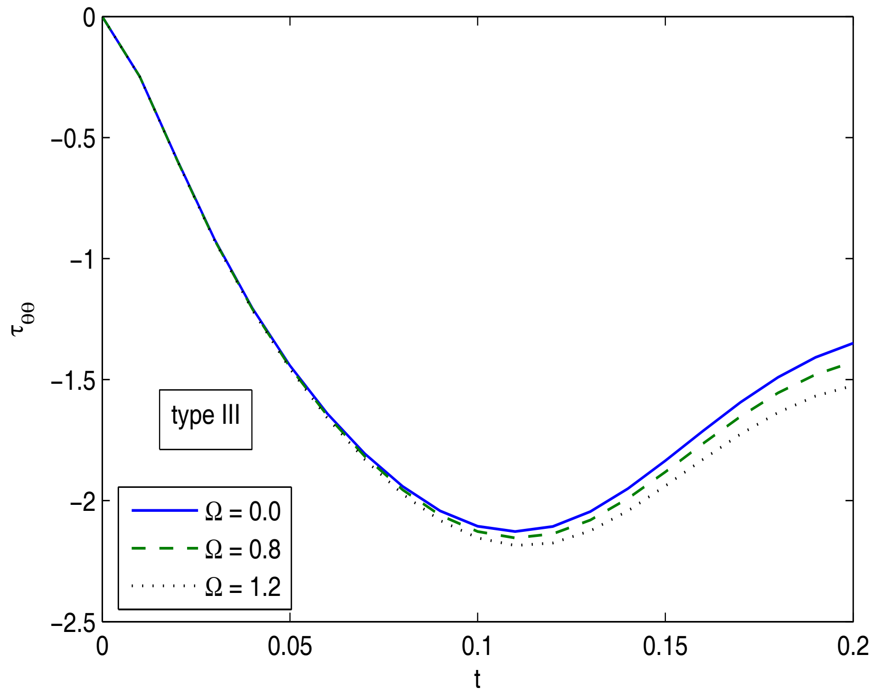

Fig. 12. The variation of hoop stress for type III at $r = 1.15$.

5 Conclusion

In this paper, we have investigated the solution of the thermal shock problem of generalized thermoelasticity of a rotating homogeneous isotropic hollow cylinder based on the Green-Naghdi theory of type II and type III by using the finite element method. The differences between the field quantities predicted by the GN theory of types II and III are remarkable in the presence and absence of rotation. We concluded that the rotation has a great effect on the field quantities. The results obtained in this paper can be used to design various homogeneous thermoelastic elements under thermal load to meet special engineering requirements.

References

[1] I. A. Abbas, Finite element analysis of the thermoelastic interactions in an unbounded body with a cavity. Forsch Ingenieurwes 71 (2007) 215-222.

[2] I. A. Abbas, A. N. Abd-alla, Effects of thermal relaxations on thermoelastic interactions in an infinite orthotropic elastic medium with a cylindrical cavity, Arch. Appl. Mech. 78 (2008) 283-293.

[3] D. E. Carlson, Linear thermoelasticity, Handbuch der Physik. Via/2 97 (1972) 346.

[4] D. S. Chandrasekharaiah, K. S. Srinath, One-dimensional waves in a thermo- elastic half-space without energy dissipation, Int. J. Engng. Sci. 34 (1996) 1447- 1455.

[5] D. S. Chandrasekharaiah, K. S. Srinath, Axisymmetric thermoelastic inter- actions without energy dissipation in an unbounded body with cylindrical cavity, J. Elasticity 46 (1997) 19-31.

[6] R. D. Cook, D. S. Malkus, M. E. Plesha, Concepts and applications of finite element analysis, Third Ed., John Wiley, New York, 1989.

[7] A. E. Green, K. A. Lindsay, Thermoelasticity, J. Elast. 2 (1972) 1-7.

[8] A. E. Green, P. M. Naghdi, A re-examination of the basic postulates of thermomechanics. Proc. Royal Society London A. 432 (1991) 171-194.

[9] A. E. Green, P. M. Naghdi, On undamped heat waves in an elastic solid. J. Thermal Stresses 15 (1992) 253-264.

[10] A. E. Green, P. M. Naghdi, Thermoelasticity without energy dissipation, J. Elasticity 31 (1993) 189-208.

[11] R. B. Hetnarski, J. Ignazack, Generalized thermoelasticity, J. Thermal Stresses 22 (1999) 541-470.

[12] H. W. Lord, Y. Shulman, A generalized dynamical theory of thermoelasticity, J. Mech. Phys. Solids 15 (1967) 299-309.

[13] M. I. A. Othman, Effect of rotation and relaxation time on thermal shock problem for a half-space in generalized thermo-visco-elasticity, Acta Mech. 74 (2005) 129-143.

[14] M. I. A. Othman, Effect of rotation on plane waves in generalized thermoelasticity with two relaxation times. Int. J. Solids and Structures 41 (2004) 2939-2956.

[15] M. I. A. Othman, Y. Q. Song, Reflection of magneto-thermoelasticity waves with two relaxation times and temperature dependent elastic moduli, Appl. Math. Modelling 32 (2008) 483-500.

[16] M. I. A. Othman, S. Y. Atwa, R. M. Farouk, Generalized magnetothermo-viscoelastic plane waves under the effect of rotation without energy dissipation, Int. J. Engng. Sci. 46 (2008) 369-653.

[17] M. I. A. Othman, Y. Q. Song, The effect of rotation on the reflection of magnetothermoelastic waves under thermoelasticity without energy dissipation, Acta Mechanica 184 (2006) 189-204.

[18] M. I. A. Othman, Y. Q. Song, Reflection of plane waves from an elastic solid halfspace under hydrostatic initial stress without energy dissipation, Int. J. Solids and Structures 44 (2007) 5651-5664.

[19] M. I. A. Othman, Y. Q. Song, Reflection and refraction of thermo-visco-elastic waves at the interface between two micropolar viscoelastic media without energy dissipation, Canadian J. Physics 85 (2007) 797-812.

[20] J. N. Reddy, An Introduction to the finite element method, Second Ed., Mc Graw-Hill, New York, 1993.

[21] H.Youssef, I. A. Abbas, Thermal shock problem of generalized thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity, Computational Methods in Science and Technology 13(2) (2007) 95-100.

[22] O. C. Zienkiewicz, R. L. Taylor, The finite element method Fluid dynamics, Fifth Ed Butterwroth-Heinemann, London, 2000.