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Research Article

Estimating most productive scale size in DEA with real and integer value data

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Abstract

For better guiding a system, senior managers should have accurate information. Using Data Envelopment analysis (DEA) help managers in this objective. Thus, many investigations have been made in order to find the most productive scale size (MPSS) for the evaluating decision making units (DMUs). In this paper we consider this case where there exist subsets of input and output variables to be integer valued. We use data envelopment analysis (DEA) technique, which is a mathematical programming for efficiency evaluation and target setting for a set of DMUs, and develop a model with which the desired MPSS point can be found. The applicability of developed model is illustrated in the context of the analysis to show the validity of the proposed model.

Keywords : Data envelopment analysis; Most productive scale size; Integer values.

1 Introduction

DAta envelopment analysis (DEA) is a mathematical programming technique for efficiency assessment of decision making units (DMUs). The CCR model, Charnes et al. [5], evaluates the relative efficiency of DMUs under constat returns to scale form of technology and Banker et al. [3] provided the BCC model under that of variable returns to scale. In DEA literature there are some methods for determining the returns to scale (RTS) situation such as Banker et al. [2], Färe et al. ([8]) and Zarepisheh et al. [?]. The type of RTS for each efficient DMU in a variable returns to scale production technology, indicates the direction of rescaling needed for improving the productivity. Banker et al. [3] reported that a new free BCC dual variable, u_o, estimates RTS. If the case of increasing returns to scale (IRS) the expansion of the DMU under evaluation is suggested, and by decreasing returns to scale (DRS), contraction is suggested. In the case of constant returns to scale (CRS) it is said that DMU under evaluation operates as a most productive scale size (MPSS). Banker [1] shows that the CCR model can be employed to test for DMU's RTS using the concept of MPSS. The concept of most productive scale size (MPSS) has been defined by Banker [1] is invariant with respect to the orientation of the model. In the production technology of the BCC model Banker [1] gave conditions for identifying increasing, constant and decreasing returns to scale. He showed that the type of RTS also serves as an indicator of the direction towards the MPSS. As Podinovski [11] states, if a DMU exhibits CRS, it operates at MPSS and if it exhibits IRS and DRS, it does not operate at MPSS but would achieve it by scaling its operations up and down, respectively.

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Also, Cooper et al. [6] proposed a model with a fractional objective function for determining most productive scale size. Jahanshahloo and Khodabakhshi [9], also, provided an input−output orientation model to estimate most productive scale size units with linear objective function. Khodabakhshi [10] stated that one of the important methods to deal with imprecise data is considering stochastic data in DEA. Thus he has studied most productive scale size considering stochastic data in DEA. Therefore, input_ output orientation model introduced in Jahanshahloo and Khodabakhshi [9] has been extended in stochastic data envelopment analysis.
Although real valued inputs and outputs are usually utilized in data envelopment analysis technique, in many occasions some inputs and/ or outputs can only take integer values. In this occasion rounding the acquired solution can lead to misleading efficiency assessments and performance targets. Therefore, some changes in conventional DEA models should be made in order to consider the real life settings. The aim of this paper is to propose a procedure for detecting the most productive scale size and guide units through the region of the most productive scale size when some inputs and/ or outputs can only take integer values.

The paper unfolds as follows: in Section 2, data envelopment analysis will be briefly reviewed. In Section 3, the proposed model will be presented. Section 4 contains an illustrative example and Section 5 concludes the paper.

2 Data envelopment analysis

Assume there are n homogeneous decision making units each of them uses m inputs x_{ij} (i=1,..., m) to produce s outputs y_{rj} (r=1,..., s). Also let $X_j \in R^m$ and $Y_j \in R^s$ be non-negative vectors. We define the set of production possibility as $T = \{(X,Y)|X \ can\ produce\ Y\}$.

When $T = T_{BCC}$ we have;

$$T_{BCC} = \{(x,y)| \quad x \geq \lambda X, y \leq \lambda Y, 1\lambda = 1, \lambda \geq 0\}$$

and when $T = T_{CCR}$ we have;

$$T_{CCR} = \{(x,y)| \quad x \geq \lambda X, y \leq \lambda Y, \lambda \geq 0\}$$

The constant returns to scale form of the enveloping problem, which was first introduced by Charnes et al. (1978), is as follows:

$$\begin{aligned} \min \quad & \theta - \varepsilon\Big(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\Big) \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io}, \quad i = 1, ..., m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1, ..., s, \\ & \lambda_j \geq 0, \quad j = 1, ..., n. \end{aligned} \tag{2.1}$$

The variable returns to scale form of the enveloping problem, which was first introduced by Banker et al. (1984), is as follows:

$$\begin{aligned} \min \quad & \theta - \varepsilon\Big(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\Big) \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io}, \quad i = 1, ..., m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1, ..., s, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \geq 0, \quad j = 1, ..., n. \end{aligned} \tag{2.2}$$

If a DMU_o is not CCR (BCC) efficient, via the following formulas which use the optimal values of the above models, we can project this DMU onto the CCR (BCC) efficiency frontier:

$$\begin{cases} \hat{x}_{io} = \theta^* x_{io} - s_i^{-*} = \sum_{j=1}^{n} \lambda_j^* x_{ij}, \quad i = 1, ..., m, \\ \hat{y}_{ro} = y_{ro} + s_r^{+*} = \sum_{j=1}^{n} \lambda_j^* y_{rj}, \quad r = 1, ..., s. \end{cases} \tag{2.3}$$

It should be noted that the DMU under assessment may be projected into the MPSS region by means of the following formulas, Banker and Morey [4]:

$$(\frac{\theta^* x_{io} - s_i^{-^*}}{\sum_{j=1}^n \hat{\lambda}_j^*}, \frac{y_{ro} + s_r^{+^*}}{\sum_{j=1}^n \hat{\lambda}_j^*}) \tag{2.4}$$

in which θ^*, s^{-^*} and s^{+^*} are optimal solution of CCR model and $\sum_{j=1}^n \hat{\lambda}_j^*$ is the optimal solution of the following model while decreasing returns to scale prevail at DMU_o.

$$\begin{aligned}
\max \quad & \sum_{j=1}^n \hat{\lambda}_j + \varepsilon(\sum_{i=1}^m \hat{s}_i^- + \sum_{r=1}^s \hat{s}_r^+) \\
s.t. \quad & \sum_{j=1}^n \hat{\lambda}_j x_{ij} + \hat{s}_i^- = \theta^* x_{io}, \quad i = 1, ..., m, \\
& \sum_{j=1}^n \hat{\lambda}_j y_{rj} - \hat{s}_r^+ = y_{ro}, \quad r = 1, ..., s, \\
& \sum_{j=1}^n \hat{\lambda}_j \leq 1, \\
& \hat{s}^- \geq 0, \quad \hat{s}^+ \geq 0, \quad \lambda_j \geq 0, \quad j = 1, ..., n.
\end{aligned} \tag{2.5}$$

It is noteworthy of attention that while increasing returns to scale prevail at DMU under evaluation the objective function of this model should be changed into minimization and also $\sum_{j=1}^n \hat{\lambda}_j \leq 1$, should be replaced by $\sum_{j=1}^n \hat{\lambda}_j \geq 1$. As mentioned in Banker and Morey [4], convexification of (2.3) provides a point in MPSS projection. That means, using the additional step provided by the benchmark projection formula one can achieve MPSS points.

3 MPSS with integer valued data

In this section, a model for finding a suitable MPSS will be introduced while there exist some inputs and/ or outputs that can only take integer values. Usually real valued data utilized in data envelopment analysis but in many occasions some inputs and/ or outputs can only take integer values. In this case rounding the real number will yield inaccurate results.
Without of loss of generality assume that DMU_o is BCC efficient. We emphasis that in order to find the maximum and minimum value of $\sum_{j=1}^n \hat{\lambda}_j$ there is no need to consider constraint (a) in model (2.5) and introduce different models for DMUs which have various RTS status. But it is sufficient to solve the following model for under assessment DMUs without consideration of its RTS type. Thus, consider the following problems which have constant returns to scale technology.

$$\begin{aligned}
\lambda^- = \quad \min \quad & \sum_{j=1}^n \hat{\lambda}_j - \varepsilon(\sum_{i=1}^m \hat{s}_i^- + \sum_{r=1}^s \hat{s}_r^+) \\
s.t. \quad & \sum_{j=1}^n \hat{\lambda}_j x_{ij} + \hat{s}_i^- = \theta^* x_{io}, i = 1, ..., m, \\
& \sum_{j=1}^n \hat{\lambda}_j y_{rj} - \hat{s}_r^+ = y_{ro}, r = 1, ..., s, \\
& \hat{s}^- \geq 0, \quad \hat{s}^+ \geq 0, \\
(a) \quad & \lambda_j \geq 0, j = 1, ..., n.
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
\lambda^+ = \quad \max \quad & \sum_{j=1}^n \hat{\lambda}_j + \varepsilon(\sum_{i=1}^m \hat{s}_i^- + \sum_{r=1}^s \hat{s}_r^+) \\
s.t. \quad & \sum_{j=1}^n \hat{\lambda}_j x_{ij} + \hat{s}_i^- = \theta^* x_{io}, i = 1, ..., m, \\
& \sum_{j=1}^n \hat{\lambda}_j y_{rj} - \hat{s}_r^+ = y_{ro}, r = 1, ..., s, \\
& \hat{s}^- \geq 0, \quad \hat{s}^+ \geq 0, \\
& \lambda_j \geq 0, j = 1, ..., n.
\end{aligned} \tag{3.7}$$

As stated in Jahanshahloo and Khodabakhshi [9], the largest and the smallest MPSS can be achieved. In regard of the obtained optimal solution of CCR model and the obtained λ^- and λ^+ through solving the aforesaid models, we can define the largest MPSS correspond with DMU_o as following:

$$(\frac{\theta^* X_o - S^{-^*}}{\lambda^-}, \frac{Y_o + S^{+^*}}{\lambda^-}) = (\bar{X}_o, \bar{Y}_o) \tag{3.8}$$

Equivalently, the smallest MPSS correspond with DMU_o can be defined as following:

$$(\frac{\theta^* X_o - S^{-^*}}{\lambda^+}, \frac{Y_o + S^{+^*}}{\lambda^+}) = (\tilde{X}_o, \tilde{Y}_o) \tag{3.9}$$

Considering CCR model, it is noteworthy of attention that since $(\theta^* X_o - s^{-*}, Y_o + s^{+*})$ belongs to the CCR frontier and due to the constant returns to scale axiom in CCR production possibility set, (3.8) also belongs to CCR frontier. As provided in Banker and Morey [4] it can be easily verified that when the projection point on CCR frontier is scaled by λ^- the obtained point is located onto the BCC frontier. Thus this convexification provides a MPSS point. The same procedure is true for the smallest MPSS. Considering the largest and the smallest MPSS points corresponding to DMU_o, for all $\gamma \in [0,1], \gamma(\bar{X}_o, \bar{Y}_o) + (1-\gamma)(\tilde{X}_o, \tilde{Y}_o)$ is MPSS as well. According to constant returns to scale axiom in CCR model, by rescaling a point located onto the frontier the projected point is located onto this frontier. Thus it is possible to find a point with integer value. But, the important issue is that we are looking for a point with integer value in the intersection of CCR and BCC frontiers. Note that the largest and smallest MPSS points correspond with DMU_o have integer coordinate in the required element then, but the aim is find a point suitable for the unit under evaluation in regards of its ability and the corresponding inputs and outputs. Since the convex combination of MPSS points is not necessarily an MPSS point, thus for finding an desired point we confine the analysis to seek in the intersection of CCR frontier and convex combination of MPSS points.

For finding an MPSS point with integer values for DMU_o we propose the following model in which E, indicates the set of MPSS points. These points can be recognized through solving the mentioned models in previous section. For DMU_o under assessment, after finding the largest and the smallest corresponding MPSS points we are seeking for the nearest point to the line segmented the largest and the smallest points. Moreover, since the aim is find a point which is MPSS thus we restrict the analysis to the intersection of CCR frontier and convex hull of MPSS points because under this condition the obtained point will be MPSS. The objective of the proposed model, (3.10), is to minimize the distance in order to find the smallest region in the intersection of CCR frontier and convex hull of MPSS points which contains a unit with integer value. In model (3.10) constraints (a) and (b) confine the analysis into the CCR frontier. Constraint (c) is also imposed to ensure that the obtained point has integer values. Moreover, Constraints (d), (e) and (f) indicate the convex combination of MPSS points. I_1 shows the set of inputs which can only take integer values. $\tilde{s}^-$ and $\tilde{s}^+$ are considered unrestricted in sign in order to increase the flexibility of this model to find a suitable MPSS point. It should be reminded that $\tilde{X}$ and $\bar{X}$ are those introduced in (3.9) and (3.8). For purpose of the remainder of this paper, we assume that in input side, a subset of variables can only take integer values.

$$
\begin{aligned}
&\min \quad \sum_{i=1}^{m} |\tilde{s}_i^-| + \sum_{r=1}^{s} |\tilde{s}_r^+| \\
&\textit{subject to} \\
&\sum_{j=1}^{n} \tilde{\lambda}_j x_{ij} = (\alpha \bar{x}_{io} + (1-\alpha)\tilde{x}_{io}) + \tilde{s}_i^-, i = 1, ..., m, (a) \\
&\sum_{j=1}^{n} \tilde{\lambda}_j y_{rj} = (\alpha \bar{y}_{ro} + (1-\alpha)\tilde{y}_{ro}) + \tilde{s}_r^+, r = 1, ..., s, (b) \\
&(\alpha \bar{x}_{io} + (1-\alpha)\tilde{x}_{io}) + \tilde{s}_i^- = w_i, i \in I_1, (c) \\
&\sum_{j \in E} \tilde{\mu}_j x_{ij} = (\alpha \bar{x}_{io} + (1-\alpha)\tilde{x}_{io}) + \tilde{s}_i^-, i = 1, ..., m, (d) \\
&\sum_{j \in E} \tilde{\mu}_j y_{rj} = (\alpha \bar{y}_{ro} + (1-\alpha)\tilde{y}_{ro}) + \tilde{s}_r^+, r = 1, ..., s, (e) \\
&\sum_{j \in E} \hat{\mu}_j = 1, (f) \\
&\tilde{s}^-, \quad \tilde{s}^+ \textit{unrestricted}, w_i \in Z_+, i \in I_1, \\
&\tilde{\lambda}_j \geq 0, \quad j = 1, ..., n, 0 \leq \alpha \leq 1.
\end{aligned}
\tag{3.10}
$$

This model can be easily converted into its linear counterpart. To this end let $\tilde{s}_r^+ = u_r - v_r, u_r \geq 0, v_r \geq 0$ for all r and $\tilde{s}_i^- = u_i - v_i, u_i \geq 0, v_i \geq 0$ for all i, where

$$
u_r = \begin{cases} 0, & \tilde{s}_r^+ \leq 0, \\ \tilde{s}_r^+, & \tilde{s}_r^+ \geq 0. \end{cases} \qquad v_r = \begin{cases} 0, & \tilde{s}_r^+ \geq 0, \\ -\tilde{s}_r^+, & \tilde{s}_r^+ \leq 0, \end{cases}
$$

and

$$
u_i = \begin{cases} 0, & \tilde{s}_i^- \leq 0, \\ \tilde{s}_i^-, & \tilde{s}_i^- \geq 0. \end{cases} \qquad v_i = \begin{cases} 0, & \tilde{s}_i^- \geq 0, \\ -\tilde{s}_i^-, & \tilde{s}_i^- \leq 0, \end{cases}
$$

which results in $u_i.v_i = 0$ and $u_r.v_r = 0$ for every r and i. Now, accordingly $|\tilde{s}_r^+| = u_r + v_r$ for all r and $|\tilde{s}_i^-| = u_i + v_i$ for all i. It should be noted that by using this variable transformation the nonlinear constraint, $u_r.v_r = 0$ and $u_i.v_i = 0$ for every r and i, are also added to the model. But nonlinear constraints, $u_r.v_r = 0$ and $u_i.v_i = 0$, are redundant due to the dependency of corresponding coefficient column vectors.

Thus, model (3.10) can be easily written in linear form. The linear counterpart of model (3.10) is as follows:

$$\begin{aligned}
&\min \quad \sum_{i=1}^{m}(u_i+v_i)+\sum_{r=1}^{s}(u_r+v_r)\\
&\textit{subjec to}\\
&\sum_{j=1}^{n}\tilde{\lambda}_j x_{ij} =\\
&(\alpha\bar{x}_{io}+(1-\alpha)\tilde{x}_{io})+u_i-v_i, i=1,...,m, (a)\\
&\sum_{j=1}^{n}\tilde{\lambda}_j y_{rj} =\\
&(\alpha\bar{y}_{ro}+(1-\alpha)\tilde{y}_{ro})+u_r-v_r, r=1,...,s, (b)\\
&(\alpha\bar{x}_{io}+(1-\alpha)\tilde{x}_{io})+u_i-v_i=w_i, i\in I_1, (c)\\
&\sum_{j\in E}\tilde{\mu}_j x_{ij} =\\
&(\alpha\bar{x}_{io}+(1-\alpha)\tilde{x}_{io})+u_i-v_i, i=1,...,m, (d)\\
&\sum_{j\in E}\tilde{\mu}_j y_{rj} =\\
&(\alpha\bar{y}_{ro}+(1-\alpha)\tilde{y}_{ro})+u_r-v_r, r=1,...,s, (e)\\
&\sum_{j\in E}\hat{\mu}_j = 1, (f)\\
&u_i\ge 0,\quad v_i\ge 0,\quad i=1,...,m, w_i\in Z_+,\quad i\in I_1,\\
&\tilde{\lambda}_j\ge 0, j=1,...,n, 0\le\alpha\le 1.
\end{aligned} \tag{3.11}$$

To clarify we proceed to develop the following examples by using the coordinate values as follows:
$A = (1,1)$ $B = (2,3)$ $C = (4,6)$ $D = (6,9)$ $E = (7,9.5)$ $F = (7,10)$

The coordinates are in the order (X, Y). In this example only input is confined to take integer value.
In this figure the BCC efficiency frontier represented by the solid lines and the CCR efficiency frontier represented by the dotted line from the origin which evaluates the technical and returns to scale performances of DMUs, simultaneously. We illustrate with A=(1, 1) in Fig. 1 for which we utilize expression (3.9) to obtain the smallest MPSS. By assessing unit A with CCR model we have $\theta^* = \frac{2}{3}$ and $s^{-^*} = s^{+^*} = 0$. Thus;

$$\frac{1\theta^* - s^{-^*}}{\frac{1}{3}} = 2, \quad \frac{1+s^{+^*}}{\frac{1}{3}} = 3$$

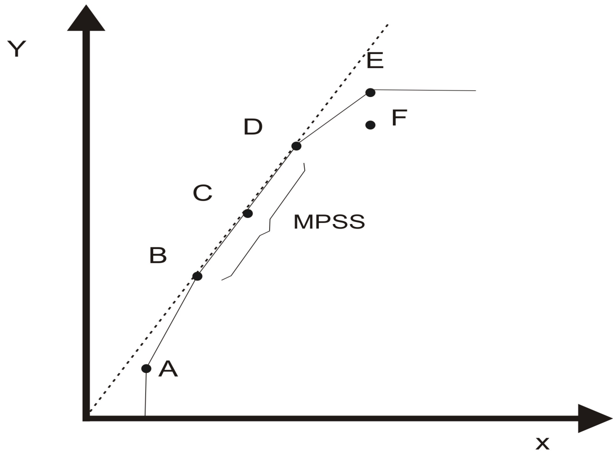

Figure 1: *Most productive scale size..*

which is unit B. Also, by Considering (3.8) the smallest and the largest MPSS points corresponding to unit A, will be unit B and D. Taking into account units B and D, as corresponding MPSS points for unit A, and solving the proposed model we will have $\alpha^* = 0.5$, $s^- = 0$ and $s^+ = 0$ which results in a point with the coordinate of $(3,4)$.

According to Fig. 1, this is the point in line segment of units B and C. This example is in R^2, one-input and one-output, thus at the first step the required MPSS is obtained but we use it just to show that how this model works. As noted, input is integer and it is needed to find a MPSS point with integer value in input. It should be mentioned that the largest and the smallest MPSS points in this example, as they are observed DMUs, have integer value in input. The proposed model has the ability to find other points in MPSS region with input being integer in accordance to the input and outputs of the unit under evaluation which is more suitable and achievable. As mentioned, considering other methods (3,4) can not be achieved.

4 Application

In this section we consider the data that were used in *benchmarking marketing productivity* written by Donthu et al.[7]. Input/Output data are gathered in Table 1.

The input variables, used by Donthu et al.[7]., include advertising and promotion expenses

Table 1: Data set of chain stores.

DMU #	θ^*_{BCC}	θ^*_{CCR}	*Input*1	*Input*2	*Output*1	*Output*2
1	0.95	1.00	3.5	24	4.1	3.6
2	1.00	1.00	2.2	32	4.0	3.2
3	0.85	0.97	3.9	30	4.4	4.2
4	0.78	0.87	4.1	35	4.4	4.4
5	1.00	1.00	2.5	25	3.6	4.0
6	0.93	1.00	3.3	24	3.9	4.2
7	0.76	0.90	5.0	33	4.5	4.5
8	1.00	1.00	2.1	32	3.6	3.7
9	1.00	1.00	2.8	20	3.6	3.8
10	0.77	0.97	4.3	36	4.5	4.6
11	0.74	0.78	4.0	33	4.0	4.1
12	0.84	0.90	3.5	32	4.2	4.2
13	0.92	1.00	4.0	26	4.3	4.0
14	0.81	0.82	3.0	38	3.9	4.0
15	0.85	1.00	4.6	30	4.6	4.5
16	0.89	0.97	3.5	28	4.2	4.0
17	0.88	0.96	3.5	29	4.2	4.2
18	0.93	0.93	2.6	30	3.7	3.9
19	0.87	0.90	3.0	24	3.5	3.7
20	0.89	0.97	3.5	25	4.0	4.0
21	0.81	1.00	4.2	30	4.3	4.6
22	0.83	0.92	3.8	32	4.3	4.4
23	0.96	1.00	3.0	35	4.5	4.6
24	0.84	0.87	3.4	30	4.0	4.1
25	0.90	0.96	3.8	24	3.9	4.0
26	0.87	0.88	2.9	28	3.7	3.6

Table 2: The obtained MPSS point after scaling.

DMU #	*Input*1	*Input*2	*Output*1	*Output*2
1	2.80	20.00	3.60	3.80
6	2.78	20.25	3.60	3.81
13	2.80	20.00	3.60	3.80
15	2.80	20.00	3.60	3.80
21	2.80	20.00	3.60	3.80
23	2.37	27.66	3.69	3.77

Table 3: MPSS points through solving the proposed model.

DMU #	*Input*1	*Input*2	*Output*1	*Output*2
1	2.80	20	3.60	3.80
6	2.80	20	3.60	3.80
13	2.80	20	3.60	3.80
15	2.80	20	3.60	3.80
21	2.80	20	3.60	3.80
23	2.35	28	3.68	3.77

(I_1) and number of employees (I_2). Output variables include customer satisfaction (O_1) and sales (O_2). As mentioned above, the number of employee (I_2) was included as an explicit input but it should be considered that number of employee is an integer value.

By solving BCC models efficient units under variable returns to scale can be identified. By solving CCR and BCC models the MPSS DMUs have been found which are DMUs 2, 5, 8 and 9. As noted in Table 1, DMUs 1, 6, 13, 15, 21 and 23 are BCC efficient but they are not MPSS. After examining the status of RTS it has been found that decreasing returns to scale prevail on all these units. Which means they do not operate at MPSS unless by scaling their operations down. As already mentioned, by projecting these units onto the CCR frontier and finding maximum and minimum of $\sum_{j=1}^{n} \hat{\lambda}_j^*$, for any DMU_j can be scaled thus the largest and the smallest MPSS points will be acquired. Finally, after this scaling if an MPSS point, with integer value in the second input, can not be found by solving the proposed model a suitable MPSS will be obtained. Consider DMU_{23} for which $\lambda^+ = \lambda^- = 1.22$ thus after scaling $(I_1, I_2) = (2.37, 27.66)$ and $(O_1, O_2) = (3.69, 3.77)$ will be obtained which is not a suitable MPSS. The result of using expressions (3.8) and (3.9) are gathered in Table 2. Noted that using these expressions the largest and the smallest MPSS points coincide into each other. Considering the units that were gathered in Table 2 and solving the proposed model we will have the following results listed in Table 3. According to the presented method MPSS points with integer value found, as indicated in Table 3. Now, in regards of the obtained MPPS points, it is possible for managers to guide units to reach the most productive scale size region in order to perform efficiently.

5 Conclusion

Real valued inputs and outputs are usually utilized in DEA technique. In many occasions some inputs and/ or outputs can only take integer values. The aim of this paper is to propose a procedure for detecting the MPSS and guide units through the region of the most productive scale size when some inputs and/ or outputs can only take integer values. In this paper we consider this case where there exist subsets of inputs variables to be integer valued. This procedure can be easily be generalized for different subsets of inputs and / or outputs. Illustrative example is also documented which confirms validity of this model as a means for determining MPSS points.

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