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Research Article

The Selection Suitable Targets in T_v on the Basis of Scores Relating to Ranking

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Abstract
In the evaluation of decision making units (DMUs), by using the ordinary data envelopment analysis models, the projected point may be considered as a target point. Various orientations lead to various targets. The selection of one of these projected points or other designated efficient points as targets, is a considerable problem in target setting. There are different criteria for the selection of such suitable targets. In this article, we introduce a method for target setting, on the basis of ranking in T_v. Since most of the designated points for targets are non-extreme, we propose a method for ranking non-extreme DMUs, based on a convex combination (with the best score) of extreme efficient DMUs, located on the minimum face containing the non-extreme DMUs.
Keywords : Data Envelopment Analysis; Target Setting; Ranking; Minimum Face.

1 Introduction

Data Envelopment Analysis (DEA) is a mathematical programming approach for the evaluation of the performance of the Decision Making Units (DMUs), especially when we intend to obtain the efficiency score of a DMU, so that it can be compared to the scores of others. One of the main applications of DEA is to set targets for the under-evaluated DMUs. A target is a point on the efficiency frontier, which, DMUs consider to reach. Every frontier point can be a target. Ordinary targets can be obtained using the DMUs'projections on the efficient frontier in usual DEA models (such as CCR and BCC models in different orientations). Various orientations lead to various targets. The question is which of these points is the suitable target? A variety of studies on the target setting can be considered in literature. For example, see [1, 2, 3, 4, 5, 6]. Aparicio, et al [7] proposed an integer linear programming model to find the closest targets for a

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given unit. Although less distance can be a good criterion for the selection of projected points, especially for conservative Decision Makers (DM), a DM may use the Ranking-Related-Score (R-R-S) to select the targets. R-R-S is a number used by various ranking models for the ranking purposes. The problem is apparent when almost every designated efficient point is non-extreme. Anderson and Peterson [8] used super-efficiency model for the ranking purposes, through the exclusion of the unit from the Production Possibility Set (PPS) and then analyzing the change in the Pareto frontier. Since the proposed model may be infeasible, and, unstable in some cases, other super efficiency methods have been proposed. For example, see [9, 10, 11, 12]. Although these models are generally interesting and useful, they are not able to rank non-extreme efficient DMUs. We know that non-extreme DMUs can be constructed by extreme DMUs. In this article, we propose a method to rank non-extreme DMUs, on the basis of a convex combination (with the best score) of extreme efficient DMUs located on the minimum face containing the non-extreme DMUs. Then, using this convex combination, we can get an R-R-S for non-extreme DMUs. By applying the R-R-S obtained for all DMUs, we can select the target among various candidate target points based on the their R-R-S. This paper is organized as follows: Section 2 briefly presents the background of DEA and reviews a mathematical basis used in this study. Sections 3 and 4 present our proposed model. Section 5 deals with a numerical example. We use our proposed method to select a target from input and output orientation projections for each of 28 cities in China. Section 6 contains our conclusions.

2 The DEA Background

We assume n DMUs, each of which consumes m inputs to produce s outputs. By X_j, we denote the input vector of DMU_j, and by Y_j, the output vector of DMU_j. The production possibility set T_v is defined as follows:

$$T_v = \left\{ (X,Y) \;\middle|\; X \geq \sum_{j=1}^{n} \lambda_j X_j \ , \ Y \leq \sum_{j=1}^{n} \lambda_j Y_j \ , \ \sum_{j=1}^{n} \lambda_j = 1 \ , \ \lambda_j \geq 0 \ ; \ j = 1, ..., n \right\}$$

Each DMU on the frontier of T_v is relatively efficient, but others are inefficient. The envelopment form of the BCC model in the input orientation for the evaluation of DMU_o in T_v is:

$$\begin{array}{ll} Min & \theta \\ s.t. & \sum_{j=1}^{n} \lambda_j X_j \leq \theta X_o \\ & \sum_{j=1}^{n} \lambda_j Y_j \geq Y_o \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geq 0 \quad j = 1, ..., n \end{array} \tag{2.1}$$

The dual of this model is the multiplier form of the BCC model in the input orientation:

$$\begin{array}{ll} Max & U^T Y_o + u_0 \\ s.t. & U^T Y_j - V^T X_j + u_0 \leq 0 \qquad j = 1, ..., n \\ & V^T X_o = 1 \\ & V \geq 0 \ , \quad U \geq 0 \\ & u_0 \ free \end{array} \tag{2.2}$$

Here, we consider the following two-phase model:

$$\begin{array}{ll} Phase\ \mathrm{I} & \\ Min & \theta \\ s.t. & \sum_{j=1}^{n}\lambda_j X_j + S^- = \theta X_o \\ & \sum_{j=1}^{n}\lambda_j Y_j - S^+ = Y_o \\ & \sum_{j=1}^{n}\lambda_j = 1 \\ & \lambda_j \geq 0 \quad j=1,...,n \\ & S^- \geq 0\ ,\ S^+ \geq 0 \end{array} \tag{2.3}$$

$$\begin{array}{ll} Phase\ \mathrm{II} & \\ Max & 1^T S^- + 1^T S^+ \\ s.t. & \sum_{j=1}^{n}\lambda_j X_j + S^- = \theta^* X_o \\ & \sum_{j=1}^{n}\lambda_j Y_j - S^+ = Y_o \\ & \sum_{j=1}^{n}\lambda_j = 1 \\ & \lambda_j \geq 0 \quad j=1,...,n \\ & S^- \geq 0\ ,\ S^+ \geq 0 \end{array} \tag{2.4}$$

Where, θ^* is the optimal solution obtained from (2.3). $(\sum_{j=1}^{n}\lambda_j^* X_j, \sum_{j=1}^{n}\lambda_j^* Y_j)$ is the input orientation projection point of DMU_o on the efficient frontier. The envelopment form of the BCC model in the output orientation, its dual form, the two phase model, and the projection points related to the envelopment form can be defined similarly. For more details, see [13]. We consider that E is the set of extreme efficient DMUs, and that $E^{'}$ is the set of non-extreme efficient DMUs. The reference set for DMU_o can be conventionally defined as $RF_o = \left\{ j \in E \bigcup E' \,\middle|\, \lambda_j^* > 0 \ on\ (\theta^*, \lambda^*)\ of\ (2.4) \right\}$.
Sueyoshi [14] indicates that we can uniquely determine a reference set that contains the maximum number of efficient DMUs. He shows that the reference set can be identified by a pair of solutions of (2.1) and (2.2) which satisfy Strong Complementary Slackness Conditions (SCSC). He proposes the following DEA model to find such a reference set for DMU_o.

$$\begin{array}{ll} \max & \eta \\ s.t. & \sum_{j=1}^{n}\lambda_j X_j \leq \theta X_o \\ & \sum_{j=1}^{n}\lambda_j Y_j \geq Y_o \\ & \sum_{j=1}^{n}\lambda_j = 1 \\ & \lambda_j \geq 0, \quad j=1,...,n\ ,\ \theta\ free \\ & V^T X_o = 1 \\ & U^T Y_j - V^T X_j + u_0 \leq 0\ , \quad j=1,...,n \\ & V \geq 0\ , \quad U \geq 0, u_0\ free \\ & \theta = U^T Y_o + u_0 \\ & \lambda + VX - WY - \sigma I \geq \eta I \\ & V^T - X\lambda + \theta X_o \geq \eta I \\ & W^T + Y\lambda - Y_o \geq \eta I \\ & \eta \geq 0 \end{array} \tag{2.5}$$

In this article, we use the above model to find the reference set for the non-extreme efficient DMUs. Sueyoshi [14] also shows that the identification of a reference set containing the maximum number of efficient DMUs, can be considered as a computation process to find the minimum face containing$(\theta^* X_o, Y_o)$. In fact, model (2.5) obtains every efficient DMU located on the minimum face containing input orientation projection of DMU_o. We do the same for the output orientation. Therefore, if DMU_o is an efficient DMU, the previous model can find the minimum face containing DMU_o. Such an efficient DMU can be obtained from various orientations or criteria.

3 The projections which need an R-R-S

Fig. 1 shows four regions in the T_v. From the probability point of view, it can be seen that the projections obtained from the two-phase model, of input and output orientations for every DMU located in (I), are non-extreme and that they are on the (A-B-C-D-E) segment of the frontier. The Output Orient Projection (O-O-P) for the DMUs in region (II), is DMU_E, and the Input Orient Projection (I-O-P) is probably a non-extreme point of A-B-C-D-E. The I-O-P for the DMUs in region (III) is DMU_A, and the I-O-P is probably a non-extreme point of A-B-C-D-E. Only for the DMUs in region (IV), both input and output orientation projections are extreme efficient DMUs of A, E. If any direction d is selected for the movement towards the Pareto efficient frontier, we can see that almost all of the projections are non-extreme. Therefore, if a DM wishes to select one of such projections with the ranking criterion in mind, a method for obtaining an R-R-S will become necessary. Using the R-R-S of extreme DMUs, we try to obtain an R-R-S for the non-extreme DMUs. Like the influence of the extreme DMUs on the construction of non-extreme efficient DMUs, extreme DMUs influence the performance and consequently the R-R-S of the non-extreme efficient DMUs. Therefore, the reference set of non-extreme DMUs, containing the maximum number of efficient DMUs, which, according to the [14] is equivalent to the minimum face that contains the projection point, is considerable. In the next section, we will present an R-R-S for the non-extreme efficient DMUs.

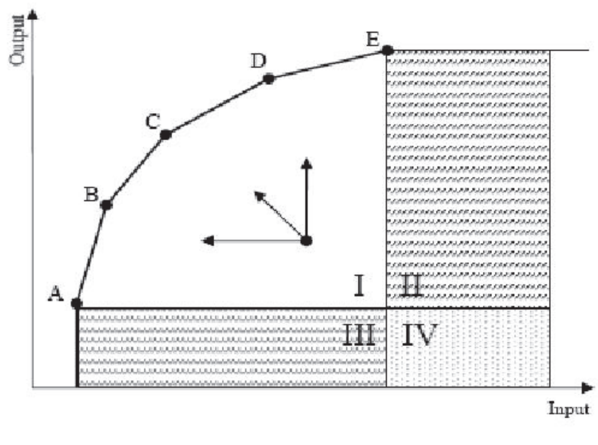

Figure 1: Four regions of T_v and different orientations

4 R-R-S for non-extreme efficient DMUs

First, we need to have an R-R-S of extreme DMUs. We can do this by ranking the DMUs by means of methods that can give us an R-R-S for extreme DMUs, for instance the L_1

norm model (see [9]for more details), as presented below (DMUo is an efficient unit):

$$\begin{array}{ll} Min & \sum_{i=1}^{m} x_i - \sum_{r=1}^{s} y_r + \alpha \\ s.t. & \sum_{j=1,j\neq o}^{n} \lambda_j x_{ij} \leq x_i, \ \ i=1,...,m \\ & \sum_{j=1,j\neq o}^{n} \lambda_j y_{rj} \geq y_r, \ \ r=1,...,s \\ & x_i \geq x_{io}, \ \ i=1,...,m \\ & 0 \leq y_r \leq y_{ro}, \ \ r=1,...,s \\ & \sum_{j=1,j\neq o}^{n} \lambda_j = 1 \\ & \lambda_j \geq 0, \ j=1,...,n \ , j \neq o \end{array} \tag{4.6}$$

Where, $\alpha = \sum_{r=1}^{s} y_{ro} - \sum_{i=1}^{m} x_{io}$is a constant number. We know that this ranking model is always feasible, but it can only give us an R-R-S for extreme efficient DMUs.
1. $DMU_o \in E$ only if the optimal value of (4.6) is greater than zero.
2. $DMU_o \in E'$ only if the optimal value of (4.6) is zero.

Suppose that $DMU_o \in E'$. This DMU can be one of the non-extreme designated DMUs. Using (2.5), we can find all the extreme efficient DMUs located on the minimum face containing DMU_o. We define RF_o^+ as follows: RF_o^+={indexes of extreme DMUs existing on the minimum face containing DMU_o}. In fact, RF_o^+can be obtained from the reference set of the $DMU_o$by excluding non-extreme DMUs. Suppose that θ_i is the R-R-S of DMU_i, obtained from the L_1norm model. Now, to determine the R-R-S of DMU_o, we use the following model:

$$\begin{array}{ll} Max & \sum_{j\in RF_o^+} \lambda_j \theta_j \\ s.t. & \sum_{j\in RF_o^+} \lambda_j X_j = X_o \\ & \sum_{j\in RF_o^+} \lambda_j Y_j = Y_o \\ & \sum_{j\in RF_o^+} \lambda_j = 1, \\ & \lambda_j \geq 0, \ j \in RF_o^+, \end{array} \tag{4.7}$$

Now, we denote $\sum_{j\in RF_o^+} \lambda_j^* \theta_j$ as the R-R-S of DMU_o. With this score, we can rank all DMUs. The greater the R-R-S, the higher the rank. The idea behind this model is that, DM wishes to select a designated point with a higher rank. Naturally, a DM having such a viewpoint wishes to find a combination of extreme DMUs which can construct a target with the highest rank. For this reason, we maximized the objective function of model (4.7).

5 Example

In this section, we use our proposed method to select a target from the input and output orientation projections, for 28 cities in china, (28 DMUs) with three inputs and three outputs as defined by Table 1(the data have been scaled). These data were originally reported by Charnes, et al [15], which consist of 28 cities in china (DMUs) in 1983. There are three outputs (gross industrial output value, profit & taxes, and retail sales) and three inputs (labor, working funds, and investment). In Table 1, the numbers on the last column show the R-R-S of extreme efficient DMUs. First, we evaluated all DMUs by models (2.3) and (2.4). In order to select target for inefficient DMUs between I-O-P and O-O-P, we found the R-R-S of efficient DMUs which were all extreme (Since the optimal value of (4.6) was greater than zero for them). Then, we obtained the I-O-P and O-O-P of the inefficient DMUs by CCR mode in input and output orientation. In Tables 2 and 3, you can see the O-O-P and the I-O-P. Simply you can see that all of the projected points are non-extreme (Since, just 4, 6, 7, 19, 21, 22, 23, 24, 25, 27, and 28 are extreme DMUs of the PPS). Their corresponding R-R-Ss were obtained from proposed model (model 4.7). You can refer to the R-R-S of the O-O-P in the last column of Table 2, and the R-R-S of the I-O-P in the last column of Table 3.

In Table 4, we can see the comparison between the R-R-S of the I-O-P and the R-R-S of the O-O-P for the selection of relevant targets. In order to set targets, we recommend I-O-P for inefficient DMUs 16,17, 18, and the O-O-P for the other DMUs, according to the greater value of R-R-Ss. The data on these tables have been rounded.

Table 1. 28 Chinese cities

DMUs	Input1	Input2	Input3	Output1	Output2	Output3	R-R-S using L_1 for efficient DMUs
1	0.26823	0.0685584	0.341941	0.2292025	0.0406947	0.0473600	
2	0.20202	0.0452713	0.117429	0.1158016	0.0135939	0.0336165	
3	0.19793	0.0471650	0.112634	0.1244124	0.0204909	0.0317709	
4	0.17896	0.0423124	0.189743	0.1187130	0.0190178	0.0605037	0.0131
5	0.14804	0.0367012	0.097004	0.0658910	0.0086514	0.0239760	
6	0.18993	0.0408311	0.111904	0.0993238	0.1411954	0.0353896	0.1045
7	0.02333	0.0245542	0.091861	0.0854188	0.0135327	0.0239360	0.0514
8	0.11691	0.0305316	0.091710	0.0606743	0.0078357	0.0208188	
9	0.12962	0.0295812	0.092409	0.0736545	0.0114365	0.0298112	
10	0.10626	0.0198703	0.053499	0.0454684	0.0067154	0.0233733	
11	0.08970	0.0210891	0.095642	0.0494196	0.0078992	0.0118553	
12	0.10926	0.0282209	0.084202	0.0842854	0.0149186	0.0243361	
13	0.08550	0.0184992	0.049357	0.0776285	0.0116974	0.0234875	
14	0.07217	0.0222327	0.073907	0.0490998	0.0117854	0.0118924	
15	0.07618	0.0161159	0.047977	0.0482448	0.067857	0.0158250	
16	0.07321	0.0144163	0.043312	0.0515237	0.0114883	0.0101231	
17	0.08672	00190043	0.055326	0.0625514	0.0173099	0.0130423	
18	0.06909	0.0158439	0.066640	0.0382880	0.0074126	0.0123968	
19	0.07769	0.0135046	0.046198	0.0867467	0.0065229	0.0262876	0.0047
20	0.09742	0.0206926	0.066120	0.0830142	0.0128279	0.0242773	
21	0.05496	0.0079563	0.043192	0.0521684	0.0037245	0.0184055	0.0013
22	0.06700	0.0144092	0.043350	0.0869973	0.0086859	0.0194416	0.0100
23	0.04630	0.0100431	0.031428	0.0604715	0.0055989	0.0012758	0.0178
24	0.06512	0.0096873	0.028112	0.0601299	0.0037088	0.0224855	0.0164
25	0.02009	0.0050717	0.054650	0.0145792	0.0011816	0.0024442	0.0279
26	0.06981	0.0117790	0.030976	0.0319218	0.0031726	0.0169051	
27	0.48301	0.1397736	0.616961	0.6785798	0.1594957	0.1088699	0.5583
28	0.37195	0.0855509	0.038545	0.2505984	0.0545140	0.0835745	0.2021

Table 2. O-O-P for Inefficient DMUs and Corresponding R-R-S by Using our Proposed Model

Inefficient DMUs	Input1	Input2	Input3	Output1	Output2	Output3	R-R-S using our proposed model for the O-O-P
1	0.257	0.069	0.298	0.320	0.068	0.066	0.217
2	0.195	0.045	0.117	0.179	0.032	0.052	0.097
3	0.198	0.047	0.113	0.198	0.037	0.050	0.122
5	0.148	0.037	0.097	0.126	0.020	0.046	0.052
8	0.117	0.031	0.092	0.114	0.017	0.039	0.043
9	0.130	0.030	0.092	0.102	0.016	0.041	0.035
10	0.104	0.020	0.053	0.082	0.010	0.032	0.029
11	0.090	0.021	0.076	0.113	0.018	0.027	0.038
12	0.109	0.028	0.084	0.121	0.021	0.035	0.053
13	0.085	0.018	0.049	0.090	0.014	0.027	0.025
14	0.072	0.022	0.074	0.099	0.024	0.024	0.046
15	0.076	0.016	0.048	0.080	0.011	0.026	0.023
16	0.073	0.014	0.043	0.077	0.017	0.018	0.020
17	0.087	0.019	0.055	0.094	0.026	0.022	0.026
18	0.069	0.016	0.052	0.077	0.015	0.025	0.024
20	0.097	0.021	0.066	0.105	0.016	0.031	0.029
26	0.070	0.012	0.031	0.065	0.005	0.024	0.021

Table 3. I-O-P for Inefficient DMUs and Corresponding R-R-S by Using our Proposed Model

Inefficient DMUs	Input1	Input2	Input3	Output1	Output2	Output3	R-R-S using our proposed model for the I-O-P
1	0.177	0.045	0.190	0.229	0.044	0.047	0.139
2	0.110	0.025	0.064	0.116	0.015	0.034	0.041
3	0.116	0.028	0.066	0.124	0.020	0.032	0.053
5	0.064	0.014	0.042	0.069	0.009	0.024	0.028
8	0.056	0.012	0.044	0.061	0.008	0.021	0.025
9	0.085	0.019	0.061	0.079	0.011	0.030	0.030
10	0.067	0.012	0.034	0.064	0.007	0.023	0.022
11	0.045	0.011	0.048	0.049	0.008	0.012	0.024
12	0.070	0.018	0.054	0.084	0.015	0.024	0.028
13	0.073	0.016	0.042	0.078	0.012	0.023	0.025
14	0.048	0.011	0.049	0.049	0.012	0.012	0.029
15	0.057	0.010	0.036	0.052	0.007	0.016	0.021
16	0.059	0.011	0.035	0.061	0.011	0.010	0.022
17	0.064	0.013	0.041	0.063	0.017	0.013	0.027
18	0.046	0.008	0.045	0.038	0.007	0.012	0.025
20	0.073	0.015	0.049	0.083	0.013	0.024	0.017
26	0.064	0.010	0.028	0.060	0.004	0.021	0.016

Table 4. Comparison Between R-R-S of I-O-P and R-R-S of O-O-P for the Selection of Targets.

Inefficient DMUs	R-R-S of O-O-P	R-R-S of I-O-P	Target
1	0.217	0.139	O-O-P
2	0.097	0.041	O-O-P
3	0.122	0.053	O-O-P
5	0.052	0.028	O-O-P
8	0.043	0.025	O-O-P
9	0.035	0.030	O-O-P
10	0.029	0.022	O-O-P
11	0.038	0.024	O-O-P
12	0.053	0.028	O-O-P
13	0.02513	0.02481	O-O-P
14	0.046	0.029	O-O-P
15	0.023	0.021	O-O-P
16	0.020	0.022	I-O-P
17	0.026	0.027	I-O-P
18	0.024	0.025	I-O-P
20	0.029	0.017	O-O-P
26	0.021	0.016	O-O-P

6 Conclusion

In this article, we introduced a target setting method on the basis of the ranking in T_v. Since most of the designated points for targets are non-extreme, we also proposed a method to rank non-extreme DMUs, based on a convex combination (with the best score) of the extreme efficient DMUs, located on the minimum face, containing the non-extreme DMU, in accordance with a managerial point of view. Although the Pareto efficient DMUs do not necessarily dominate the under-evaluated DMU, these DMUs can be designated as target. In the numerical example, however, we only compared the input orientation projections targets to the output orientation projections. Applying our proposed method, other efficient points obtained from various target setting models or various directions can be ranked. Using this approach, DM can select an appropriate target among different designated targets considering ranking criterion. This paper was organized on the VRS assumption. With minor revisions, the proposed method can be expanded on CRS assumption.

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