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Research Article

Transient free convective flow of a micropolar fluid between two vertical walls

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Abstract

The aim of this paper is to present numerically the transport mechanism of an incompressible micropolar fluid between two vertical walls when the motion occurs due to the buoyancy force. The governing equations in non-dimensional form are solved numerically using the Matlab software. The obtained numerical solutions for the velocity and micro-rotation profiles are displayed using the graphs for various values of the vortex viscosity parameter and material parameter. It is found that the material parameter has retarding effect on the velocity of micropolar fluid and to make the flow steady state earlier for both thermal conditions. The effect of the vortex viscosity parameter is to decrease the steady state time of the velocity and micro-rotation.

Keywords : Micropolar fluid; Matlab R2008a; Vertical walls; Unsteady flow.

Nomenclature

g *Accelerationduetogravity*
G˙r *Grashofnumber*
L *Distancebetweentwoverticalwalls*
P˙r *Prandtlnumber*
R *Materialparameter*
t *Timeinnon − dimensionalform*
t' *Time*
T¨c *Temperatureofthewallaty*'=L
T¨h *Temperatureofthewallaty*'=0
T¨m *Initialtemperatureofthefluid*
u *Velocityinnon − dimensionalform*
u' *Velocityoffluid*
y *Dimensionlessco − ordinateperpendiculartothewalls*
y' *Co − ordinateperpendicularofthewall*

Greek symbols

ω Dimensionless angular velocity
j Micro-inertia density
k Vortex viscosity
μ Dynamic viscosity
θ Temperature of the fluid in non-dimensional form
u Kinematic viscosity of the fluid

1 Introduction

The dynamics of micropolar fluids has attracted considerable attention by research workers during the last few decades because traditional Newtonian fluids cannot precisely describe the

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characteristics of fluid flow with suspended particles. The basic idea about micropolar fluids has originated from the need to model the flow of fluids containing rotating micro-constituents. Beside the usual equations for Newtonian flow, this theory introduces some new material parameters, an additional independent vector field the microrotation and new constitutive equations that must be solved simultaneously with the usual Newtonian flow equations. In the history of fluid mechanics, Eringen [1, 2], a pioneering researcher, has first formulated the theory of micropolar fluids. This theory in formulating the governing equations takes into account the effects arising from the local structure and micro-motions of the fluid elements, and is able to describe the behavior of the polymeric additives, animal blood, lubricants, liquid crystals, dirty oils, solutions of colloidal suspensions, etc.. The micropolar fluids exhibit certain microscopic effects arising from local structure and microrotation of fluid elements.

Sastry and Rao [3] have focused the numerical solution of a micropolar fluid flow in a channel with porous walls. Further, Bhargava and Rani [4] have found the numerical solution on the heat transfer phenomenon of the micropolar fluid flowing in a channel having porous walls. Agarwal and Dhanapal [5] have analyzed numerically the free convective flow of the micropolar fluid between two parallel porous vertical plates while Gorla et al. [6] have studied on mixed convective flow of a micropolar fluid. Kim [7] considered unsteady convection flow of micropolar fluids past a vertical porous medium. Srinivasacharya et al. [8] have investigated the unsteady Stokes flow of a micropolar fluid between two parallel porous plates. Kim [9] has studied the unsteady MHD convective flow of the polar fluids past a vertical moving porous plate in a porous medium. Further, Kim and Fedorov [10] have discussed on the transient mixed radiative convection of the micropolar fluid past a moving semi-infinite vertical porous plate while Bhargava et al. [11] have studied the numerical solution of MHD free convective flow of the micropolar fluid flow between two parallel porous vertical plates. Chamkha et al. [12] reported the solution of fully developed free convection of a micropolar fluid in a vertical channel.

Singh and Paul [13] have investigated the transient natural convective flow between two vertical walls heated/cooled asymmetrically. Chen [14] has studied the non-linear stability characterization of the thin micropolar liquid film flowing down the inner surface of a rotating vertical cylinder. Recently, Rahman [15] has investigated the convective flows of the micropolar fluids from isothermal porous surfaces with viscous dissipation and Joule heating where as Ishak et al. [16] have shown the dual solutions in mixed convective boundary layer flow of the micropolar fluids. Sajid et al. [17] have considered the exact solutions for thin film flows of a micropolar fluid. Further, Ishak et al. [18] have conducted the MHD boundary-layer flow of a micropolar fluid past a wedge with constant wall heat flux. Pal and Chatterjee [19] have presented the heat and mass transfer in MHD non-Darcian flow of a micropolar fluid over a stretching sheet embedded in a porous media with non-uniform heat source and thermal radiation.

In this paper, we have solved the unsteady free convective flow formation of a micropolar fluid between two infinitely long vertical walls as a result of symmetric and asymmetric heating. A non-dimensional parameter is used in order to characterize the temperature of the vertical walls with respect to the fluid temperature. The partial differential equations of governing flow and heat transfer have been solved by using the Matlab R2008a software. Finally, the results are displayed using the graphs and table.

2 Governing Equations

We consider the unsteady free-convective flow of a micro-polar fluid between two vertical walls separated by a distance L apart. For mathematical formulation, we take the x'-axis along one of the vertical walls in the upward direction and the y'-axis normal to it. Initially, the temperatures of walls and the fluid are same says T'_m . At time $t' > 0$, the temperature of the walls at $y' = 0$ and L is instantaneously raised or lowered to T'_h and T'_c respectively such that $(T'_h > T'_c)$ which is there after maintained constant. As the walls are of infinite extent, the flow depends only on the transverse co-ordinate y' and time t'. We have assumed that the fluid properties are not affected by the temperature difference except the density in the body force term. Under these assumptions, the governing equations corresponding to the considered model are derived as follows:

$$\rho\frac{\partial u'}{\partial t'} = (\mu+k)\frac{\partial^2 u'}{\partial y'^2}+k\frac{\partial \omega'}{\partial y'}+\rho g\beta(T'-T'_m), \quad (2.1)$$

$$\rho\frac{\partial \omega'}{\partial t'} = (\mu+0.5k)\frac{\partial^2 \omega'}{\partial y'^2}-k\left(2\omega'+\frac{\partial u'}{\partial y'}\right), \quad (2.2)$$

$$\rho\frac{\partial T}{\partial t'} = \alpha\frac{\partial^2 T'}{\partial y'^2}. \quad (2.3)$$

The initial and boundary conditions for the velocity, angular velocity field and temperature are as follows:

$$\begin{array}{llll} t \leq 0 & u' = \omega' = 0 & T' = T'_m, & 0 \leq y' \leq L; \\ t > 0 & u' = \omega' = 0 & T' = T'_h, & y' = 0; \\ & u' = \omega' = 0 & T' = T'_c, & y = L. \end{array} \quad (2.4)$$

In order to non-dimensionalize the above equations, we introduce the following non dimensional variables and physical parameters defined as follows:

$$\begin{array}{lll} Y & = & y'/L, \qquad t = vt'/L^2, \\ u & = & u'v/\beta g L^2(T'_h - T'_m), \\ \theta & = & (T' - T'_m)/(T'_h - T'_m), \\ \omega & = & \omega' v/\beta g L(T'_h - T'_m), \qquad Pr = v/\alpha, \\ m & = & (T'_c - T'_m)/(T'_h - T'_m), \qquad b = L^2/j, \\ R & = & k/\mu. \end{array} \quad (2.5)$$

The physical quantities used in the above equations are defined in the Nomenclature. Use of non-dimensional variables and parameters defined in (2.5) into Equations (2.1) - (2.3), have resulted the following equations in non-dimensional form:

$$\frac{\partial u}{\partial t} = (1+R)\frac{\partial^2 u}{\partial y^2}+\theta+R\frac{\partial \omega}{\partial y}, \quad (2.6)$$

$$\frac{\partial \omega}{\partial t} = (1+0.5R)\frac{\partial^2 \omega}{\partial y^2}-Rb\left(\frac{\partial u}{\partial y}+2\omega\right), \quad (2.7)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^1}. \quad (2.8)$$

The boundary conditions corresponding to the considered model in dimensionless form are derived as follows:

$$\begin{array}{llll} t \leq 0 & u = \omega = \theta = 0, & 0 \leq y \leq 1; \\ t > 0 & u = \omega = 0, & \theta = 1 \quad y = 0; \\ & u = \omega = 0, & \theta = m \quad y = 1. \end{array} \quad (2.9)$$

3 Numerical solution

The linear parabolic partial differential equations (2.6), (2.7) and (2.8) with their appropriate initial and boundary conditions are solved numerically by using Matlab R2008a software. The numerical solution is obtained on a mesh produced by 20 equally spaced points for the spatial interval [0, 1] starting from initial value of time to steady state value.

4 Steady state solution

In order to check the accuracy of the numerical solutions obtained with MATLAB software, we compare the steady-state numerical solution with the analytical solutions of the corresponding steady flow. Denoting these solutions as u_s and ω_s, it can be shown that

$$\begin{array}{lll} u_s & = & -2c_1 y + c_2 k_{11} e^{\sqrt{k_4}y} - c_3 k_{11} e^{-\sqrt{k_4}y} \\ & & -k_{12}y^3 - k_8 y^2 + k_{13}y + c_4, \end{array} \quad (4.10)$$

$$\begin{array}{lll} \omega_s & = & c_1 + c_2 e^{\sqrt{k_4}y} + c_3 e^{-\sqrt{k_4}y} \\ & & +k_7 y^2 + k_8 y. \end{array} \quad (4.11)$$

where

$k_1 = 2Rb/(2+R)$, $k_2 = 1/k_1$,
$k_3 = (1+R)k_2$, $k_4 = (2+R)/k_3$,
$k_5 = (m-1)/k_3$, $k_6 = 1/k_3$,
$k_7 = k_5/2k_4$, $k_8 = k_6/k_4$,
$k_9 = k_2 k_4$, $k_{10} = (k_9 - 2)$,
$k_{11} = k_{10}/\sqrt{k_4}$, $k_{12} = 2k_7/3$,
$k_{13} = 2k_2 k_7$,
$k_{14} = k_{13} - k_8 - k_{12}$,
$k_{15} = 2 + k_{11}e^{\sqrt{k_4}} - k_{11}$,
$k_{16} = 2 - k_{11}e^{-\sqrt{k_4}} + k_{11}$, $k_{17} = e^{\sqrt{k_4}} - 1$,
$k_{18} = e^{-\sqrt{k_4}} - 1$, $k_{19} = k_7 + k_8$,
$k_{20} = -k_{16}/\sqrt{k_{15}}$, $k_{21} = -k_{14}/k_{15}$,
$k_{22} = k_{20}k_{17} + k_{18}$, $k_{23} = k_{21}k_{17} + k_{19}$,
$k_{24} = -k_{23}/k_{22}$, $k_{25} = k_{20}k_{24} + k_{21}$,
$k_{26} = -k_{25} - k_{24}$,
$k_{27} = k_{24}k_{11} - k_{25}k_{11}$,
$c_1 = k_{26}$, $c_2 = k_{25}$.
$c_3 = k_{24}$, $c_4 = k_{27}$.

In Table 1, we have given the numerical values of the steady-state velocity and microrotation obtained by using Matlab software with u_s and ω_s for $m = 0$ and 1. One can see that the numerical and analytical results agree very well. The

Table 1: Numerical values of steady state velocity field and microrotation profile for m=0 and m = 1.0.

M	R	b	y	velocity field Matlab-R2008a By Eq. (4.10)		microrotation profile Matlab-R2008a By Eq. (4.11)	
0.0	0.5	1.0	0.0	0.000000	0.000000	0.000000	0.000000
			0.2	0.032050	0.032063	-7.94E-04	-0.000790
			0.4	0.042770	0.042781	6.84E-05	0.000074
			0.6	0.037410	0.037429	0.001110	0.001121
			0.8	0.021350	0.021364	0.001300	0.001308
			1.0	0.000000	0.000000	0.000000	0.000000
1.0	0.5	0.1	0.0	0.000000	0.000000	0.000000	0.000000
			0.2	0.053330	0.053343	-2.13E-04	-0.000210
			0.4	0.080000	0.080021	-1.06E-04	-0.000110
			0.6	0.080000	0.080021	1.06E-04	0.000106
			0.8	0.053330	0.053343	2.13E-04	0.000213
			1.0	0.000000	0.000000	0.000000	0.000000
0.0	0.5	2.0	0.0	0.000000	0.000000	0.000000	0.000000
			0.2	0.032110	0.032123	-0.001600	-0.001590
			0.4	0.042870	0.042892	8.35E-05	0.000093
			0.6	0.037500	0.037521	0.002140	0.002148
			0.8	0.021380	0.021394	0.002530	0.002538
			1.0	0.000000	0.000000	0.000000	0.000000
1.0	0.5	2.0	0.0	0.000000	0.000000	0.000000	0.000000
			0.2	0.053510	0.053518	-0.004130	-0.004130
			0.4	0.080390	0.080413	-0.002050	-0.002060
			0.6	0.080300	0.080413	0.002050	0.002055
			0.8	0.053510	0.053518	0.004130	0.004131
			1.0	0.000000	0.000000	0.000000	0.000000
0.0	0.5	0.5	0.0	0.000000	0.000000	0.000000	0.000000
			0.2	0.032020	0.032032	-3.42E-04	-0.000390
			0.4	0.042700	0.042724	-1.14E-04	0.000045
			0.6	0.037360	0.037381	5.93E-04	0.000573
			0.8	0.021330	0.021348	6.40E-04	0.000665
			1.0	0.000000	0.000000	0.000000	0.000000
1.0	0.5	0.5	0.0	0.000000	0.000000	0.000000	0.000000
			0.	0.053340	0.053380	-0.001060	-0.001060
			0.4	0.080040	0.080106	-5.28E-04	-0.000530
			0.6	0.080040	0.080106	-5.28E-04	0.000528
			0.8	0.053340	0.053380	0.001060	0.001058
			1.0	0.000000	0.000000	0.000000	0.000000
0.0	2.0	0.5	0.0	0.000000	0.000000	0.000000	0.000000
			0.2	0.016070	0.016079	-5.00E-04	-0.000500
			0.4	0.021470	0.021476	3.79E-05	0.000042
			0.6	0.018780	0.018786	6.92E-04	0.000696
			0.8	0.010700	0.010705	8.12E-04	0.000814
			1.0	0.000000	0.000000	0.000000	0.000000
1.0	2.0	0.5	0.0	0.000000	0.000000	0.000000	0.000000
			0.2	0.026750	0.026783	-0.001310	-0.001310
			0.4	0.040210	0.040262	-6.54E-04	-0.000650
			0.6	0.040210	0.040262	-6.54E-04	0.000654
			0.8	0.026750	0.026783	0.001310	0.001312
			1.0	0.000000	0.000000	0.000000	0.000000

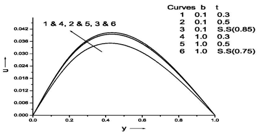

Figure 1: Velocity profile for different values of t at $m = 0.0, R = 0.5, Pr = 1.0$.

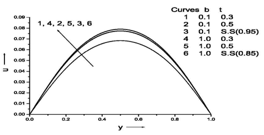

Figure 2: Velocity Profile for different values of t at $m = 1.0, R = 0.5, Pr = 1.0$.

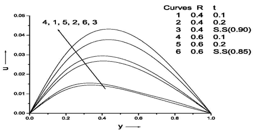

Figure 3: Velocity Profile for different values of t at $m = 0.0, b = 0.5, Pr = 1.0$.

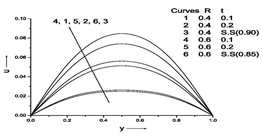

Figure 4: Velocity Profile for different values of t at $m = 1.0, b = 0.5, Pr = 1.0$.

physical parameters appearing into the model are vortex viscosity parameter R, material parameter b, Prandtl number Pr and temperature ratio parameter m having values 0 and 1 for asymmetric and symmetric heating of the vertical walls, respectively. In this study, we have focused our attention on the physical parameters R, b and Pr and presented their influences through Figs. 1-12 on the velocity as well as micro-rotation profiles for asymmetric and symmetric heating of the vertical walls.

Figures 1 and 2 show the effect of material parameter b on the velocity profiles for different values of t when $Pr = 1.0$ and $R = 0.5$ for the cases of asymmetric and symmetric heating, respectively. In both thermal cases, we can observe that the velocity of fluid has attained steady state by increasing with the time and steady state times are 0.85 and 0.75 for asymmetric heating case while 0.95 and 0.85 for symmetric heating case when $b = 0.1$ and 1.0, respectively. Thus the steady state is achieved in the case of asymmetrical heating faster than in the case of symmetric heating. The effect of material parameter b is to make the flow steady state earlier. The influence of the vortex viscosity as well as time parameter on magnitude of the velocity profile for $b = 0.5$ and $Pr = 1.0$ is presented in Figs. 3 and 4 corresponding to the asymmetric and symmetric thermal cases. For both thermal cases, the velocity profiles have increasing tendency with time and have attained steady state profiles at $t = 0.90$ and 0.85 for $R = 0.4$ and 0.6, respectively. Hence, the vortex viscosity parameter has decreasing tendency on the velocity profiles and as well as on the steady state time. Figures 5 and 6 present the effect of Prandtl number Pr on the velocity profile for

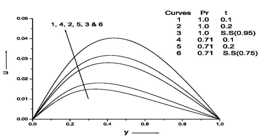

Figure 5: Velocity Profile for different values of t at $m = 0.0, R = 0.5, b = 0.5$.

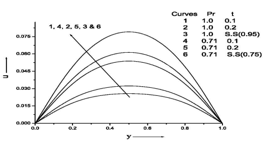

Figure 6: Velocity Profile for different values of t at $m = 1.0, R = 0.5, b = 0.5$.

the cases of asymmetric and symmetric heating at $R = 0.5$ and $b = 0.5$. From these figures we can observed that the magnitude of the velocity profile decreases with increase in the value of Prandtl number for both cases. The steady state time is increasing with the Pr.

Figures 7 and 8 illustrate that the effect of material parameter b on the micro-rotation profiles for different values of t when $Pr = 1.0$ and $R = 0.5$ in the cases of asymmetric and symmetric heating, respectively. In both thermal cases, the steady state time and the magnitude of the micro-rotation have increasing tendency with the material parameter. A close study of these figures reveals that the steady state time in case of asymmetric heating is more than the symmetric heating. Figures 9 and 10 show the micro rotation profiles corresponding to the asymmetric and symmetric thermal cases for

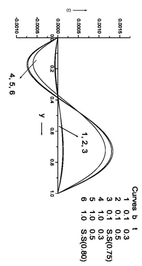

Figure 7: Micro-rotation profile for different values of t at $m = 0.0, R = 0.5, Pr = 1.0$.

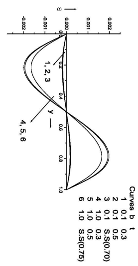

Figure 8: Micro-rotation profile for different values of t at $m = 1.0, R = 0.5, Pr = 1.0$.

$Pr = 1.0$ and $b = 0.5$. From these figures we find that steady state time are $0.90(R = 0.4)$ and $0.85(R = 0.6)$ for both thermal cases and thus the steady state time decreases as R increases. Also, the magnitudes of the micro-rotation profiles show the increasing tendency with R. Figures 11 and 12 show the effect of Prandtl number Pr at $R = b = 0.5$ for asymmetric and symmetric heating, respectively. As discussed earlier, the variation in the magnitude of micro-rotation profile decreases. We found that the steady state times are 0.95 and 0.80 for $Pr = 1.0$ and 0.71 for both thermal conditions and thus the steady state time of micro-rotation increases with the Prandtl number Pr for both thermal cases.

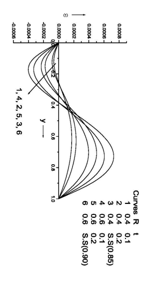

Figure 9: Micro-rotation profile for different values of t at $m = 0.0, Pr = 1.0, b = 0.5$.

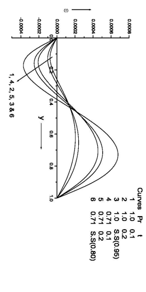

Figure 11: Micro-rotation profile for different values of t at $m = 0.0, R = 0.5, b = 0.5$.

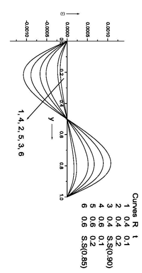

Figure 10: Micro-rotation profile for different values of t at $m = 1.0, Pr = 1.0, b = 0.5$.

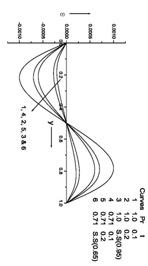

Figure 12: Micro-rotation profile for different values of t at $m = 1.0, R = 0.5, b = 0.5$.

5 Conclusion

Here, we have investigated the effects of vortex viscosity parameter, material parameter and Prandtl number on free convective flow along vertical walls in case of asymmetric and symmetric heating or cooling of walls by obtaining the numerical solutions using the Matlab R2008a programming language code. It is seen that the steady state time of velocity profile is more for symmetric case than asymmetric case while the steady state time of micro-rotation profile for asymmetric heating is more than the symmetric heating with respect to the material parameter. The velocity profile decreases and micro-rotation profile of fluid increases at any point of fluid regime with vortex viscosity parameter. As the Prandtl number increases the steady state time for the velocity profile and micro-rotation profile increases for both thermal cases.

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