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Research Article

Piecewise Constant Controlled Linear Fuzzy Systems

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Abstract
In this work we prove that for any measurable admissible control $w(\cdot)$ and for any $\varepsilon > 0$ there exists piecewise constant admissible control $\bar{w}(\cdot)$ such that for fuzzy solutions of control fuzzy linear system are ε-closed.
Keywords : fuzzy system; control; linear; piecewise constant

1 Introduction

In recent years, the fuzzy set theory introduced by Zadeh [48] has emerged as an interesting and fascinating branch of pure and applied sciences. The applications of fuzzy set theory can be found in many branches of science as physical, mathematical, differential equations and engineering sciences. Recently there have been new advances in the theory of fuzzy differential equations [8, 9, 10, 11, 12, 15, 16, 20, 21, 22, 23, 28, 37, 44, 46], fuzzy integrodifferential equations [2, 5, 6, 7, 18, 42], differential inclusions with fuzzy right-hand side [1, 3, 4, 14, 20, 21, 34, 35] and fuzzy differential inclusions [36, 45, 47] as well as in the theory of control fuzzy differential equations [17, 26, 27, 29, 30], control fuzzy integrodifferential equations [19, 24, 25], control fuzzy differential inclusions [31, 32, 33], and control fuzzy integrodifferential inclusions [43].

In many engineering control systems piecewise constant controls, instead of measurable controls are applied. In this article we prove that for any measurable admissible control $w(\cdot)$ and for any $\varepsilon > 0$ there exists piecewise constant admissible control $\bar{w}(\cdot)$ such that for fuzzy solutions of control fuzzy linear system are ε-closed.

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2 Preliminaries

Let $\mathcal{CC}(\mathbb{R}^n)$ be the family of all nonempty compact convex subsets of $\mathbb{R}^n$ with the Hausdorff metric $h(A,B) = \max\{\max_{a\in A}\min_{b\in B}\|a-b\|,\ \max_{b\in B}\min_{a\in A}\|a-b\|\}$, where $\|\cdot\|$ denotes the usual Euclidean norm in $\mathbb{R}^n$.

Let $\mathbb{E}^n$ be the family of mappings $x : \mathbb{R}^n \to [0,1]$ satisfying the following conditions:

(i) x is normal, i.e. there exists an $\xi_0 \in \mathbb{R}^n$ such that $x(\xi_0) = 1$;

(ii) x is fuzzy convex, i.e. $x(\lambda\xi + (1-\lambda)\zeta) \geq \min\{x(\xi), x(\zeta)\}$ whenever $\xi, \zeta \in \mathbb{R}^n$ and $\lambda \in [0,1]$;

(iii) x is upper semicontinuous, i.e. for any $\xi_0 \in \mathbb{R}^n$ and $\varepsilon > 0$ exists $\delta(\xi_0, \varepsilon) > 0$ such that $x(\xi) < x(\xi_0) + \varepsilon$ whenever $||\xi - \xi_0|| < \delta,\ \xi \in \mathbb{R}^n$;

(iv) the closure of the set $cl\{\xi \in \mathbb{R}^n : x(\xi) > 0\}$ is compact.

Let $\widehat{0}$ be the fuzzy mapping defined by $\widehat{0}(\xi) = 0$ if $\xi \neq 0$ and $\widehat{0}(0) = 1$.

Definition 2.1. *The set* $\{y \in \mathbb{R}^n : x(y) \geq \alpha\}$ *is called the* α *– level* $[x]^\alpha$ *of a mapping* $x \in \mathbb{E}^n$ *for* $0 < \alpha \leq 1$. *The closure of the set* $\{y \in \mathbb{R}^n : x(y) > 0\}$ *is called the 0 - level* $[x]^0$ *of a mapping* $x \in \mathbb{E}^n$.

Define the metric $D : \mathbb{E}^n \times \mathbb{E}^n \to \mathbb{R}_+$ by the equation $D(x,y) = \sup_{\alpha\in[0,1]} h([x]^\alpha, [y]^\alpha)$.

Using the results of [40], we know that

(i) $(\mathbb{E}^n, D)$ is a complete metric space,

(ii) $D(x+z, y+z) = D(x,y)$ for all $x, y, z \in \mathbb{E}^n$,

(iii) $D(kx, ky) = |k|D(x,y)$ for all $x, y \in \mathbb{E}^n$, $k \in \mathbb{R}$.

Let A, B, C be in $\mathcal{CC}(\mathbb{R}^n)$. The set C is the Hukuhara difference of A and B, if $B + C = A$, i.e. $C = A \overset{H}{-} B$. From Rådström's Cancellation Lemma [41], it follows that if this difference exists, then it is unique.

Definition 2.2. *[13] A mapping* $F : [0,T] \to \mathcal{CC}(\mathbb{R}^n)$ *is differentiable in the sense of Hukuhara at* $t \in [0,T]$ *if for some* $h > 0$ *the Hukuhara differences*

$$F(t+\Delta t) \overset{H}{-} F(t),\ \ F(t) \overset{H}{-} F(t-\Delta t)$$

exists in $\mathcal{CC}(\mathbb{R}^n)$ *for all* $0 < \Delta t < h$ *and there exists an* $D_H F(t) \in \mathcal{CC}(\mathbb{R}^n)$ *such that*

$$\lim_{\Delta t\to 0+} h(\Delta t^{-1}(F(t+\Delta t) \overset{H}{-} F(t)), D_H F(t)) = 0$$

and

$$\lim_{\Delta t\to 0+} h(\Delta t^{-1}(F(t) \overset{H}{-} F(t-\Delta t)), D_H F(t)) = 0.$$

Here $D_H F(t)$ is called the Hukuhara derivative of $F(t)$ at t.

Definition 2.3. *[15] A mapping $x : [0, T] \to \mathbb{E}^n$ is called differentiable at $t \in [0, T]$ if, for any $\alpha \in [0, 1]$, the set-valued mapping $x_\alpha(t) = [x(t)]^\alpha$ is is differentiable in the sense of Hukuhara at point t with $D_H x_\alpha(t)$ and the family $\{D_H x_\alpha(t) : \alpha \in [0, 1]\}$ define a fuzzy number $\dot{x}(t) \in E^n$.*

If $x : [0, T] \to \mathbb{E}^n$ is differentiable at $t \in [0, T]$, then we say that $\dot{x}(t)$ is the fuzzy derivative of $x(\cdot)$ at the point $t \in [0, T]$.

Consider the fuzzy Cauchy problem

$$\dot{x} = A(t)x + g(t), \ x(0) = x_0, \tag{2.1}$$

where $A(t)$ is $n \times n$-dimensional matrix-valued function; $g(t)$ is the fuzzy map, $x_0 \in \mathbb{E}^n$.

Definition 2.4. *A fuzzy mapping $x : [0, T] \to \mathbb{E}^n$ is a solution to the problem (2.1) if and only if it is continuous and satisfies the integral equation $x(t) = x_0 + \int_0^t [A(s)x(s) + g(s)]ds$ for all $t \in [0, T]$.*

Theorem 2.1. *[37, 46] Let the following conditions are true:*

1) *$A(t)$ is measurable on $[0, T]$;*

2) *There exists $a > 0$ such that $\|A(t)\| \le a$ for almost every $t \in [0, T]$;*

3) *The fuzzy map $g(s)$ is measurable on $[0, T]$;*

4) *There exists $\bar{g}(t) \in L_2[0, T]$ such that $D\left(g(t), \widehat{0}\right) \le \bar{g}(t)$ almost everywhere on $t \in [0, T]$.*

Then problem (2.1) has on exactly one solution.

3 The control fuzzy differential equation

Now we consider following control fuzzy differential equation

$$\dot{x} = A(t)x + B(t)w + f(t), \ x(0) = x_0 \tag{3.2}$$

where $w \in \mathbb{R}^m$ is the control, $B(t)$ is $n \times m$-dimensional matrix-valued function; $f : \mathbb{R}_+ \to \mathbb{E}^n$ is the fuzzy map.

Let $W : \mathbb{R}_+ \to \mathbb{R}^m$ be the measurable set-valued map.

Definition 3.1. *The set LW of all measurable single-valued branches of the set-valued map $W(t)$ is the set of the admissible controls.*

Obviously, the control fuzzy differential equation (3.2) turns into the ordinary fuzzy differential equation (2.1) if the control $\tilde{w}(\cdot) \in LW$ is fixed and $g(t) \equiv B(t)\tilde{w}(t) + f(t)$.

Let $x(t)$ denotes the fuzzy solution of the differential equation (2.1), then $x(t, w)$ denotes the fuzzy solution of the control differential equation (3.2) for the fixed $w(\cdot) \in LW$.

Definition 3.2. *The set $Y(T) = \{x(T, w) : w(\cdot) \in LW\}$ be called the attainable set of the system (3.2).*

Theorem 3.1. *[30] Let the following conditions are true:*

1) *$A(t)$ is measurable on $[0,T]$;*

2) *There exists $a>0$ such that $\|A(t)\| \le a$ for almost every $t \in [0,T]$;*

3) *$B(t)$ is measurable on $[0,T]$;*

4) *There exists $b>0$ such that $\|B(t)\| \le a$ for almost every $t \in [0,T]$;*

5) *The set-valued map $W:[0,T] \to \mathcal{CC}(\mathbb{R}^m)$ is measurable on $[0,T]$;*

6) *The fuzzy map $f:[0,T] \to \mathbb{E}^n$ is measurable on $[0,T]$;*

7) *There exist $v(\cdot) \in L_2[0,T]$ and $\bar{f}(\cdot) \in L_2[0,T]$ such that $h(W(t),\{0\}) \le v(t)$, $D(f(t),\widehat{0}) \le \bar{f}(t)$ almost everywhere on $[0,T]$.*

Then for every $w(\cdot) \in LW$ there exists the fuzzy solution $x(\cdot,w)$ on $[0,T]$ and the attainable set $Y(T)$ is compact and convex.

Let $U = \prod_{i=1}^{m} [u^i_{min}, u^i_{max}]$ and $W(t) \equiv U$ on $[0,T]$.

Now, we need to establish that for any measurable admissible control $w(\cdot)$ and for any $\varepsilon > 0$ there exists piecewise constant admissible control $\bar{w}(\cdot)$ such that for fuzzy solutions of system (3.2) holds $D(x(t,w),x(t,\bar{w})) < \varepsilon$ for all $t \in [0,T]$.

Theorem 3.2. *Let the conditions of the theorem 3.1 are true.*

Then for every $w(\cdot) \in LW$ there exists $\bar{w}(\cdot) \in LW$ such that

1) *$\bar{w}(t)$ is constant on every $\left[(i-1)\frac{T}{k}, i\frac{T}{k}\right)$, $i=\overline{1,k}$;*

2) *$\bar{w}_i(t) = \{(\bar{w}^1_i(t),...,\bar{w}^m_i(t))^T \,|\, \bar{w}^j_i(t) \in \{u^j_{min}, u^j_{max}\}, i=\overline{1,k}, j=\overline{1,m}\}$ for every $t \in [0,T]$;*

3) *$D(x(t,w),x(t,\bar{w})) \le be^{aT}\frac{T}{2k}\|u_{max}-u_{min}\|$ for all $t \in [0,T]$,*

where $u_{min} = \left(u^1_{min},...,u^m_{min}\right)^T$, $u_{max} = \left(u^1_{max},...,u^m_{max}\right)^T$.

Proof. We have any $w(\cdot) \in LW$ and any $k \in N$. Let $W_i = (W^1_i,...,W^m_i)^T$, where $W^j_i = \int_0^{i\frac{T}{k}} w^j(s)ds$, $i=\overline{1,k}$, $j=\overline{1,m}$.

Obviously, $W^j_{i+1} - W^j_i = \int_{\frac{iT}{k}}^{\frac{(i+1)T}{k}} w^j(s)ds$, $u^j_{min}\frac{T}{k} \le W^j_{i+1} - W^j_i \le u^j_{max}\frac{T}{k}$, $j=\overline{1,m}$, and

$$\|W_{i+1} - W_i\| \le \|u_{max} - u_{min}\|\frac{T}{k}.$$

Now we take

$$\bar{w}(t) = \begin{cases} \bar{w}_1, & t \in [0,\frac{T}{k}), \\ \vdots & \vdots \\ \bar{w}_{k-1}, & t \in [\frac{(k-2)T}{k}, \frac{(k-1)T}{k}), \\ \bar{w}_k, & t \in [\frac{(k-1)T}{k}, T], \end{cases}$$

such that

1) $\bar{w}_1 = (\bar{w}_1^1, ..., \bar{w}_1^m)^T$, where $\bar{w}_1^j = \begin{cases} u^j_{max}, & \text{if } W_1^j \geq \frac{T}{2k}(u^j_{max} + u^j_{min}), \\ u^j_{min}, & \text{if } W_1^j < \frac{T}{2k}(u^j_{max} + u^j_{min}), \end{cases}$ $\quad j = \overline{1, m}$;

2) $\bar{w}_i = (\bar{w}_i^1, ..., \bar{w}_i^m)^T$, $i = \overline{2, k}$,

where $\bar{w}_i^j = \begin{cases} u^j_{max}, & \text{if } W_i^j - \sum\limits_{l=1}^{i-1} \bar{w}_j^l \frac{T}{k} \geq \frac{T}{2k}(u^j_{max} + u^j_{min}), \\ u^j_{min}, & \text{if } W_i^j - \sum\limits_{l=1}^{i-1} \bar{w}_j^l \frac{T}{k} < \frac{T}{2k}(u^j_{max} + u^j_{min}), \end{cases}$ $\quad j = \overline{1, m}$;

Obviously, for $i = 1$ and $j = \overline{1, m}$ we have

a) if $\bar{w}_1^j = u^j_{max}$, when $-\frac{T}{2k}(u^j_{max} - u^j_{min}) \leq W_1^j - \bar{w}_1^j \frac{T}{k} \leq 0$,

b) if $\bar{w}_1^j = u^j_{min}$, when $\frac{T}{2k}(u^j_{max} - u^j_{min}) > W_1^j - \bar{w}_1^j \frac{T}{k} \geq 0$.

Hence we obtain $|W_1^j - \bar{w}_1^j| \leq \frac{T}{2k}(u^j_{max} - u^j_{min})$, $j = \overline{1, m}$, and $\|W_1 - \bar{w}_1\| \leq \frac{T}{2k}\|u_{max} - u_{min}\|$.

Thus, by induction, we obtain that, for $i = \overline{2, k}$

$$|W_i^j - \sum_{l=1}^{i} \bar{w}_l^j \frac{T}{k}| \leq \frac{T}{2k}(u^j_{max} - u^j_{min}), \; j = \overline{1, m},$$

and

$$\|W_i - \sum_{l=1}^{i} \bar{w}_l \frac{T}{k}\| \leq \frac{T}{2k}\|u_{max} - u_{min}\|. \tag{3.3}$$

Therefore, if $t_i = \frac{iT}{k}$, $i = \overline{1, k}$; then $\left\| \int\limits_0^{t_i} w(s)ds - \int\limits_0^{t_i} \bar{w}(s)ds \right\| \leq \frac{T}{2k}\|u_{max} - u_{min}\|$.

Now, we take $t \in \left(\frac{(i-1)T}{k}, \frac{iT}{k}\right)$. Then

$$\left\| \int_0^t w(s)ds - \int_0^t \bar{w}(s)ds \right\| \leq \left\| W_{i-1} - \sum_{l=1}^{i-1} \bar{w}_l \frac{T}{k} + \int_{\frac{(i-1)T}{k}}^{t} (w(s) - \bar{w}_i)ds \right\|.$$

As for all $j = \overline{1, m}$

$$W_i^j - \sum_{l=1}^{i} \bar{w}_l^j \frac{T}{k} \geq W_{i-1}^j - \sum_{l=1}^{i-1} \bar{w}_l^j \frac{T}{k} + \int_{\frac{(i-1)T}{k}}^{t} (w^j(s) - \bar{w}_i^j)ds \geq W_{i-1}^j - \sum_{l=1}^{i-1} \bar{w}_l^j \frac{T}{k};$$

then

$$\left\| \int_0^t w(s)ds - \int_0^t \bar{w}(s)ds \right\| \leq \max\{\|W_i - \sum_{l=1}^{i} \bar{w}_l \frac{T}{k}\|, \|W_{i-1}^j - \sum_{l=1}^{i-1} \bar{w}_l^j \frac{T}{k}\|\}.$$

By (3.3), we get for all $t \in [0, T]$

$$\left\| \int_0^t w(s)ds - \int_0^t \bar{w}(s)ds \right\| \leq \frac{T}{2k}\|u_{max} - u_{min}\|. \tag{3.4}$$

Now, applying definition 2.4 and conditions of the theorem, we obtain

$$D(x(t, w), x(t, \bar{w})) =$$

$$= D(\int_0^t [A(s)x(s,w) + B(s)w(s)]ds, \int_0^t [A(s)x(s,\bar{w}) + B(s)\bar{w}(s)]ds) \leq$$

$$\leq \int_0^t D(A(s)x(s,w), A(s)x(s,\bar{w}))ds + \| \int_0^t B(s)w(s)ds - \int_0^t B(s)\bar{w}(s)ds\| \leq$$

$$\leq a \int_0^t D(x(s,w), x(s,\bar{w}))ds + b \left\| \int_0^t w(s)ds - \int_0^t \bar{w}(s)ds \right\|.$$

Using Gronwall-Bellman's inequality, we obtain

$$D(x(t,w), x(t,\bar{w})) \leq be^{aT} \left\| \int_0^t w(s)ds - \int_0^t \bar{w}(s)ds \right\|.$$

By (3.4), we have $D(x(t,w), x(t,\bar{w})) \leq be^{aT}\frac{T}{2k}\|u_{max} - u_{min}\|$. Theorem is proved. □

Remark 3.1. *Obviously, if we take* $k > be^{aT}\frac{T}{2\varepsilon}\|u_{max} - u_{min}\|$*; then* $D(x(t,w), x(t,\bar{w})) < \varepsilon$ *for all* $t \in [0,T]$.

4 Conclusion

We remark that this result helps to build ε-optimal piecewise constant controls for optimal control fuzzy system (fuzzy Mayer problem [26], fuzzy time-optimal problem [30, 31, 32, 33] and other).

We can as will receive that for any measurable admissible control $w(\cdot)$ and for any $\varepsilon > 0$ there exists piecewise constant admissible control $\bar{w}(\cdot)$ such that for fuzzy R-solutions of control linear differential inclusion with fuzzy right-hand side

$$\dot{x} \in A(t)x + B(t)w + f(t), \ x(0) = x_0 \tag{4.5}$$

holds $D(X(t,w), X(t,\bar{w})) < \varepsilon$ for all $t \in [0,T]$, where $x \in R^n$, $x_0 \in R^n$, $\dot{x} = \frac{dx}{dt}$, $X(\cdot,w)$ is fuzzy R-solution of system (4.5).

Also, the given result as can be received if to take the generalized derivative [9, 38, 39].

References

[1] S. Abbasbandy, T. Allahviranloo, Oscar Lopez-Pouso, Juan J. Nieto, Numerical methods for fuzzy differential inclusions, Journal of Computer and Mathematics with Applications 48 (2004) 1633-1641.

[2] T. Allahviranloo, A. Amirteimoori, M. Khezerloo, S. Khezerloo, A new method for solving fuzzy volterra integro-differential equations, Australian Journal of Basic and Applied Sciences 5 (2011) 154-164.

[3] J.P. Aubin, Fuzzy differential inclusions, Probl. Control Inf. Theory 19 (1990) 55-67.

[4] VA. Baidosov, Fuzzy differential inclusions, J. Appl. Math. Mech. 54 (1990) 8-13, doi:10.1016/0021-8928(90)90080-T.

[5] K. Balachandran, K. Kanagarajan, Existence of solutions of fuzzy delay integrodifferential equations with nonlocal condition, Journal of Korea Society for Industrial and Applied Mathematics 9 (2005) 65-74.

[6] P. Balasubramaniam, S. Muralisankar, Existence and uniqueness of fuzzy solution for the nonlinear fuzzy integrodifferential equations, Appl. Math. Lett. 14(4) (2001) 455-462, doi:10.1016/S0893-9659(00)00177-4.

[7] P. Balasubramaniam, S. Muralisankar, Existence and uniqueness of fuzzy solution for semilinear fuzzy integrodifferential equations with nonlocal conditions, Comput. Math. Appl. 47 (2004) 1115-1122, doi:10.1016/S0898-1221(04)90091-0.

[8] M. Barkhordari Ahmadi, NA. Kiani, Solving Two-Dimensional Fuzzy Partial Differential Equation by the Alternating Direction Implicit Method, Int. J. Industrial Mathematics 1 (2009) 105-120.

[9] B. Bede, SG. Gal, Generalizations of the differentiability of fuzzynumber-valued functions with applications to fuzzy differential equations, Fuzzy Sets Syst. 151 (2005) 581-599, doi:10.1016/j.fss.2004.08.001.

[10] Y. Chalco-Cano, H. Roman- Flores, On new solutions of fuzzy differential equations, Chaos Solitons Fractals 38 (2006) 112-119, doi:10.1016/j.chaos.2006.10.043.

[11] R. Ezzati, S. Siah mansouri, Numerical Solution of Hybrid Fuzzy Differential Equation (IVP) by Improved Predictor-Corrector Method, Int. J. Industrial Mathematics 1 (2009) 147-161.

[12] N. Ghanbari, Numerical solution of fuzzy initial value problems under generalized differentiability by HPM, Int. J. Industrial Mathematics 1 (2009) 19-39.

[13] M. Hukuhara, Integration des applications mesurables dont la valeur est un compact convexe, Funkcial. Ekvac. 10 (1967) 205-223.

[14] E. Hullermeier, An approach to modelling and simulation of uncertain dynamical system, Int. J. Uncertain. Fuzziness Knowl.-Based Syst. 7 (1997) 117-137.

[15] O. Kaleva, Fuzzy differential equations, Fuzzy Sets Syst. 24 (1987) 301-317, doi:10.1016/0165-0114(87)90029-7.

[16] O. Kaleva, A note on fuzzy differential equations, Nonlinear Anal. 64 (2006) 895-900, doi:10.1016/j.na.2005.01.003.

[17] YC. Kwun, DG. Park, Optimal control problem for fuzzy differential equations, In: Proceedings of the Korea-Vietnam Joint Seminar 5 (1998) 103-114.

[18] YC. Kwun, MJ. Kim, BY. Lee, JH. Park, Existence of solutions for the semilinear fuzzy integrodifferential equations using by succesive iteration, Journal of Korean Institute of Intelligent Systems 18 (2008) 543-548, doi:10.5391/JKIIS.2008.18.4.543.

[19] YC. Kwun, JS. Kim, MJ. Park, JH. Park, Controllability for the impulsive semilinear nonlocal fuzzy integrodifferential equations in n-dimensional fuzzy vector space, Adv. Difference Equ. 22 (2010) 480-483 , doi:10.1155/2010/983483.

[20] V. Lakshmikantham, Gnana T. Bhaskar, Devi J. Vasundhara, Theory of set differential equations in metric spaces, Cambridge Scientific Publishers, Cambridge 2006.

[21] V. Lakshmikantham, R. Mohapatra, Theory of fuzzy differential equations anf inclusions, Taylor - Francis (2003).

[22] JY. Park, HK. Han, Existence and uniqueness theorem for a solution of fuzzy differential equations, Int. J. Math. Math. Sci. 22 (1999) 271-279, doi:10.1155/S0161171299222715.

[23] JY. Park, HK. Han, Fuzzy differential equations, Fuzzy Sets Syst. 110 (2000) 69-77, doi:10.1016/S0165-0114(98)00150-X.

[24] JH. Park, JS. Park, YC. Kwun, Controllability for the semilinear fuzzy integrodifferential equations with nonlocal conditions, Fuzzy Systems and Knowledge Discovery. Lecture Notes in Computer Science 4 (2006) 221-230, doi:10.1007/11881599_25.

[25] JH. Park, JS. Park, YC. Ahn, YC. Kwun, Controllability for the impulsive semilinear fuzzy integrodifferential equations, Adv. Soft Comput. 40 (2007) 704-713, doi:10.1007/978-3-540-71441-5_76.

[26] AV. Plotnikov, The averaging of control linear fuzzy differential equations, J. Adv. Res. Appl. Math. 3 (2011) 1-20, doi:10.5373/jaram.664.120610.

[27] AV. Plotnikov, TA. Komleva, Linear problems of optimal control of fuzzy maps, Intelligent Information Management 1 (2009) 139-144, doi:10.4236/iim.2009.13020.

[28] AV. Plotnikov, TA. Komleva, The full averaging of linear fuzzy differential equations, J. Adv. Res. Differ. Equ. 2 (2010) 21-34.

[29] AV. Plotnikov, TA. Komleva, The averaging of control linear fuzzy 2π-periodic differential equations, Dyn. Contin. Discrete Impuls. Syst., Ser. B, Appl. Algorithms 18 (2011) 833-847.

[30] AV. Plotnikov, TA. Komleva, AV. Arsiry, Necessary and sufficient optimality conditions for a control fuzzy linear problem, Int. J. Industrial Mathematics 3 (2009) 197-207.

[31] AV. Plotnikov, TA. Komleva, IV. Molchanyuk, Linear control problems of the fuzzy maps, J. Software Engineering & Applications 3 (2010) 191-197.

[32] AV. Plotnikov, TA. Komleva, IV. Molchanyuk, The Time-Optimal Problems for Controlled Fuzzy R-Solutions, Intelligent Control and Automation, 2 (2011) 152-159, doi:10.4236/ica.2011.22018.

[33] AV. Plotnikov, TA. Komleva, IV. Molchanyuk, Linear control differential inclusions with fuzzy right-hand side and some optimal problems, J. Adv. Res. Dyn. Control Syst. 3 (2011) 34-46.

[34] AV. Plotnikov, TA. Komleva, LI. Plotnikova, The partial averaging of differential inclusions with fuzzy right-hand side, J. Adv. Res. Dyn. Control Syst. 2 (2010) 26-34.

[35] AV. Plotnikov, TA. Komleva, LI. Plotnikova, On the averaging of differential inclusions with fuzzy right-hand side when the average of the right-hand side is absent, Iranian Journal of Optimization 3 (2010) 506-517.

[36] AV. Plotnikov, NV. Skripnik, The generalized solutions of the fuzzy differential inclusions, Int. J. Pure Appl. Math. 56 (2009) 165-172.

[37] AV. Plotnikov, NV. Skripnik, Differential equations with "clear" and fuzzy multivalued right-hand sides. Asymptotics Methods, AstroPrint, Odessa (2009), in Russian.

[38] AV. Plotnikov, NV. Skripnik, Set-valued differential equations with generalized derivative, J. Adv. Res. Pure Math. 3 (2011) 144-160, doi:10.5373/jarpm.475.062210.

[39] AV. Plotnikov, NV. Skripnik, Existence and Uniqueness Theorems for Generalized Set Differential Equations, International Journal of Control Science and Engineering 2 (2012) 1-6, doi:10.5923/j.Control.20120201.01.

[40] ML. Puri, DA. Ralescu, Fuzzy random variables, J. Math. Anal. Appl. 114(2) (1986) 409-422, doi:10.1016/0022-247X(86)90093-4.

[41] H. Rådström, An embeldding theorem for spaces of convex sets, Proc. Amer. Math. Soc. 3 (1952) 165-169.

[42] HR. Rahimi, S. Khezerloo, M. Khezerloo, Approximating the Fuzzy Solution of the Non-linear Fuzzy Volterra Integro-differential Equation Using Fixed Point Theorems, Int. J. Industrial Mathematics 3 (2011) 227-236.

[43] VS. Vasil'kovskaya, AV. Plotnikov, Integrodifferential systems with fuzzy noise, Ukr. Math. J. 59 (2007) 1482-1492, doi:10.1007/s11253-008-0005-z.

[44] S. Seikkala, On the fuzzy initial value problem, Fuzzy Sets Syst. 24 (1987) 319-330, doi:10.1016/0165-0114(87)90030-3.

[45] NV. Skripnik, The full averaging of fuzzy differential inclusions, Iranian Journal of Optimization 1 (2009) 302-317.

[46] NV. Skripnik, Linear fuzzy differential equations, Journal of Uncertain Systems 5(4) (2011) 305-313.

[47] NV. Skripnik, The Partial Averaging of Fuzzy Differential Inclusions, J. Adv. Res. Differ. Equ. 3 (2011) 52-66.

[48] LA. Zadeh, Fuzzy sets, Inf. Control 8 (1965) 338-353, doi:10.1016/S0019-9958(65)90241-X.