Available online at http://ijim.srbiau.ac.ir/
**Int. J. Industrial Mathematics (ISSN 2008-5621)**
Vol. 5, No. 1, 2013 Article ID IJIM-00223, 5 pages
Research Article

Science and Research Branch (IAU)

# Approximating solution of fully fuzzy linear systems in dual form

S. Salahshour [*] [†] , M. Homayoun nejad [‡]

**Abstract**

In this paper, we propose a method to obtain fuzzy solutions of duality fully fuzzy linear system (DFFLS) of the form $\tilde{A}\tilde{X} = \tilde{B}\tilde{X} + \tilde{C}$, where $\tilde{A}$, $\tilde{B}$ are fuzzy matrices and $\tilde{C}$, $\tilde{X}$ are fuzzy number vectors. To this end, we solve the 1-cut of DFFLS (which is a crisp system here) ,then some unknown spreads are allocated to any row of a 1-cut of dual fully fuzzy linear system in 1-cut position. Also, by using this method we determine fuzzy solutions will be placed in the tolerable solution set (TSS) and in the controllable solution set (CSS).

*Keywords* : Fuzzy solutions; fully fuzzy linear systems in dual form; Fuzzy number.

## 1 Introduction

Linear systems of equations, are used for solving many problems in various areas of applied sciences. Fuzzy methods constitute an important mathematical and computational tool for modeling real-world systems with uncertainties of parameters.
The system of linear equations $\tilde{A}\tilde{X} = \tilde{b}$ ,where the elements, $\tilde{a}_{ij}$ , of the coefficient matrix $\tilde{A}$ and the elements, $\tilde{b}_i$ , of the vector $\tilde{b}$ are fuzzy numbers, is called Fully Fuzzy Linear System (FFLS).
Fully fuzzy linear systems have been studied by several authors, like Allahviranloo et al. [1, 2], Buckley and Qu [3, 4, 5], Dehghan et al. [6, 7, 8], Muzzioli and Reynaerts [9] and Vroman et al. [10, 11, 12], have presented new models.
Allahviranloo et al. in [1] proposed an analytical method for obtaining non-zero solutions from the FFLS, and in [2], a practical method for solving the FFLSs was suggested. They obtained some symmetric solutions based on the 1-cut expansion. Clearly, obtained solutions can be used as bounded solutions. Buckley and Qu [3, 4, 5], have proposed various solutions for FFLS. Using their works, Muzzioli and Reynaerts have investigated DFFLS in [9]. Dehghan et al. [6, 7, 8], has used famous numerical methods for solving a FFLS. Vroman et al. in their continuous works [10, 11, 12], have presented methods for solving the FSLEs. In [11], they have suggested the Cramer's rule, which had better solutions from Buckley and Qu's solution, also in [10, 11, 12], an algorithm for improving is presented which is a better method for solving FFLS.
The rest of paper is organized as follows: In Section 2, we discuss some basic concepts and results on fuzzy numbers and fuzzy linear system. In Section 3, we suggest our method to solve DFFLS. In Section 4, The proposed algorithm is illustrated by solving numerical example to show the efficiency of method. Conclusions are drawn in Section 5.

## 2 Preliminaries

The basic concepts are given as follows [2, 9]:

**Definition 2.1** *A fuzzy number is a fuzzy set* $u : R \to [0, 1]$ *which satisfies*
*1. u is upper semi continuous;*
*2.* $u(x) = 0$ *outside some interval* $[a, b]$ *;*
*3. there are real numbers* $a, b$ *such as* $a \le b \le c \le d$ *and*
*3.1.* $u(x)$ *is monotonic increasing on*$[c,\ d]$ *;*
*3.2.* $u(x)$ *is monotonic decreasing on*$[b,\ d]$*;*
*3.3.* $u(x) = 1$ *for all* $x \in [b,\ c]$.
*The set of all fuzzy numbers is denoted by* $E$ *. Another definition of fuzzy number is:*

**Definition 2.2** *A fuzzy number u in parametric form is a pair* $(\underline{u}, \overline{u})$ *of functions* $\underline{u}(r), \overline{u}(r)$ , $0 \le r \le 1$ ,

*Corresponding author. soheilsalahshour@yahoo.com
†Young Researchers Club, Mobarakeh Branch, Islamic Azad University, Mobarakeh, Iran.
‡Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.

*which satisfies the following requirements:*
*1.* $\underline{u}(r)$ *is a bounded monotonic increasing left continuous function;*
*2.* $\overline{u}(r)$ *is a bounded monotonic decreasing left continuous function;*
*3.* $\underline{u}(r) \leq \overline{u}(r)$, $0 \leq r \leq 1$ *A popular fuzzy number is the symmetric triangular fuzzy number* $S[x_0, \sigma]$ *centered at* $x_0$ *with basic* $2\sigma$

$$u(x) = \begin{cases} \frac{1}{\sigma}(x - x_0 + \sigma), & x_0 - \sigma \leq x \leq x_0, \\ \frac{1}{\sigma}(x_0 + \sigma - x), & x_0 \leq x \leq x_0 + \sigma, \\ 0 & otherwise. \end{cases}$$

*We define for arbitrary* $u = (\underline{u}(r), \overline{u}(r)), v = (\underline{v}(r), \overline{v})(r))$ *, addition, subtraction , multiplication:*

*1. Addition:* $\underline{u+v}(r) = \underline{u}(r) + \underline{v}(r)$, $\overline{u+v}(r) = \bar{u}(r) + \bar{v}(r)$

*2. Subtracyion:* $\underline{u-v}(r) = \underline{u}(r) - \bar{v}(r)$, $\overline{u-v}(r) = \bar{u}(r) - \underline{v}(r)$

*3. Multiplication:*

$$\underline{uv}(r) = \min\{\underline{u}(r)\underline{v}(r), \underline{u}(r)\bar{v}(r), \bar{u}(r)\underline{v}(r), \bar{u}(r)\bar{v}(r)\},$$

$$\overline{uv}(r) = \min\{\underline{u}(r)\underline{v}(r), \underline{u}(r)\bar{v}(r), \bar{u}(r)\underline{v}(r), \bar{u}(r)\bar{v}(r)\},$$

**Definition 2.3** *The linear system of equations*

$$\begin{cases} \tilde{a}_{11}\tilde{x}_1 + \tilde{a}_{12}\tilde{x}_2 + \ \dots \ + \tilde{a}_{1n}\tilde{x}_n = \\ \qquad \tilde{b}_{11}\tilde{x}_1 + \tilde{b}_{12}\tilde{x}_2 + \ \dots \ + \tilde{b}_{1n}\tilde{x}_n + \ \tilde{c}_1 \\ \tilde{a}_{21}\tilde{x}_1 + \tilde{a}_{22}\tilde{x}_2 + \ \dots \ + \tilde{a}_{2n}\tilde{x}_n = \\ \qquad \tilde{b}_{21}\tilde{x}_1 + \tilde{b}_{22}\tilde{x}_2 + \ \dots \ + \tilde{b}_{2n}\tilde{x}_n + \tilde{c}_2 \\ \vdots \\ \tilde{a}_{n1}\tilde{x}_1 + \tilde{a}_{n2}\tilde{x}_2 + \ \dots \ + \tilde{a}_{nn}\tilde{x}_n = \\ \qquad \tilde{b}_{n1}\tilde{x}_1 + \tilde{b}_{n2}\tilde{x}_2 + \ \dots \ + \tilde{b}_{nn}\tilde{x}_n + \tilde{c}_n \end{cases} \tag{2.1}$$

*where the elements* $\tilde{a}_{ij}$, $\tilde{b}_{ij}$ *and* $\tilde{c}_j$, $1 \leq i, j \leq n$ *are fuzzy numbers is so-called dual fully fuzzy linear systems (DFFLS).*

**Definition 2.4** *The united solution set (USS), the tolerable solution set (TSS) and controllable solution set (CSS) for the system (2.1) are respectively as follows:*

$$X_{\exists\exists} = \{x' \in R^n : (\exists A' \in A)(\exists b' \in b) \ s.t. \ A'x' = b'\} = \{x' \in R^n : Ax' \cap b' \neq \emptyset\},$$

$$X_{\forall\exists} = \{x' \in R^n : (\exists A' \in A), (\exists b' \in b) \ s.t. A'x' = b'\} = \{x' \in R^n : Ax' \subseteq b\}$$

$$X_{\exists\forall} = \{x' \in R^n : (\exists b' \in b)(\exists A' \in A) \ s.t. \ A'x' = b'\} = \{x' \in R^n : Ax' \supseteq b\}$$

**Definition 2.5** *A fuzzy vector* $\tilde{X} = (\tilde{x}_1, ..., \tilde{x}_n)^t$ *given by* $\tilde{x}_i = [\underline{x_i}(r), \bar{x}_i(r)]$, $1 \leq i \leq n$, $0 \leq r \leq 1$ *is called the minimal symmetric solution of (2.1) which is placed in the CSS if for any arbitrary symmetric solution* $\tilde{Y} = (\tilde{y}_1, \dots, \tilde{y}_n)^t$ *which is place in the CSS that is* $\tilde{Y}(1) = \tilde{X}(1)$ *we have*

$$\left(\tilde{Y} \supseteq \tilde{X}\right), i.e., \ (\tilde{y}_i \supseteq \tilde{x}_i), i.e., \ (\sigma_{\tilde{y}_i} \geq \sigma_{\tilde{x}_i}), \forall i = 1, \dots, n$$

*where* $\sigma_{\tilde{y}_i}$ *and* $\sigma_{\tilde{x}_i}$ *are symmetric spreads of* $\tilde{y}_i$ *and* $\tilde{x}_i$ *, respectively.*

**Definition 2.6** *A fuzzy vector* $\tilde{X} = (\tilde{x}_1, \dots, \tilde{x}_n)^t$ *given by* $\tilde{x}_i = [\underline{x_i}(r), \bar{x}_i(r)]$, $1 \leq i \leq n$, $0 \leq r \leq 1$ *is called the minimal symmetric solution of (2.1) which is placed in the TSS if for any arbitrary symmetric solution* $\tilde{z} = (\tilde{z}_1, \dots, \tilde{z}_n)^t$ *which is place in the TSS that is* $\tilde{Z}(1) = \tilde{X}(1)$ *we have*

$$\left(\tilde{X} \supseteq \tilde{Z}\right), i.e., \ (\tilde{x}_i \supseteq \tilde{z}_i), i.e., \ (\sigma_{\tilde{x}_i} \geq \sigma_{\tilde{z}_i}), \forall i = 1, \cdots, n$$

*where* $\sigma_{\tilde{x}_i}$ *and* $\sigma_{\tilde{z}_i}$ *are symmetric spreads of* $\tilde{x}_i$ *and* $\tilde{z}_i$ *, respectively.*

## 3 Fuzzy solutions of DFFLS

In this section, we propose a practical method to obtain solutions of DFFLSs. First, we obtain the crisp solution from 1-cut of original system. So, we have the following crisp system:

$$\sum_{j=1}^{n} \tilde{a}_{ij}(1)x_j = \sum_{j=1}^{n} \tilde{b}_{ij}(1)x_j + \tilde{c}_i(1), \quad i = 1, ..., n \tag{3.2}$$

Where $\tilde{b}_i(1)$, $\tilde{a}_{ij}(1) \in R$ and $x_j$, $j = 1, \dots, n$ are unknown real variables. Then, we fuzzify 1-cut solution with unknown unsymmetric spreads.
Consequently, the crisp system (3.2) is changed to linear equations, which we only consider its i-th row as following:

$$\begin{aligned} &[\underline{a}_{i1}(r), \bar{a}_{i1}(r)][x_1 - \alpha_i(r), x_1 + \beta_i(r)] + \dots \\ &\quad + [\underline{a}_{in}(r), \bar{a}_{in}(r)][x_n - \alpha_i(r), x_n + \beta_i(r)] = \\ &\qquad [\underline{b}_{i1}(r), \bar{b}_{i1}(r)][x_1 - \alpha_i(r), x_1 + \beta_i(r)] + \dots \\ &\qquad\quad + [\underline{b}_{in}(r), \bar{b}_{in}(r)][x_n - \alpha_i(r), x_n + \beta_i(r)] \\ &\qquad\quad + [\underline{c}_i(r), \bar{c}_i(r)]. \end{aligned} \tag{3.3}$$

in above row, $x_j$, $j = 1, \dots, n$ are solutions of crisp system (3.2) and $\alpha_i(r) > 0$, $\beta_i(r) > 0$, are unknown unsymmetric spreads, if in the above row spread $\alpha_i = \beta_i$, the solutions are symmetric and otherwise, we calculate non-symmetric solutions of the DFFLS.
Furthermore, the obtained solutions using our proposed method are approximating solutions. Then, for

solving the system on the elements of matrices $\tilde{A}$, $\tilde{B}$ , we consider nine following types:

1. $\mathrm{I}_1 = \left\{(i,j) \in \mathrm{N}_n \times \mathrm{N}_n \middle| \tilde{a}_{ij} > 0\,,\, \tilde{b}_{ij} > 0\right\}$,

2. $\mathrm{I}_2 = \left\{(i,j) \in \mathrm{N}_n \times \mathrm{N}_n \middle| \tilde{a}_{ij} > 0\,,\, \tilde{b}_{ij} < 0\right\}$,

3. $\mathrm{I}_3 = \{(i,j) \in \mathrm{N}_n \times \mathrm{N}_n \,|\, \tilde{a}_{ij} > 0\,,\, \tilde{b}_{ij} > 0,$
$\forall j \in \mathrm{N}_s,\ s < n,\quad \tilde{b}_{ij} < 0,\ \forall j \in \mathrm{N}_n - \mathrm{N}_s\}$,

4. $\mathrm{I}_4 = \left\{(i,j) \in \mathrm{N}_n \times \mathrm{N}_n \middle| \tilde{a}_{ij} < 0\,,\, \tilde{b}_{ij} > 0\right\}$,

5. $\mathrm{I}_5 = \left\{(i,j) \in \mathrm{N}_n \times \mathrm{N}_n \middle| \tilde{a}_{ij} < 0\,,\, \tilde{b}_{ij} < 0\right\}$,

6. $\mathrm{I}_6 = \{(i,j) \in \mathrm{N}_n \times \mathrm{N}_n \,|\, \tilde{a}_{ij} < 0\,,\quad \tilde{b}_{ij} > 0,$
$\forall j \in \mathrm{N}_s,\ s < n,\quad \tilde{b}_{ij} < 0,\ \forall j \in \mathrm{N}_n - \mathrm{N}_s\}$,

7. $\mathrm{I}_7 = \{(i,j) \in \mathrm{N}_n \times \mathrm{N}_n \,|\tilde{a}_{ij} > 0,\ \forall j \in \mathrm{N}_s,$
$s < n,\quad \tilde{a}_{ij} < 0,\ \forall j \in \mathrm{N}_n - \mathrm{N}_s,\ \tilde{b}_{ij} > 0\ \}$,

8. $\mathrm{I}_8 = \{(i,j) \in \mathrm{N}_n \times \mathrm{N}_n \,|\tilde{a}_{ij} > 0,\ \forall j \in \mathrm{N}_s,$
$s < n,\quad \tilde{a}_{ij} < 0,\ \forall j \in \mathrm{N}_n - \mathrm{N}_s,\ \tilde{b}_{ij} < 0$ ,

9. $\mathrm{I}_9 = \{(i,j) \in \mathrm{N}_n \times \mathrm{N}_n \,|\tilde{a}_{ij} > 0,\ \forall j \in \mathrm{N}_s,$
$s < n,\quad \tilde{a}_{ij} < 0,\ \forall j \in \mathrm{N}_n - \mathrm{N}_s,\quad \tilde{b}_{ij} > 0,$
$\forall j \in \mathrm{N}_s,\ s < n,\quad \tilde{b}_{ij} < 0,\ \forall j \in \mathrm{N}_n - \mathrm{N}_s$

where $\mathrm{N}_n = \{1\,,\,\ldots\,,\,n\}$.
Type 1. $\mathrm{I}_1 = \left\{(i,j) \in \mathrm{N}_n \times \mathrm{N}_n \middle| \tilde{a}_{ij} > 0\,,\, \tilde{b}_{ij} > 0\right\}$
Since, elements of matrices in the i-th row of interval system are positive, then we have:

$$\sum_{j=1}^{n} \underline{a}_{ij}(r)(x_j - \alpha_i(r)) \ = \sum_{j=1}^{n} b_{ij}(r)(x_j - \alpha_i(r)) \ + \underline{c}_i(r)$$
$$\Rightarrow \alpha_i(r) = f_1(x_1, \cdots, x_n, \underline{a}_{i1}(r), \cdots, \underline{a}_{in}(r), \underline{b}_{i1}(r), \cdots$$
$$, \underline{b}_{in}(r), \underline{c}_i(r))$$

$$\sum_{j=1}^{n} \bar{a}_{ij}(r)(x_j + \beta_i(r)) \ =$$

$$\sum_{j=1}^{n} \bar{b}_{ij}(r)(x_j + \beta_i(r)) \ + \bar{c}_i(r) \Rightarrow \beta_i(r) = f_2(x_1, \cdots$$
$$, x_n,\ \bar{a}_{i1}(r), \cdots, \bar{a}_{in}(r),\ \bar{b}_{i1}(r), \cdots, \bar{b}_{in}(r),\ \bar{c}_i(r))$$

Hence, we set:

$$\alpha_i(r) = \frac{\sum_{j=1}^{n} \underline{b}_{ij}(r)x_j - \sum_{j=1}^{n} \underline{a}_{ij}(r)x_j + \underline{c}_i(r)}{\sum_{j=1}^{n} \underline{b}_{ij}(r) - \sum_{j=1}^{n} \underline{a}_{ij}(r)},$$

and

$$\beta_i(r) = \frac{\sum_{j=1}^{n} \bar{b}_{ij}(r)x_j - \sum_{j=1}^{n} \bar{a}_{ij}(r)x_j + \bar{c}_i(r)}{\sum_{j=1}^{n} \bar{a}_{ij}(r) - \sum_{j=1}^{n} \bar{b}_{ij}(r)}.$$

But, concerning $\alpha_i(r)$, $\beta_i(r)$ and the matter that we need linear spreads the following conditions are considered:
Let $\underline{a}_{ij}(r) \in \left[\underline{a}_{ij}(0), a_{ij}(1)\right]$ and $\bar{a}_{ij}(r) \in \left[a_{ij}(1), \bar{a}_{ij}(0)\right]$, then:

$$\min_{0 \le r \le 1} \left\{\sum_{j=1}^{n} \underline{b}_{ij}(r) - \sum_{j=1}^{n} \underline{a}_{ij}(r)\right\}$$
$$= \sum_{j=1}^{n} \min_{0 \le r \le 1} \{\underline{b}_{ij}(r)\} - \sum_{j=1}^{n} \max_{0 \le r \le 1} \{\underline{a}_{ij}(r)\}$$
$$= \sum_{j=1}^{n} \underline{b}_{ij}(0) - \sum_{j=s+1}^{n} \underline{a}_{ij}(1),$$

$$\max_{0 \le r \le 1} \left\{\sum_{j=1}^{n} \underline{b}_{ij}(r) - \sum_{j=1}^{n} \underline{a}_{ij}(r)\right\}$$
$$= \sum_{j=1}^{n} \max_{0 \le r \le 1} \{\underline{b}_{ij}(r)\} - \sum_{j=1}^{n} \min_{0 \le r \le 1} \{\underline{a}_{ij}(r)\}$$
$$= \sum_{j=1}^{n} \underline{b}_{ij}(1) - \sum_{j=s+1}^{n} \underline{a}_{ij}(0),$$

$$\min_{0 \le r \le 1} \left\{\sum_{j=1}^{n} \bar{a}_{ij}(r) - \sum_{j=1}^{n} \bar{b}_{ij}(r)\right\}$$
$$= \sum_{j=1}^{n} \min_{0 \le r \le 1} \{\bar{a}_{ij}(r)\} - \sum_{j=1}^{n} \max_{0 \le r \le 1} \{\bar{b}_{ij}(r)\}$$
$$= \sum_{j=1}^{n} \bar{a}_{ij}(1) - \sum_{j=1}^{n} \bar{b}_{ij}(0),$$

$$\max_{0 \le r \le 1} \left\{\sum_{j=1}^{s} \bar{a}_{ij}(r) - \sum_{j=1}^{n} \bar{b}_{ij}(r)\right\}$$
$$= \sum_{j=1}^{n} \max_{0 \le r \le 1} \{\bar{a}_{ij}(r)\} - \sum_{j=1}^{n} \min_{0 \le r \le 1} \{\bar{b}_{ij}(r)\}$$
$$= \sum_{j=1}^{n} \bar{a}_{ij}(0) - \sum_{j=1}^{n} \bar{b}_{ij}(1).$$

Then, we get:

$$\alpha_i^l(r) = \frac{\sum_{j=1}^{n} \underline{b}_{ij}(r)x_j - \sum_{j=1}^{n} \underline{a}_{ij}(r)x_j + \underline{c}_i(r)}{\sum_{j=1}^{n} \underline{b}_{ij}(1) - \sum_{j=1}^{n} \underline{a}_{ij}(0)}, \quad i = 1,\ \ldots,\ n$$

$$\alpha_i^u(r) = \frac{\sum_{j=1}^{n} \underline{b}_{ij}(r)x_j - \sum_{j=1}^{n} \underline{a}_{ij}(r)x_j + \underline{c}_i(r)}{\sum_{j=1}^{n} \underline{b}_{ij}(0) - \sum_{j=1}^{n} \underline{a}_{ij}(1)}, \quad i = 1,\ \dots,\ n$$

$$\beta_i^l(r) = \frac{\sum_{j=1}^{n} \bar{b}_{ij}(r)x_j - \sum_{j=1}^{n} \bar{a}_{ij}(r)x_j + \bar{c}_i(r)}{\sum_{j=1}^{n} \bar{a}_{ij}(0) - \sum_{j=1}^{n} \bar{b}_{ij}(1)}, \quad i = 1,\ \dots,\ n$$

$$\beta_i^u(r) = \frac{\sum_{j=1}^{n} \bar{b}_{ij}(r)x_j - \sum_{j=1}^{n} \bar{a}_{ij}(r)x_j + \bar{c}_i(r)}{\sum_{j=1}^{n} \bar{a}_{ij}(1) - \sum_{j=1}^{n} \bar{b}_{ij}(0)}. \quad i = 1,\ \dots,\ n$$

Now, some conditions are considered on the obtained linear spreads of DFFLS as following:

$$\alpha_{un}^{-,l,u}(r) = \min_{0\le r\le 1} \left\{ \left|\alpha_i^l(r)\right|,\ \left|\alpha_i^u(r)\right| \right\}, \tag{3.4}$$

$$\beta_{un}^{-,l,u}(r) = \min_{0\le r\le 1} \left\{ \left|\beta_i^l(r)\right|,\ \left|\beta_i^u(r)\right| \right\}, \tag{3.5}$$

$$\alpha_{un}^{+,l,u}(r) = \max_{0\le r\le 1} \left\{ \left|\alpha_i^l(r)\right|,\ \left|\alpha_i^u(r)\right| \right\}, \tag{3.6}$$

$$\beta_{un}^{+,l,u}(r) = \max_{0\le r\le 1} \left\{ \left|\beta_i^l(r)\right|,\ \left|\beta_i^u(r)\right| \right\}, \tag{3.7}$$

Therefore, by using the unsymmetric spreads (3.4)-(3.7), the fuzzy non-symmetric solutions of DFFLS are calculated as following:

$$\tilde{X}_L^{-,\,l}(r) = (\tilde{x}_1^{-,\,l}(r),\ \dots,\ \tilde{x}_n^{-,\,l}(r))^t, \quad s.t.\ \tilde{x}_i^{-,\,l}(r) = \left[x_i - \alpha_{us}^{-,\,l}(r),\ x_i + \beta_{us}^{-,\,l}(r)\right], \tag{3.8}$$

$$\tilde{X}_U^{+,\,u}(r) = (\tilde{x}_1^{+,\,u}(r),\ \dots,\ \tilde{x}_n^{+,\,u}(r))^t, \quad s.t.\ \tilde{x}_i^{+,\,u}(r) = \left[x_i - \alpha_{us}^{+,\,u}(r),\ x_i + \beta_{us}^{+,\,u}(r)\right], \tag{3.9}$$

**Proposition 3.1** *Let us consider the crisp solution of system (2.1) as and the linear spreads and solutions of the DFFLS are obtained by Eqs.(3.4)-(3.7), and by Eqs.(3.8)-(3.9), respectively then:*

1. $\alpha_{us}^{-,l,u}(1) = \alpha_{us}^{+,l,u}(1) = \beta_{us}^{-,l,u}(1) = \beta_{us}^{+,l,u}(1) = 0,$

2. $\tilde{X}_L^{-,\,l,\,u}(1) = \tilde{X}_U^{+,\,l,\,u}(1) = X^c.$

**Theorem 3.1** *Our proposed solutions of the DFFLS is always gives fuzzy vector.*

**Theorem 3.2** *Let us consider spreads (3.4)-(3.7) and solutions (3.8)-(3.9), of the DFFLS, then we get:*

1. $\tilde{X}_L^{-,\,l,\,u} \in TSS,$
2. $\tilde{X}_U^{+,\,l,\,u} \in CSS.$

**Theorem 3.3** *Consider $\tilde{X}_L^{-,\,l,\,u}$ and $\tilde{X}_U^{+,\,l,\,u}$ are defined in Theorem 3.2, then we have the following:*
*1. $\tilde{X}_L^{-,\,l,\,u}$ is maximal unsymmetric solution in TSS.*
*2. $\tilde{X}_U^{+,\,l,\,u}$ is minimal unsymmetric solution in CSS.*

# 4 Numerical examples

**Example 4.1** *Consider the fuzzy system*

$$\tilde{A} = \begin{pmatrix} (5+r, 7-r) & (3+r,\ 5-r) \\ (3+r, 5-r) & (3+2r, 6-r) \end{pmatrix},$$

$$\tilde{B} = \begin{pmatrix} (3+r, 5-r) & (1,\ 2-r) \\ (2+r, 4-r) & (1,\ 2-r) \end{pmatrix},$$

$$\tilde{b} = \begin{pmatrix} (6-r,\ 7-2r) \\ (6-r,\ 7-2r) \end{pmatrix}$$

*The 1-cut solution of DFFLS is $X^c = (x_1,\ x_2) = (1, 1)$ . We fuzzify this crisp solution by the use of $\alpha, \beta$ spreads, which for each row are determined like this:*

$$\begin{aligned} \alpha_1(r) &= \tfrac{2-2r}{(4+r)-(8+2r)}, \\ \beta_1(r) &= \tfrac{2-2r}{(12-2r)-(7-2r)}, \\ \alpha_2(r) &= \tfrac{3-3r}{(3+r)-(6+3r)}, \\ \beta_2(r) &= \tfrac{2-2r}{(11-2r)-(6-2r)}. \end{aligned}$$

*Then, various forms of crisp solution spreads are obtained:*

$$\alpha_{un}^{-,\,l,\,u}(r) = \frac{2-2r}{6},$$

$$\beta_{un}^{-,l,u}(r) = \frac{2-2r}{7},$$

$$\alpha_{un}^{+,l,u}(r) = \frac{3-3r}{2},$$

$$\beta_{un}^{+,l,u}(r) = \frac{2-2r}{3}.$$

*And finally, fuzzy solutions of the according to linear spreads above are:*

$$\tilde{X}_L^{-,\,l,u}(r) = \left(\left[1 - \frac{2-2r}{6},\ 1 + \frac{2-2r}{7}\right]\right.$$

$$\left., \left[1 - \frac{2-2r}{6},\ 1 + \frac{2-2r}{7}\right]^t,\right.$$

$$\tilde{X}_U^{+,\,l,\,u}(r) = \left(\left[1-\frac{3-3r}{2},\ 1+\frac{2-2r}{3}\right],\left[1-\frac{3-3r}{2},\ 1+\frac{2-2r}{3}\right]\right)^t.$$

# 5 Conclusion

In this paper, we offered a simple method to obtain solutions of duality fully fuzzy linear system. For determining the solutions, first, we solved 1-cut from DF-FLS to obtain the crisp solution, then solution from 1-cut are fuzzified with some non-symmetric spreads. In order to, obtain the spreads of system solution by the use of this new system. Also, our method led to the determining some solutions which are in the TSS and CSS and decides this method grantees that always give us fuzzy vector solution.

# References


[1] T. Allahviranloo, N. Mikaelvand, *Non-zero solutions of fully fuzzy linear systems*, Int. J. Appl. Comput. Math. 10 (2011) 271-282 .

[2] T. Allahviranloo, S. Salahshour, M. Khezerloo, *Maximal- and minimal symmetric solutions of fully fuzzy linear systems*, Journal of Computational and Applied Mathematics, In Press.

[3] J. J. Buckley, Y. Qu, *Solving linear and quadratic fuzzy equations*, Fuzzy Sets and Systems 38 (1990) 43-59.

[4] J. J. Buckley, Y. Qu, *Solving system of linear fuzzy equations*, Fuzzy Sets and Systems 43 (1991) 33-43.

[5] J. J. Buckley, Y. Qu, *Solving fuzzy equations: a new concept*, Fuzzy Sets and Systems 39 (1991) 291-301.

[6] M. Dehghan, B. Hashemi, M. Ghatee, *Computational methods for solving fully fuzzy linear systems*, Applied Mathematics and Computation 179 (2006) 328-343.

[7] M. Dehghan, B. Hashemi, *Solution of the fully fuzzy linear systems using the decomposition procedure*, Applied Mathematics and Computation 182 (2006) 1568-1580.

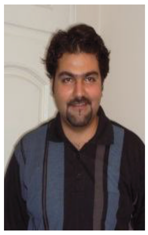
[8] M. Dehghan, B. Hashemi, M. Ghatee, *Solution of the fully fuzzy linear systems using iterative techniques*, Chaos Solitons Fractals 34 (2007) 316-336.

[9] S. Muzzilio, H. Reynaerts, *Fuzzy linear system of the form*, Fuzzy Sets and Systems 157 (2006) 939-951.

[10] A. Vroman, G. Deschrijver, E. E. Kerre, *A solution for systems of linear fuzzy equations in spite of the non-existence of a field of fuzzy numbers*, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 13 (2005) 321 - 335

[11] A. Vroman, G. Deschrijver, E. E. Kerre, *Solving systems of linear fuzzy equations by parametric functions-an improved algorithm*, Fuzzy Sets and Systems 14 (2007) 1515-1534.

[12] A. Vroman, G. Deschrijver, E. E. Kerre, *Solving systems of linear fuzzy equations by parametric functions*, IEEE Transactions on Fuzzy Systems 15 (2007) 370-384.

Soheil Salahshour, born in 1981, holds M.S. degree in Applied Mathematics form the Sistan and Baluchestan University. From the Applied Mathematics Department of Science and Research Branch, IAU, he obtained his Ph.D degree in 2012. Dr. Salahshour's research interests include fuzzy differential equations, fuzzy linear systems, and applications in engineering. Dr. Salahshour is Editor-in-Chief of the Journal of Soft Computing and Applications. Moreover, he is an editor in the Iranian Journal of Fuzzy Systems, and a board member of Advances in Fuzzy Systems (Hindawi Co.) and Applied Computational Intelligence and Soft Computing (Hindawi Co.). Moreover, he is a member of the editorial board of International Journal of Applications of Fuzzy Sets and Artificial Intelligence.

Mohsen Homayoun-nejad, born in 1984, received his B.Sc. degree in Applied Mathematics form Payam-e-Noor University of Semnan, his M.Sc. degree in Applied Mathematics from Science and Research Branch, IAU, in 2012. His research interests include fuzzy set theory, fuzzy linear system of equations and similar topics.