Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 14, No. 2, 2022 Article ID IJIM-1504, 6 pages
DOI: http://dx.doi.org/10.30495/ijim.2022.59557.1504
Research Article

Science and Research Branch (IAU)

Parameters Estimation of Extended Burr XII Distribution Using Principle of Maximum Entropy based on K-Records

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Received Date: 2021-02-22 Revised Date: 2021-10-02 Accepted Date: 2021-11-17

Abstract

In this paper a new method of parameter estimation was employed for extended Burr XII parameters using the principle of maximum entropy (POME) based on k-record values. Exact solutions for expectations were obtained. Monte Carlo simulation method were applied to assess the performance of this method.

Keywords : Shanoon Entropy; Principle of Maximum Entropy; K-Records Value; Extended Burr XII Distribution; Monte Carlo Simulation; Parameter Estimation.

1 Introduction

SHannon [4] founded information theory and introduced entropy as a measure of uncertainty and formulated it as follows

$$H(f) = -\int_{-\infty}^{\infty} f(X;\vartheta) log\left(f(X;\vartheta)\right) dx \quad (1.1)$$

where f is the probability distribution function (PDF) of random variable X with parameters ϑ. Many of authors have used shannon entropy. Jaynes [2] applied it for choosing PDF that maximizing entropy given information in the form of constraints, which is called POME. Also Levine and Tribus [7] stated that, if anyone needs a particular probability distribution to be appropriate with a sample of data, POME can uniquely determine the constraints that are appropriate for extracting the distribution. On the other hand, distribution parameters can be related to these constraints. Using this sort of relation, the estimation of parameters will be realized. Given that C_i be m linear independent constraint at the form of

$$C_i = \int w_i(x) f(X;\vartheta) dx, i = 1, 2, , m \quad (1.2)$$

where $w_i(x)$'s are the functions which their expectation is achieved for $f(X;\vartheta)$, then maximum of H is as follows

$$f(X;\vartheta) = exp\left(-a_0 - \sum_{i=1}^{m} a_i w_i(x)\right) \quad (1.3)$$

where a_i's are lagrange multipliers. Substituting recent equation (1.3) in the entropy (1.1) gives

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following equation

$$H(f) = a_0 + \sum_{i=1}^{m} a_i C_i. \tag{1.4}$$

By maximizing the relation (1.4) we get to required relation of constraints and parameters. Extended Burr XII has introduced by Shao [5] as generalization of some well-known distributions. Its PDF is as follows

$$\begin{aligned} f(x|\alpha,\lambda,c) &= \frac{c}{\lambda}\left(\frac{x}{\lambda}\right)^{c-1}\left[1-\alpha\left(\frac{x}{\lambda}\right)^{c}\right]^{\frac{1}{\alpha}-1} \alpha \neq 0 \\ &= \frac{c}{\lambda}\left(\frac{x}{\lambda}\right)^{c-1}\exp\left\{-\left(\frac{x}{\lambda}\right)^{c}\right\} \alpha = 0, \end{aligned}$$

also CDF of EBXII is given by

$$\begin{aligned} F(x|\alpha,\lambda,c) &= 1-\left[1-\alpha\left(\frac{x}{\lambda}\right)^{c}\right]^{\frac{1}{\alpha}} \alpha \neq 0 \\ &= 1-\exp\left\{-\left(\frac{x}{\lambda}\right)^{c}\right\} \alpha = 0. \end{aligned}$$

Upper k-record values extend ordinary upper record values. Let $T_{1(k)} = k, R_{1(k)} = X_{1:k}$ and for $n \geq 2$, let

$$\begin{aligned} T_{n(k)} = min\{j : j > T_{n-1(k)}, X_j \\ > X_{T_{n-1(k)}-k+1:T_{n-1(k)}},\} \end{aligned}$$

where $X_{i:n}$ indicates $i-th$ order statistics in a sample of size n. For $n \geq 1$, the sequence of upper k-records is defined by $R_{n(k)} = X_{T_{n(k)}-k+1:T_{n(k)}}$. Note that for $k = 1$ the ordinary upper record values can be recovered. The PDF of $n-th$ upper k-record value $R_{n(k)}$ for $n \geq 1$ is given by

$$\begin{aligned} f_{n(k)}(r) = \frac{k^n}{\Gamma(n)}\left[-\ln(\bar{F}(r))\right]^{n-1} \\ \bar{F}^{k-1}(r)f(r), r \geq 0, \end{aligned} \tag{1.5}$$

and joint pdf of $R_{1(k)}, R_{2(k)}, ..., R_{n(k)}$ is given by

$$\begin{aligned} f(r_1, r_2, ..., r_m) = k^m \bar{F}^k(x_m)) \prod_{i=1}^{m} \frac{f(x)}{\bar{F}(x)}, \\ r_1 < r_2 < \cdots < r_m. \end{aligned} \tag{1.6}$$

For more details one can refer to [8], [1] and [9].

2 Specification of Constraints

Distribution of nth k-record value of EBXII distribution will be attained by substituting (1.5) and (1.5) in (1.5), which is as follows:

$$\begin{aligned} f_{n(k)}(x) &= \frac{ck^n x^{c-1}}{\lambda^c \alpha^{n-1}\Gamma(n)\left(1-\alpha\left(\frac{x}{\lambda}\right)^c\right)} \\ &\times \left(1-\alpha\left(\frac{x}{\lambda}\right)^c\right)^{k/\alpha}\left(-\ln\left(1-\alpha\left(\frac{x}{\lambda}\right)^c\right)\right)^{n-1}, \\ &0 \leq x \leq \frac{\lambda}{\alpha^{1/c}}, \ \alpha > 0. \end{aligned} \tag{2.7}$$

Taking logarithm of equation (2.7) and integrating after multiplying at $(-f_{n(k)}(x))$ yields

$$\begin{aligned} S &= -n\ln k + \ln\Gamma(n) - \ln c + \ln\lambda + (n-1)\ln\alpha \\ &- (c-1)E\left(\ln\left(\frac{X}{\lambda}\right)\right) + E\left(\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)\right) \\ &- (n-1)E\left(\ln\left(-\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)\right)\right) \\ &- \frac{k}{\alpha}E\left(\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)\right) \end{aligned} \tag{2.8}$$

in which $E[*]$ denotes expectation of bracketed quantity. For maximization of function S, (2.8) the following constraints must satisfied.

$$\int_0^{\frac{\lambda}{\alpha^{1/c}}} f_{n(k)}(x)\,dx = 1, \tag{2.9}$$

$$\int_0^{\frac{\lambda}{\alpha^{1/c}}} \ln\left(\frac{x}{\lambda}\right) f_{n(k)}(x)\,dx = E\left(\ln\left(\frac{X}{\lambda}\right)\right), \tag{2.10}$$

$$\begin{aligned} &\int_0^{\frac{\lambda}{\alpha^{1/c}}} \ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right) f_{n(k)}(x)\,dx \\ &= E\left(\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)\right), \end{aligned} \tag{2.11}$$

$$\begin{aligned} &\int_0^{\frac{\lambda}{\alpha^{1/c}}} \ln\left(-\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)\right) f_{n(k)}(x)\,dx \\ &= E\left(\ln\left(-\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)\right)\right), \end{aligned}$$

the information which is sufficient to specify of EBXII distribution uniquely are in these constraints. The distribution parameters are related to these constraints.

3 Construction of Entropy Function

The form of PDF for EBXII distribution corresponding to the maximum entropy function which is consistent with constraints (2.9)-(2.12) is as follows:

$$\begin{aligned} f(X) &= exp(-w_0 - w_1 \ln\left(\frac{x}{\lambda}\right) \\ &- w_2 \ln\left(1-\alpha\left(\frac{x}{\lambda}\right)^c\right) \\ &- w_3 \ln\left(-\ln\left(1-\alpha\left(\frac{x}{\lambda}\right)^c\right)\right)), \end{aligned}$$

where w_0 to w_3 are lagrangian multipliers. By applying equation (3.12) to the equation (2.9) we have

$$\begin{aligned} e^{w_0} &= \int_0^{\frac{\lambda}{\alpha^{1/c}}} e^{-w_1 \ln\left(\frac{x}{\lambda}\right) - w_2 \ln\left(1-\alpha\left(\frac{x}{\lambda}\right)^c\right)} \\ &\times e^{-w_3 \ln\left(-\ln\left(1-\alpha\left(\frac{x}{\lambda}\right)^c\right)\right)} dx. \end{aligned} \tag{3.12}$$

By taking change of the variables $Y = -\ln\left(1-\alpha\left(\frac{x}{\lambda}\right)^c\right)$ and $u = e^{-y}$ we obtain

$$\begin{aligned} e^{w_0} &= \frac{\lambda}{c\alpha^{\frac{1-w_1}{c}}} \\ &\times \int_0^1 (-\ln u)^{-w_3} u^{-w_2} (1-u)^{\frac{1-w_1}{c}-1} du. \end{aligned}$$

Using relation (2.4) page 18 [3] and by setting

$$r = \frac{1-w_1}{c} - 1, \tag{3.13}$$

we get to this integration,

$$\begin{aligned} e^{w_0} &= \frac{\lambda}{c\alpha^{\frac{1-w_1}{c}}} \sum_{i=0}^{\infty} \frac{(-r)_i}{i!} \\ &\times \int_0^1 (-\ln u)^{-w_3} u^{i-w_3} du, \end{aligned} \tag{3.14}$$

where $(a)_n = a(a-1)(a-2)\ldots(a+n-1)$ in which $(a)_0 = 1$ is pochhammer symbol. Using change of the variable $u = e^{-z}$ alongside with relation (3.13) and taking logarithm, zeroth Lagrange multiplier is given as

$$\begin{aligned} w_0 &= \ln\lambda - \ln c + \ln\Gamma(1-w_3) - \left(\frac{1-w_1}{c}\right)\ln\alpha \\ &+ \ln\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i}{i!\,(1+i-w_2)^{1-w_3}}. \end{aligned} \tag{3.15}$$

Inserting equation (3.15) in equation (3.12) yields:

$$\begin{aligned} f(x) &= \frac{c\alpha^{\frac{1-w_1}{c}}}{\lambda\Gamma(1-w_3)} \left(\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i}{i!\,(1+i-w_2)^{1-w_3}}\right)^{-1} \\ &\times \left(\frac{x}{\lambda}\right)^{-w_1} \left(1-\alpha\left(\frac{x}{\lambda}\right)^c\right)^{-w_2} \\ &\times \left(-\ln\left(1-\alpha\left(\frac{x}{\lambda}\right)^c\right)\right)^{-w_3}. \end{aligned} \tag{3.16}$$

By comparing equations (3.16) and (1.5)one obtains $w_1 = 1-c, w_2 = 1-\frac{k}{\alpha}, w_3 = 1-n$. Taking logarithm from both sides of equation (3.16) and using relation (1.1) yields:

$$\begin{aligned} S(f) &= \ln\lambda + \ln\Gamma(1-w_3) - \ln c - \frac{1-w_1}{c}\ln\alpha \\ &+ \ln\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_3}{c}\right)_i}{i!\,(1+i-w_2)^{1-w_3}} + w_1 E\left(\ln\frac{X}{\lambda}\right) \\ &+ w_2 E(\ln\left(1-\alpha\left(\frac{X^c}{\lambda}\right)\right) \\ &+ w_3 E\left(\ln\left(-\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)\right)\right). \end{aligned} \tag{3.17}$$

4 Relation Between Distribution Parameters and Constraints

In order to obtain relations between distribution parameters and constraints it is enough to taking partial derivatives of entropy function (3.17)with respect to Lagrangian multipliers as well as parameters and equating them to the zero, and applying constraints. To that goal, taking partial derivatives of equation (3.17) with respect to $w_1, w_2, w_3, \alpha, \lambda$ and c separately and equating each of them to the zero gives:

$$\begin{aligned} &\frac{\partial S}{\partial w_1} = \frac{\ln\alpha}{c} \\ &+ \frac{\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i \left(\Psi\left(1+i-\frac{1-w_1}{c}\right) - \Psi\left(1-\frac{1-w_1}{c}\right)\right)}{i!(1+i-w_2)^{1-w_3}}}{c\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i}{i!(1+i-w_2)^{1-w_3}}} \\ &+ E\left(\ln\left(\frac{X}{\lambda}\right)\right) = 0, \end{aligned} \tag{4.18}$$

$$\frac{\partial S}{\partial w_2} = \frac{\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i (1-w_3)}{i!(1+i-w_2)^{2-w3}}}{\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i}{i!(1+i-w_2)^{1-w3}}} + E\left(\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)\right) = 0,$$

$$\frac{\partial S}{\partial w_3} = \frac{\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i (\ln(1+i-w_2))}{i!(1+i-w_2)^{1-w3}}}{\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i}{i!(1+i-w_2)^{1-w3}}} - \Psi(1-w_3) + E\left(\ln\left(-\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)\right)\right) = 0,$$

$$\frac{\partial S}{\partial \alpha} = \frac{w_1-1}{c\alpha} - w_2 E\left(\frac{\left(\frac{X}{\lambda}\right)^c}{1-\alpha\left(\frac{X}{\lambda}\right)^c}\right) - w_3 E\left(\frac{\frac{\left(\frac{X}{\lambda}\right)^c}{1-\alpha\left(\frac{X}{\lambda}\right)^c}}{\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)}\right) = 0,$$

$$\frac{\partial S}{\partial \lambda} = \frac{1-w_1}{\lambda} + \frac{\alpha c w_2}{\lambda} E\left(\frac{\left(\frac{X}{\lambda}\right)^c}{1-\alpha\left(\frac{X}{\lambda}\right)^c}\right) + \frac{\alpha c w_3}{\lambda} E\left(\frac{\frac{\left(\frac{X}{\lambda}\right)^c}{1-\alpha\left(\frac{X}{\lambda}\right)^c}}{\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)}\right) = 0,$$

$$\begin{aligned}\frac{\partial S}{\partial c} &= -\frac{1}{c} + \frac{1-w_1}{c^2}\ln\alpha - w_2\alpha E\left(\frac{\left(\frac{X}{\lambda}\right)^c \ln\left(\frac{X}{\lambda}\right)}{1-\alpha\left(\frac{X}{\lambda}\right)^c}\right) \\ &- w_3\alpha E\left(\frac{\frac{\left(\frac{X}{\lambda}\right)^c \ln\left(\frac{X}{\lambda}\right)}{1-\alpha\left(\frac{X}{\lambda}\right)^c}}{\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)}\right) + (1-w_1) \\ &\frac{\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i \left(\Psi\left(1+i-\frac{1-w_1}{c}\right)-\Psi\left(1-\frac{1-w_1}{c}\right)\right)}{i!(1+i-w_2)^{1-w3}}}{c^2\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i}{i!(1+i-w_2)^{1-w3}}} \\ &= 0. \end{aligned} \quad (4.27)$$

By simple calculation for equations (4.18)-(4.19)it's been obtained that

$$E\left(\ln\left(\frac{X}{\lambda}\right)\right) = \frac{\Psi\left(1-\frac{1-w_1}{c}\right)}{c} - \frac{\ln\alpha}{c} - \frac{\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i \Psi\left(1+i-\frac{1-w_1}{c}\right)}{i!(1+i-w_2)^{1-w3}}}{c\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i}{i!(1+i-w_2)^{1-w3}}},$$

$$E\left(\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)\right) = \frac{(w_3-1)\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i}{i!(1+i-w_2)^{2-w3}}}{\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i}{i!(1+i-w_2)^{1-w3}}},$$

$$E\left(\ln\left(-\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)\right)\right) = \Psi(1-w_3) - \frac{\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i \ln(1+i-w_2)}{i!(1+i-w_2)^{1-w3}}}{\sum_{i=0}^{\infty} \frac{\left(1-\frac{1-w_1}{c}\right)_i}{i!(1+i-w_2)^{1-w3}}},$$

$$\frac{w_1-1}{c\alpha} = w_2 E\left(\frac{\left(\frac{X}{\lambda}\right)^c}{1-\alpha\left(\frac{X}{\lambda}\right)^c}\right) + w_3 E\left(\frac{\frac{\left(\frac{X}{\lambda}\right)^c}{1-\alpha\left(\frac{X}{\lambda}\right)^c}}{\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)}\right),$$

$$\frac{w_1-1}{\lambda} = \frac{\alpha c w_2}{\lambda} E\left(\frac{\left(\frac{X}{\lambda}\right)^c}{1-\alpha\left(\frac{X}{\lambda}\right)^c}\right) + \frac{\alpha c w_3}{\lambda} E\left(\frac{\frac{\left(\frac{X}{\lambda}\right)^c}{1-\alpha\left(\frac{X}{\lambda}\right)^c}}{\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)}\right),$$

which is similar to relation (4.19). Considering equation (4.19) we get

$$\begin{aligned}
& w_2 \alpha E\left(\frac{\left(\frac{X}{\lambda}\right)^c \ln\left(\frac{X}{\lambda}\right)}{1-\alpha\left(\frac{X}{\lambda}\right)^c}\right) \\
+ \;& w_3 \alpha E\left(\frac{\frac{\left(\frac{X}{\lambda}\right)^c \ln\left(\frac{X}{\lambda}\right)}{1-\alpha\left(\frac{X}{\lambda}\right)^c}}{\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)}\right) \\
= \;& \frac{1-w_1}{c^2}\ln\alpha - \frac{1}{c} \\
- \;& \frac{1-w_1}{c^2}\Psi\left(1-\frac{1-w_1}{c}\right) \\
+ \;& \frac{(1-w_1)\sum_{i=0}^{\infty}\frac{\left(1-\frac{1-w_1}{c}\right)_i \Psi\left(1+i-\frac{1-w_1}{c}\right)}{i!(1+i-w_2)^{1-w3}}}{c^2\sum_{i=0}^{\infty}\frac{\left(1-\frac{1-w_1}{c}\right)_i}{i!(1+i-w_2)^{1-w3}}}.
\end{aligned}$$

Substituting obtained values for w_1, w_2 and w_3, in equalities (4.19) to (4.19) and we get the $(0)_0$. Srivastava, et. al. [6] showed that $(0)_0 = 1$. Therefore parameter estimation equations for the POME are given by

$$E\left(\ln\left(\frac{X}{\lambda}\right)\right) = -\frac{\ln\alpha}{c}, \qquad (4.19)$$

$$E\left(\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)\right) = -\frac{n\alpha}{k}, \qquad (4.20)$$

$$\begin{aligned}
& E\left(\ln\left(-\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)\right)\right) \\
= \;& \Psi(n) - \ln\frac{k}{\alpha}, \qquad (4.21)
\end{aligned}$$

$$\begin{aligned}
& (k-\alpha)E\left(\frac{\left(\frac{X}{\lambda}\right)^c}{1-\alpha\left(\frac{X}{\lambda}\right)^c}\right) \\
+ \;& (n-1)\alpha E\left(\frac{\frac{\left(\frac{X}{\lambda}\right)^c}{1-\alpha\left(\frac{X}{\lambda}\right)^c}}{\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)}\right) = 1
\end{aligned}$$

$$\begin{aligned}
& (k-\alpha)E\left(\frac{\left(\frac{X}{\lambda}\right)^c \ln\left(\frac{X}{\lambda}\right)}{1-\alpha\left(\frac{X}{\lambda}\right)^c}\right) \\
+ \;& (n-1)\alpha E\left(\frac{\frac{\left(\frac{X}{\lambda}\right)^c \ln\left(\frac{X}{\lambda}\right)}{1-\alpha\left(\frac{X}{\lambda}\right)^c}}{\ln\left(1-\alpha\left(\frac{X}{\lambda}\right)^c\right)}\right) \\
= \;& \frac{1}{c} - \ln\alpha. \qquad (4.22)
\end{aligned}$$

Table 1: POME based parameter estimation

n	k	r	POME α (MSE)	POME λ (MSE)	POME c (MSE)
10	2	2	1.32 (0.46)	2.45 (0.73)	4.17 (1.71)
		3	1.13 (0.43)	2.29 (0.58)	3.02 (3.64)
	6	2	1.84 0.73	2.69 (0.82)	3.89 (1.5)
		3	1.79 0.85	2.84 (1.13)	3.69 (2.08)
25	2	2	1.29 (0.28)	2.37 (0.29)	4.15 (1.44)
		3	1.12 (0.33)	2.36 (0.47)	3.56 (3.39)
	6	2	1.84 (0.75)	2.76 (0.87)	3.53 (1.09)
		3	1.53 0.5	2.55 (0.56)	3.72 (1.34)
50	2	2	1.3 (0.22)	2.38 0.17	4.47 (1.01)
		3	1.14 (0.23)	2.29 (0.19)	5.14 (2.75)
	6	2	1.89 (0.7)	2.75 (0.74)	3.41 (0.87)
		3	1.49 (0.42)	2.53 (0.49)	3.83 (0.93)

Relations (4.19) to (4.22) will be taken for estimating unknown parameters of EBXII distribution based on k-record values.

5 Simulation Study and Conclusions

In order to assessing the performance of POME method for parameter estimation, the monte carlo method has been used. Different sample sizes applied. Also different k and record number have been used. Parameters of mentioned distribution for assessment has considered as: $\alpha = 1.5, \lambda = 2.5, c = 3.5$. The record numbers 2 and 3 and also values of k were 2 and 6. Matlab R2019b with 10000 replications has been used for simulation. The numerical results are presented at table 1. The numerical values obtained at table

1 showed followings results:

- While sample size increases, estimation of λ performs better than two other parameter estimators.
- As k decreases(increases) estimator of $\lambda(c)$ performs good.
- The estimator of α performs better than estimators of two other parameters when record number decreases.

Acknowledgment

I would like to express my special thanks to the islamic azad university of Urmia for financial support of my research project which led to this article.

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