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Research Article

Exact Closed-Form Result for the Heat Transfer From Convecting-Radiating Fin of Rectangular Shape

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Abstract

In this paper, the problem of determining heat transfer from convecting-radiating fin of rectangular shape is investigated. We consider one-dimensional, steady conduction in the fin and neglect radiative exchange between adjacent fins and between the fin and its primary surface. It is demonstrated that the governing fin equation, a nonlinear second-order differential equation, is exactly solvable. The exact, closed-form analytical solutions in implicit form are convenient for physical interpretation and optimization for maximum heat transfer. Additionally, exact analytical expressions for heat transfer rate and the fin efficiency are provided.

Keywords : Exact analytical solution; Unique solution; Temperature distribution; Fin efficiency; Heat transfer rate.

1 Preliminaries and problem formulation

THe heat dissipation mechanism considered in literature is either pure convection or pure radiation. In applications where fins operate in a free or natural convection environment, the contribution of radiation is equally significant, and therefore the design must allow for occurring both convection and radiation. As an application, it can be mentioned to stamped heat sink or extruded heat sink designed for cooling a transistor. Even if forced convection is employed for cooling, radiation is significant if the operating temperatures are high as is the case with a finned regenerator [15, 17]. Enhancement of heat transfer employing fins is important in a multitude of heat exchange equipment [6, 7, 16]. It is clear from the literature review that the research has been greatly focused on the theoretical and experimental thermal analysis of both solid fins and porous fins with different profiles and thermophysical properties due to wide range of applications [1, 2, 3, 5, 8, 9, 10, 11, 12, 13, 14, 19, 20, 23, 24, 25].

Assuming one-dimensional conduction, constant thermal parameters, and neglecting fin-to-base and fin-to-fin radiation interaction, the governing equation for a unit depth of the fin is as

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Table 1: Problem formulation.

Nomenclature	
T	temperature
T_b	fin base temperature
T_s	effective sink temperature for radiation
T_∞	environment temperature for convection
L	fin length
h	convective heat transfer coefficient
k	thermal conductivity
N_r	radiation-conduction number
X	dimensional space coordinate
q	heat transfer rate

Table 1. Continue.

Q	dimensionless heat transfer rate
x	non-dimensional space coordinate
x	non-dimensional space coordinate
w_b	fin thickness at the base
w_t	fin thickness at the tip
Greeks symbols	
σ	Stefan-Boltzmann constant
η	fin efficiency
α	ratio of length to one-half base thickness
ε	surface emissivity
θ	dimensionless temperature
θ_s	dimensionless effective sink temperature for radiation
θ_∞	dimensionless environment temperature for convection

follows [17]:

$$\frac{\mathrm{d}}{\mathrm{d}X}\left[w(X)\frac{\mathrm{d}T}{\mathrm{d}X}\right] = \frac{2h}{k}\left(T - T_\infty\right) + \frac{2\varepsilon\sigma}{k}\left(T^4 - T_s^4\right), \tag{1.1}$$

where h, k, ε, σ, T_∞ and T_s denote convective heat transfer coefficient, thermal conductivity of fin material, surface emissivity, Stefan-Boltzmann constant, environment temperature for convection and effective sink temperature for radiation, respectively. In rectangular fin, for fin thickness at distance X, we have $w(X) = 1$, also, introducing the dimensionless variables $\theta = \frac{T}{T_b}$, $\theta_\infty = \frac{T_\infty}{T_b}$, $\theta_s = \frac{T_s}{T_b}$, $x = \frac{X}{L}$, $\alpha = \frac{2L}{w_b}$, $Bi = \frac{hw_b}{2k}$, and $N_r = \frac{\varepsilon\sigma w_b T_b^3}{2k}$, Eq. (1.1) is converted to

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} = \alpha^2 Bi\left(\theta - \theta_\infty\right) + \alpha^2 N_r\left(\theta^4 - \theta_s^4\right), \tag{1.2}$$

with the boundary conditions of prescribed temperature at the base and insulated tip

$$\begin{aligned} &\theta\Big|_{x=0} = 1, \\ &\frac{\mathrm{d}\theta}{\mathrm{d}x}\Big|_{x=1} = -\alpha Bi\left(\theta(1) - \theta_\infty\right) \\ &-\alpha N_r\left(\theta(1)^4 - \theta_s^4\right). \end{aligned} \tag{1.3}$$

The problem formulated by Eqs. (1.2)-(1.3) has been investigated numerically and semi-analytically by many researchers, see [4, 17, 18, 21, 22] and references therein.

The purpose of this study is to demonstrate that the problem formulated by Eqs. (1.2) and (1.3) is exactly solvable for the entire range of the parameters of the model, and more importantly its solution can be determined implicitly in exact, analytic closed-form. Furthermore, we provide exact analytical expressions for the fin efficiency and heat transfer rate with respect to fin base temperature.

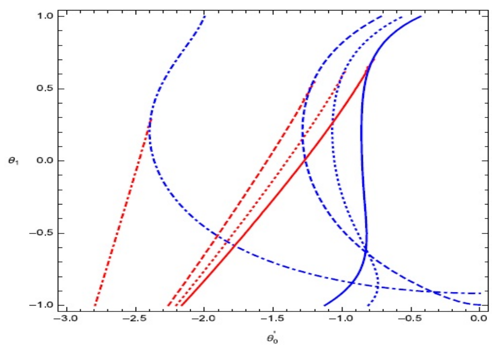

Figure 1: Diagram of θ_1 versus θ_0' for $\alpha = 4$, $N_r = 0.1$ and $\theta_s = \theta_\infty = 0.2$, $Bi = 0.01$: Bold; $Bi = 0.05$: Dotted; $Bi = 0.1$: Dashed; $Bi = 0.5$: DotDashed.

2 The exact analytical solution

With the variable transformation $u = \frac{\mathrm{d}\theta}{\mathrm{d}x}$, we have

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} = u\frac{\mathrm{d}u}{\mathrm{d}\theta}. \tag{2.4}$$

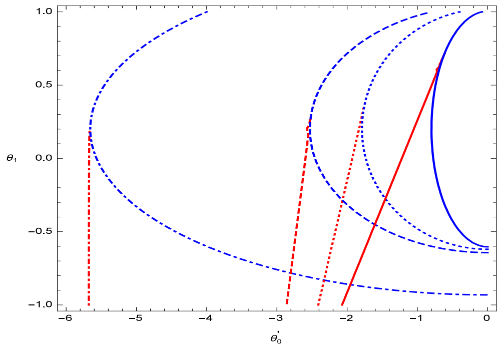

Figure 2: Diagram of θ_1 versus θ'_0 for $\alpha = 10$, $N_r = 0$ and $\theta_s = \theta_\infty = 0.2$, $Bi = 0.01$: Bold; $Bi = 0.05$: Dotted; $Bi = 0.1$: Dashed; $Bi = 0.5$: Dot Dashed.

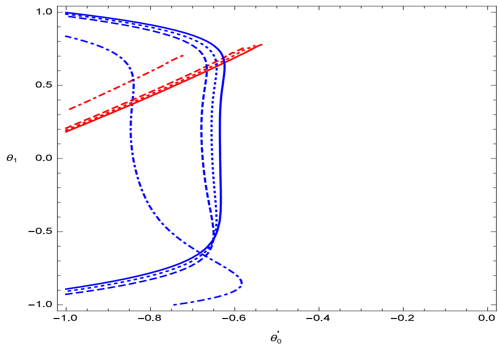

Figure 3: Diagram of θ_1 versus θ'_0 for $\alpha = N_r = 1$ and $\theta_s = \theta_\infty = 0.2$, $Bi = 0.01$: Bold; $Bi = 0.05$: Dotted; $Bi = 0.1$: Dashed; $Bi = 0.5$: Dot Dashed.

Correspondingly, Eq. (1.2) is reduced to first order equation

$$udu = \alpha^2 Bi\,(\theta - \theta_\infty) + \alpha^2 N_r \left(\theta^4 - \theta_s^4\right) \mathrm{d}\theta, \quad (2.5)$$

Owing that Eq. (2.5) is separable, integrating it and later replacing u by $\frac{\mathrm{d}\theta}{\mathrm{d}x}$, takes the form

$$\frac{1}{2}\left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)^2 = \frac{1}{2}\alpha^2 Bi\,(\theta - \theta_\infty)^2 + \alpha^2 N_r \left(\frac{\theta^5}{5} - \theta_s^4\theta\right) + C. \quad (2.6)$$

Here C is the integral constant, which is evaluated with the first boundary condition in Eq. (1.3) as

$$C = \frac{1}{2}(\theta'_0)^2 - \frac{1}{2}\alpha^2 Bi\,(1 - \theta_\infty)^2 - \alpha^2 N_r \left(\frac{1}{5} - \theta_s^4\right), \quad (2.7)$$

where $\theta'_0 = \theta'(0)$. Substituting Eq. (2.7) into Eq. (2.6) implies

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)^2 = (\theta'_0)^2 + \alpha^2 Bi \left[(\theta - \theta_\infty)^2 - (1 - \theta_\infty)^2\right] + 2\alpha^2 N_r \left[\left(\frac{\theta^5}{5} - \theta_s^4\theta\right) - \left(\frac{1}{5} - \theta_s^4\right)\right], \quad (2.8)$$

or equivalently

$$\mathrm{d}x = \frac{\mathrm{d}\theta}{\sqrt{(\theta'_0)^2 + \alpha^2 Bi \left[(\theta - \theta_\infty)^2 - (1 - \theta_\infty)^2\right] + 2\alpha^2 N_r \left[\left(\frac{\theta^5}{5} - \theta_s^4\theta\right) - \left(\frac{1}{5} - \theta_s^4\right)\right]}}. \quad (2.9)$$

Let us define a new non-algebraic function as

$$\Psi\left(\theta; z_1, z_2, z_3, z_4, z_5, z_6\right) = \int_\theta^1 \frac{\mathrm{d}z}{\sqrt{z_1^2 + z_2^2 z_3 \left[(z - z_4)^2 - (1 - z_4)^2\right] + 2z_2^2 z_5 \left[\left(\frac{z^5}{5} - z_6^4 z\right) - \left(\frac{1}{5} - z_6^4\right)\right]}}, \quad (2.10)$$

therefore, integrating both sides of Eq. (2.10) and imposing the condition $\theta(0) = 1$ in Eq. (1.3) leads to

$$x = \Psi\left(\theta; \theta'_0, \alpha, Bi, \theta_\infty, N_r, \theta_s\right) \quad (2.11)$$

where the function $\Psi(\cdot)$ can be treated as the same as other known functions by today's powerful software's programmes such as Mathematical and Matlab, we have used Mathematica during this work.

It is recognizable here that there is still an unknown quantity in the implicit solution of Eq. (2.11) namely $\theta'_0 = \theta'(0)$. This quantity and simultaneously $\theta_1 = \theta(1)$ can be easily obtained with the second boundary condition in Eq. (1.3) as follows: Eq. (1.3), together with Eq. (2.9), using the boundary condition at $x = 1$ yields

$$\alpha^2\left[Bi\,(\theta_1 - \theta_\infty) + N_r\left(\theta_1^4 - \theta_s^4\right)\right]^2 = (\theta'_0)^2 + \alpha^2 Bi \left[(\theta_1 - \theta_\infty)^2 - (1 - \theta_\infty)^2\right] + 2\alpha^2 N_r \left[\left(\frac{\theta_1^5}{5} - \theta_s^4\theta_1\right) - \left(\frac{1}{5} - \theta_s^4\right)\right], \quad (2.12)$$

on the other hand, setting $x = 1$ in the solution given by Eq. (2.11) yields

$$1 = \Psi\left(\theta_1; \theta_0', \alpha, Bi, \theta_\infty, N_r, \theta_s\right). \qquad (2.13)$$

As soon as θ_0' and θ_1 are obtained by solving the coupled system of Eqs. (2.13)-(2.13) for any given combination of α, Bi, θ_∞, N_r and θ_s, the exact solution is presented by Eq. (2.11).

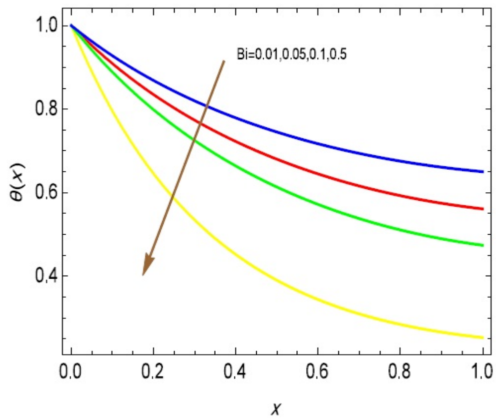

Figure 4: Profile of temperature distribution corresponding to the values of Fig. 1.

3 Results and discussions

In the previous section, we have developed the exact analytical solutions by way of Eqs. (2.11) and (2.13)-(2.13) of the nonlinear fin problem formulated by Eqs. (1.2)-(1.3). The implicit solution in Eq. (2.11) expressed in terms of non-algebraic functions, is obtainable with symbolic computer software, like Mathematica and Maple.

Figs. 1-3 show θ_1, fin base temperature, versus θ_0' through Eqs. (2.13)-(2.13) for different values of Biot number $Bi = 0.01, 0.05, 0.1, 0.5$ and $\alpha = 1, 4, 10$ and $N_r = 0, 0.1, 1$ when $\theta_s = \theta_\infty = 0.2$. Intersection of these two curves is obtained value for θ_0' and θ_1, then their corresponding solutions are plotted in Figs. 4-6. By inspection in these figures, we may conclude the following remarkable features:

1. In the case of $\alpha = 0$, the original Eqs. (1.2)-(1.3) turns to

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} = 0, \qquad \theta(0) = 1, \frac{\mathrm{d}\theta}{\mathrm{d}x}(1) = 0, \qquad (3.14)$$

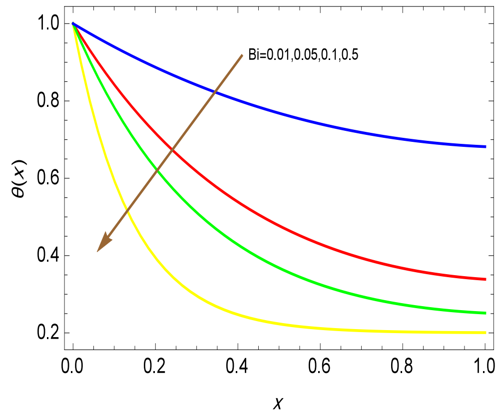

Figure 5: Profile of temperature distribution corresponding to the values of Fig. 2.

whose solution $\theta(x) = 1$ is easily obtained therefore, $\theta_0' = 0$.

2. In the case of $N_r = 0$, the original Eqs. (1.2)-(1.3) turns to

$$\begin{aligned} &\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} = \alpha^2 Bi\left(\theta - \theta_\infty\right), \qquad (3.15)\\ &\theta(0) = 1, \theta'(1) = -\alpha Bi\left(\theta(1) - \theta_\infty\right), \end{aligned}$$

which is linear and then it can be solved easily. The solution obviously is

$$\begin{aligned} \theta(x) = \theta_\infty + & \qquad (3.16)\\ &\frac{\left(\sqrt{\mathrm{Bi}}-1\right)\left(\theta_\infty-1\right)e^{\alpha\sqrt{\mathrm{Bi}}x}}{\sqrt{\mathrm{Bi}}e^{2\alpha\sqrt{\mathrm{Bi}}}+e^{2\alpha\sqrt{\mathrm{Bi}}}-\sqrt{\mathrm{Bi}}+1}\\ &-\frac{\left(\sqrt{\mathrm{Bi}}+1\right)\left(\theta_\infty-1\right)e^{2\alpha\sqrt{\mathrm{Bi}}-\alpha\sqrt{\mathrm{Bi}}x}}{\sqrt{\mathrm{Bi}}e^{2\alpha\sqrt{\mathrm{Bi}}}+e^{2\alpha\sqrt{\mathrm{Bi}}}-\sqrt{\mathrm{Bi}}+1}, \end{aligned}$$

Therefore, we have

$$\begin{aligned} \theta_0' = & \qquad (3.17)\\ &\frac{\alpha\left(\sqrt{\mathrm{Bi}}+1\right)\sqrt{\mathrm{Bi}}e^{2\alpha\sqrt{\mathrm{Bi}}}\left(\theta_\infty-1\right)}{\sqrt{\mathrm{Bi}}e^{2\alpha\sqrt{\mathrm{Bi}}}+e^{2\alpha\sqrt{\mathrm{Bi}}}-\sqrt{\mathrm{Bi}}+1}\\ &+\frac{\alpha\left(\sqrt{\mathrm{Bi}}-1\right)\sqrt{\mathrm{Bi}}\left(\theta_\infty-1\right)}{\sqrt{\mathrm{Bi}}e^{2\alpha\sqrt{\mathrm{Bi}}}+e^{2\alpha\sqrt{\mathrm{Bi}}}-\sqrt{\mathrm{Bi}}+1}, \end{aligned}$$

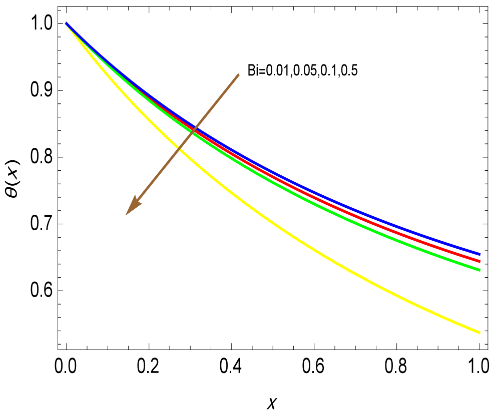

Figure 6: Profile of temperature distribution corresponding to the values of Fig. 3.

and

$$\theta_1 = \theta_\infty + \frac{\left(\sqrt{\mathrm{Bi}}-1\right)e^{\alpha\sqrt{\mathrm{Bi}}}\left(\theta_\infty-1\right)}{\sqrt{\mathrm{Bi}}e^{2\alpha\sqrt{\mathrm{Bi}}}+e^{2\alpha\sqrt{\mathrm{Bi}}}-\sqrt{\mathrm{Bi}}+1} - \frac{\left(\sqrt{\mathrm{Bi}}+1\right)e^{\alpha\sqrt{\mathrm{Bi}}}\left(\theta_\infty-1\right)}{\sqrt{\mathrm{Bi}}e^{2\alpha\sqrt{\mathrm{Bi}}}+e^{2\alpha\sqrt{\mathrm{Bi}}}-\sqrt{\mathrm{Bi}}+1}. \tag{3.18}$$

Then, by setting $\alpha = 10$ and $\theta_s = \theta_\infty = 0.2$, some calculations through (3.18)-(3.19) imply

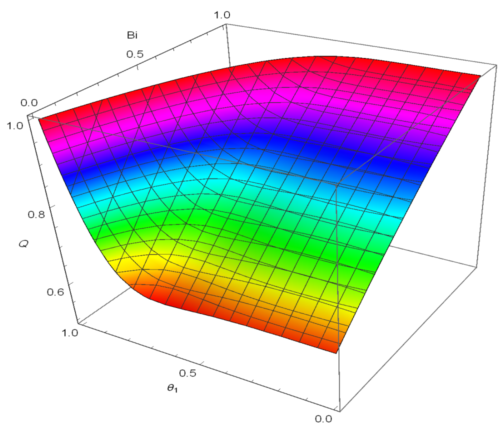

Figure 7: Heat transfer rate with respect to fin base temperature and Biot number with $N_r = 1$ for any value of α.

$\theta_0' = -0.640496 \qquad \theta_1 = 0.681753$
for $\quad Bi = 0.01$
$\theta_0' = -1.76311 \qquad \theta_1 = 0.338749$
for $\quad Bi = 0.05$
$\theta_0' = -2.52512 \qquad \theta_1 = 0.251407$
for $\quad Bi = 0.1$
$\theta_0' = -5.65685 \qquad \theta_1 = 0.200796$
for $\quad Bi = 0.5$

Furthermore, it is verified that $\lim_{Bi\to\infty}\theta_1 = \theta_s = 0.2$ and $\lim_{Bi\to\infty}\theta_0' = -\infty$ which are in full agreement with Figs. 2 and 5.

3. Fin base temperature decrease while Biot number increase for any given combination of α, N_r, θ_s and θ_∞.

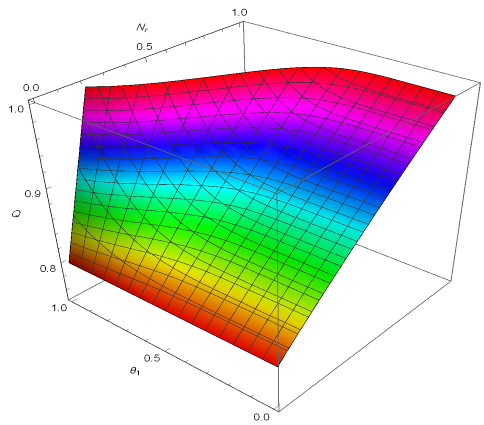

Figure 8: Heat transfer rate with respect to fin base temperature and radiation-conduction number with $Bi = 1$ for any value of α.

The heat transfer rate q (per unit depth) is given by

$$q = -kw_b\frac{\mathrm{d}T(0)}{\mathrm{d}X}, \tag{3.19}$$

which is in dimensionless form as

$$Q = \frac{q}{kT_b} = -\frac{1}{\alpha}\frac{\mathrm{d}\theta(0)}{\mathrm{d}x} = -\frac{\theta_0'}{\alpha}. \tag{3.20}$$

Using Eq. (3.20) in Eq. (2.13) yields

$$Q^2 = \left[Bi\left(\theta_1-\theta_\infty\right)+N_r\left(\theta_1^4-\theta_s^4\right)\right]^2 - Bi\left[\left(\theta_1-\theta_\infty\right)^2-\left(1-\theta_\infty\right)^2\right] - 2N_r\left[\left(\frac{\theta_1^5}{5}-\theta_s^4\theta_1\right)-\left(\frac{1}{5}-\theta_s^4\right)\right]. \tag{3.21}$$

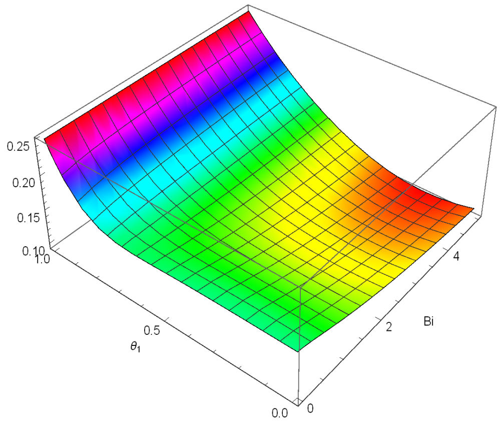

Figure 9: Fin efficiency with respect to fin base temperature and Biot number with $N_r = \alpha = 1$.

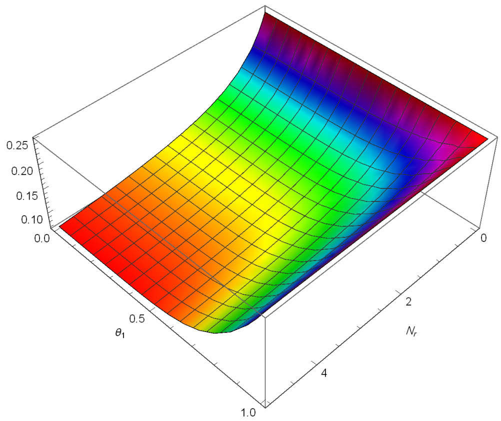

Figure 10: Fin efficiency with respect to fin base temperature and radiation-conduction number with $Bi = \alpha = 1$.

Figs. 7-8 shows the heat transfer rate with respect to fin base temperature, Biot number and radiation-conduction number when $\theta_s = \theta_\infty = 0.2$.

Fin efficiency is the ratio of the real heat transfer rate to the ideal heat transfer rate for a fin of infinite thermal conductivity

$$\eta = \frac{q}{h\left(2L + w_t\right)\left(T_b - T_\infty\right) + \left(2L + w_t\right)\varepsilon\sigma\left(T_b^4 - T_s^4\right)}, \tag{3.22}$$

which can be rewritten in dimensionless form as

$$\eta = \frac{Q}{2\left(\alpha+1\right)\left[Bi\left(1-\theta_\infty\right) + N_r\left(1-\theta_s^4\right)\right]}, \tag{3.23}$$

so, we conclude from Eq. (3.22)

$$\eta = \frac{\sqrt{\begin{array}{c}\left[Bi\left(\theta_1-\theta_\infty\right) + N_r\left(\theta_1^4-\theta_s^4\right)\right]^2 \\ -Bi\left[\left(\theta_1-\theta_\infty\right)^2 - \left(1-\theta_\infty\right)^2\right] - \\ 2N_r\left[\left(\frac{\theta_1^5}{5} - \theta_s^4\theta_1\right) - \left(\frac{1}{5} - \theta_s^4\right)\right]\end{array}}}{2\left(\alpha+1\right)\left[Bi\left(1-\theta_\infty\right) + N_r\left(1-\theta_s^4\right)\right]}. \tag{3.24}$$

We have shown the fin efficiency with respect to fin base temperature, Biot number and radiation-conduction number in Figs. 9-10 when $\theta_s = \theta_\infty = 0.2$ and $\alpha = 1$.

4 Conclusions

In this paper, we have studied accurately the governing equation of determining heat transfer from convecting-radiating fin of rectangular shape. It has been neglected radiative exchange between adjacent fins and between the fin and its primary surface. We have shown that governing differential equation is exactly solvable. The exact, closed-form analytical solutions in implicit form as well as exact analytical expressions for heat transfer rate and the fin efficiency have been extracted.

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