Available online at http://ijim.srbiau.ac.ir/

Int. J. Industrial Mathematics (ISSN 2008-5621)

Vol. 14, No. 1, 2022 Article ID IJIM-1189, 13 pages

DOI: http://dx.doi.org/10.30495/ijim.2021.19031

Research Article

Science and Research Branch (IAU)

Maximizing total efficiency by resource reallocation in DEA: A case study on Tehran Stock Exchange

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Received Date: 2020-04-29 Revised Date: 2021-08-21 Accepted Date: 2021-12-01

Abstract

Data Envelopment Analysis is a technique based on mathematical planning for specifying the efficiency of decision making units (DMUs). DMUs are units which produce similar outputs using similar inputs. So far, the issue of allocating a new resource to decision-making units has been discussed as a challenging issue in several articles. In some cases, manager is not going to add a new resource but to reallocate one of the previous resources. Reallocating a resource may be done for different purposes and has different benefits. For example, without adding a new resource and only using the same resources, is it possible to increase the efficiency of one unit or even increase the efficiency of the whole system? This is the main idea of the present article. In this paper, a mathematical model is presented that can be used to reallocate one of the previous available resources between units in such a way that the total efficiency of decision-making units reaches the maximum possible value. In this model, in order to prevent excessive reduction of the share of each unit of the desired source, restrictions have been considered. In these constraints, a lower bound for the share of each unit is specified. Also, reallocating a resource is likely to lead some changes in output values of decision-making units. Fortunately, this issue has not been neglected in our model. In the presented model, some constraints are considered that specify an upper bound for outputs produced by the units. There are other restrictions in this model. The first is that the total share of units from the desired resource should not exceed the amount available of it and the second is that the total output produced by all units should be at least equal to the total output produced before reallocation. The model presented in this article, in addition to considering the restrictions described, all of which are unavoidable, has been transformed into a linear programming model that can be solved by many existing software. Finally the mentioned model has been utilized in two examples. The first example is numerical and the second one is related to Tehran stock exchange. In both examples, after the reallocation of the resource in question, the total efficiency of the units has increased significantly.

Keywords : Data envelopment analysis; Reallocation; Efficiency, DEA, DMU.

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1 Introduction

DAta envelopment analysis (DEA) has become an effective tool for performance evaluation since it is first introduced by Charnes et al. [13].

Following the first CCR model, several kinds of DEA models have been proposed (Banker et al. [10]; Cook and Seiford [16]) and widely applied to different fields. As examples can be mentioned fixed cost allocation and resource allocation (Cooper et al. [17]) . Many DEA-based approaches have been proposed to deal with the fixed cost allocation and resource allocation problems (Cook and Kress [14], Cook and Zhu [15], Athanassopoulos [7, 8, 9]) . Färe [21] focus on reallocation for some or all inputs. Resource allocation is an important issue that plays an important role in corporate planning. (Amirteimoori & Shafiei [2], Wu, An, Ali, & Liang [50]). Usually, resource allocation is used to improve the performance of organizations (Golany [23], Golany and Tamir [22]).Resource allocation is a classic application in management science and plays an important role in practical issues (Korhonen & Syrjänen [31]). In most cases, the problem of resource allocation arises in environments where decisions are centralized, such as supermarkets, banks, and so on Bi et al. [12] proposed a methodology based on data envelopment analysis for resource allocation in a parallel production system. Afterward an allocation mechanism is introduced by Hosseinzadeh Lotfi et al. [25] that the mechanism is based on a common dual weights approach. In another study pachkova [44] presents a way for producing a function with regard to some cost of total reallocation. Through the mentioned function we can reach to the highest possible performance resources. Another article that we can mention it is Huaqing et al. [27] presented an innovative DEA approach for reallocation of emission permits to improve the overall environmental efficiency for a system with undesirable outputs. Furthermore Resource allocation is important in energy saving and emission reduction. In terms of economics China is one the developing countries which produces a great deal of environmental pollution during its rapid economic development process. It takes a lot of energy to eliminate the pollution. Therefore, more attention is needed to save energy and reduce emissions. So, reasonable allocation of resources and reduction of greenhouse gas emissions in various regions of China is very important. So Li et al. [26] have presented some resource allocation models as a multiple objective linear problem where the reduction of desirable outputs and the reduction of inputs are objectives and the changes amount of undesirable outputs are constraints. After that Wu et al. [51] developed the previous ideas by considering both the economic and environmental factors for the resource allocation Problem. The cross-efficiency concept in DEA is used for approaching resource allocation problems by Du et al. [19]. Another article on reallocation is presented by Khatibi et al. [33]. They proposed the idea of reallocation inputs by using data envelopment analysis as a way to improve organizations' productivity instead of decreasing the inputs. This proposed idea was investigated about data related to some fire department stations. In addition, Amirteimoori [1] proved that the Beasley [11] method would be impractical in many cases. Kordrostami [32] used a set of common weights to keep performance unchanged when creating a fixed cost allocation scheme, but it was not necessarily satisfactory.Both Amirteimoori [1] and Kordrostami [32] and Lin [40] introduced a ratio parameter which is used to guarantee that the allocations are commensurate with the input usages and output productions in terms of size. Afterthat jahanshahloo et al. [30] used several examples to show that the principle of performance immutability in Amirteimoori and Kordrostami [1] is not necessarily met. Therefore Jahanshahloo et al. [30] used the two principles of common weight and no change in efficiency to allocate fixed costs.

In addition, Li et al. [37] proposed a method for allocating new resources that based on the two principles of common weight and no change in efficiency to allocate fixed costs. They considered two allocations, one allocation is in such a way that a set of common weights is used to minimize efficiency deviations and another one exactly preyields Maintains allocation and integrates relative weights throughout.

Li et al. [39] in one of their articles paper, have proposed a DEA-game cross-efficiency approach for allocating the fixed cost, where all DMUs centralize more on the crossefficiency betterments than the allocated costs. Also Yu et al. [54], Zhu et al. [37] and Li et al. [38] developed the fixed cost allocation problem to network situations by considering the internal two-stage processes. Li et al. [36] in their paper, have suggested a new nonegoistic principle, which states that each DMU should propose its allocation proposal in such a way that the maximal cost would be allocated to itself. Also, the optimal allocation is maximized the efficiency scores for all DMUs.

Du et al. [19] proposed a novel approach for allocating the fixed cost based on the game cross-efficiency method by taking the game relations among users in efficiency evaluation. After that Li Y et al. [34] have proved that the new approach of Du et al. [19] is equivalent to the efficiency maximization approach of Li et al. [35], and may exist multiple optimal cost allocation plans. Therefore they have proposed a cooperative game approach by Taking into account the game relations in the allocation process.

Another application of data envelopment analysis in reducing environmental pollution can be mentioned. Today, sustainable development is one of the most important challenges in the world.

There are many articles on this topic, for example Wang and Wu, [53] and Tanner and Wolfingcast, [48].The close relationship between environmental pollution and a country's economy causes Sustainable development has been recognized as an important issue in environmental policy and economic development [45]. Therefore, in order to achieve sustainable development, the reduction of environmental pollution has received much attention [55]. Greenhouses affect the quality of the environmental climate [47].

The most economical way to reduce greenhouse gas emissions is through restrictions and trade policies. Countries can buy or sell emissions licenses to achieve their goals. In this trade, the total amount of allowable emissions is considered as a fixed limit. Therefore, an alternative approach to the fair allocation of publishing licenses using data envelopment analysis is very important. In many articles, this method has been used to solve allocation problems, including Wu et al., [52]; Fang [20]; An et al., [3]...can be mentioned. Also Momeni et al. [43] presented a Centralized DEA-based reallocation model to reduce the total release rate and fair allocation of publishing licenses according to the restriction and trade policy.

In addition, another propose model for measuring optimal production resource reallocation using data envelopment analysis for reducing the effects of climate change was represented by Hidemichi et al. [29]. Afterwards Hatami-Marbini et al. [28] proposed an alternative common-weights DEA model to determine the amount of input and output reduction needed for each DMU to increase score of all the DMUs. Wu et al. [53] designed an effective resource allocation mechanism, which can bring the greatest benefits for the central organization. In addition, they suggested the multi-objective linear programming (MOLP) approach to optimize the DMUs with multiple objectives. Afterwards Many studies have been conducted in reallocation field such as [4, 5, 6, 11, 18, 19, 21, 24, 32, 41, 42, 46].

The existing DEA models for resource allocation are mostly based on one of these assumptions: 1) The efficiency of each DMU may be change after resource allocation. In most cases all of the DMUs become efficient after reallocation, that is unreasonable because a DMU with low efficiency should not become efficient in one stage.

2) The efficiency of each DMU is constant regardless of resource allocation. This issue is unreasonable too, because reallocation of a resource between units can follow various goals. For example, consider a bank Manager may want to reallocate available man power source in all branches so that the summation of DMU's efficiency is maximized. In this paper a model is presented that by using it, can reallocate a source between all units so that the summation of DMU's efficiency is maximized. Furthermore, in our model a lower bound for portion of each DMU for the input that is reallocated between them is assumed. This lower bound can be defined by the manager. This constraint doesn't leave the DMUs utilize mentioned input arbitrarily.

This article is organized like this. In the next section, the basic models of data envelopment analysis are given. In the third part, the main model of this article is presented, which allows the reallocation of one of the available resources. Solving this, in addition to reallocating one of the available resources, maximizes the performance of the entire system. In this section, in addition to linearizing the presented model, its feasibility has also been proven. In the fourth and fifth section, in addition to using the model presented in a numerical example, a practical example related to the Tehran stock exchange is given along with statistical analysis. In the last section, the results are analyzed.

2 Preliminaries

Consider n DMUs each of them consumes m inputs to produce s outputs. Suppose that DMU_o is evaluated. If $u_1, \ldots u_s$ are weights of outputs and $v_1, \ldots, v_m$ are weights of inputs, then the ratio $I_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$ Will show the efficiency score of DMU_j for $j = 1, \ldots, n$. So I_o is the efficiency score of DMU_o. Typically, the weights should be determined in a way that the examined efficiency I_o could be maximized. Ratio efficiency I_o is obtained from solving the following model:

$$\begin{aligned} Max \quad & \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ S.t \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \qquad j = 1, \ldots, n \\ & u_r \geq \varepsilon \qquad r = 1, \ldots, s \\ & v_i \geq \varepsilon \qquad i = 1, \ldots, m \end{aligned} \tag{2.1}$$

The above model is converted in to model (2.2) which is known as multiplier form of CCR model in input oriented [13].

$$\begin{aligned} Max \quad & \sum_{r=1}^{s} u_r y_{ro} \\ S.t. \quad & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad j = 1, \ldots, n \\ & u_r \geq \varepsilon \qquad r = 1, \ldots, s \\ & v_i \geq \varepsilon \qquad i = 1, \ldots, m \end{aligned} \tag{2.2}$$

Definition 2.1. *(Efficiency): DMU_o is CCR efficient if and only if $\frac{\sum_{r=1}^{S} u_r^* y_{ro}}{\sum_{i=1}^{m} v_i^* x_{io}} = 1$ in the model (2.1) and DMU_O is inefficient if and only if $\frac{\sum_{r=1}^{S} u_r^* y_{ro}}{\sum_{i=1}^{m} v_i^* x_{io}} < 1$. In this paper the meaning of efficiency is CCR efficiency.*

3 The proposed model for resource reallocation

Suppose n DMUs are evaluated, each of them consumes m inputs to produce s outputs. Assume that the i(th) input of the DMU_j is x_{ij} for $j = 1, \ldots, n$ and the r(th) output of the DMU_j is y_{rj} for $j = 1, \ldots, n$. Suppose the efficiency score of DMU_j is θ_j^* with this amount of inputs and outputs obtained by Model (2.2). But the goal we pursue in this article is reallocation one of available inputs between this n units, So that the summation of efficiency scores is maximized. Without lost of generality suppose that the input which is wanted to reallocate, is m(th) input. Suppose $\widehat{x}_{mj}$ is the portion of DMU_j ($j = 1, \ldots, n$) of input m (after reallocation). In addition, the reallocation one of the inputs may influence output values. So consider $\widehat{y}_{rj}$ as r(th) output after reallocation. Furthermore u_r^p and v_i^p are sequentially the weights of r(th) output and i(th) input for DMU_p ($p = 1, \ldots, n$). Here we present a model whose solution gives the values $\widehat{x}_{mj}$ and $\widehat{y}_{rj}$ in such a way that the efficiency of each unit is not less than before, also maximizes total efficiency. In addition, some restrictions apply to the new values of inputs and outputs in this model.
Taking into account the above assumptions, the following model is proposed:

$$Max \sum_{p=1}^{n} d_p \tag{3.3}$$

$$S.t \quad \frac{\sum_{r=1}^{S} u_r^p \hat{y}_{rj}}{\sum_{i=1}^{m-1} v_i^p x_{ij} + v_m^p \hat{x}_{mj}} \leq 1 \qquad j = 1, \ldots, n; \quad p = 1, \ldots, n \tag{3.4}$$

$$\frac{\sum_{r=1}^{S} u_r^p \hat{y}_{rp}}{\sum_{i=1}^{m-1} v_i^p x_{ip} + v_m^p \hat{x}_{mp}} - d_p = \theta_p^* \qquad p = 1, \ldots, n \tag{3.5}$$

$$\sum_{j=1}^{n} \hat{x}_{mj} \leq \sum_{j=1}^{n} x_{mj} \tag{3.6}$$

$$\sum_{j=1}^{n} \hat{y}_{rj} \geq \sum_{j=1}^{n} y_{rj} \quad r = 1, \ldots, s \tag{3.7}$$

$$(1 - \gamma_j)\, x_{mj} \leq \hat{x}_{mj} \quad j = 1, \ldots, n \tag{3.8}$$

$$\hat{y}_{rj} \leq (1 + w_j) y_{rj} \qquad j = 1, \ldots, n; r = 1, \ldots, s \tag{3.9}$$

$$u_r^p \geq \varepsilon \quad r=1,\ldots,s; \quad p=1,\ldots,n$$
$$v_i^p \geq \varepsilon \quad i=1,\ldots,m; \quad p=1,\ldots,n$$
$$d_p \geq 0 \quad p=1,\ldots,n$$
$$\hat{x}_{mj} \geq \varepsilon \quad j=1,\ldots,n$$
$$\hat{y}_{rj} \geq \varepsilon \quad r=1,\ldots, \quad s; j=1,\ldots,n$$

d_p is the difference between the efficiency score of DMU_j with taking into account the values of inputs and outputs before and after reallocation . Also $\frac{\sum_{r=1}^{S} u_r^p \hat{y}_{rj}}{\sum_{i=1}^{m-1} v_i^p x_{ij}+v_m^p \hat{x}_{mj}} \leq 1$ is the necessary constraint which restrict DMU_j to be in the PPS with the weights of DMU_P. The constraint $\frac{\sum_{r=1}^{S} u_r^p \hat{y}_{rp}}{\sum_{i=1}^{m-1} v_i^p x_{ip} + v_m^p \hat{x}_{mp}} - d_p = \theta_p^*$ indicates that the efficiency of DMU_P after reallocation should not be lower than before, for $p=1,\ldots,n$. According to the above explanations, the objective function and constraints of Model (3.3) can be briefly explained as follows:

1. All units must be in the PPS produced by weights of DMU_p for $(p=1,\ldots,n)$. (constraint (3.4))
2. The efficiency of DMU_p should not be less than before (for $(p=1,\ldots,n)$). (constraint (3.5) with $d_p \geq 0$)
3. Total consumption of m(th) input by all units should not be greater than before. (constraint (3.6))
4. Total production of r(th) output by all units should not be less than before (for $(r=1,\ldots,s)$). (constraint (3.7))
5. The maximum percent that manager allows to be decreased from m(th) input of DMU_j is γ_j. (constraint (3.8))
6. The maximum percent that manager allows to be added to each outputs of DMU_j is w_j. (constraint (3.9))
7. In objective function of model (3.3), maximizing the summation of deviation variables $(\sum_{p=1}^{n} d_p)$ is equivalent to maximizing the summation of efficiencies after reallocation, because according to constraint (3.5) we have: $Max \ \sum_{p=1}^{n} d_p = Max \sum_{p=1}^{n} (\frac{\sum_{r=1}^{S} u_r^p \hat{y}_{rp}}{\sum_{i=1}^{m-1} v_i^p x_{ip}+v_m^p \hat{x}_{mp}} - \theta_p^*)$ and since θ_p^* for $p=1,\ldots,n$ are constant, then maximizing $\sum_{p=1}^{n} d_p$ means maximizing the summation of efficiency scores after reallocation.
8. $\hat{x}_{mj} \geq \varepsilon$ and $\hat{y}_{rj} \geq \varepsilon$ state that the new value of inputs and outputs must be strictly positive. Also to avoid week efficiency, all weights are selected positive $(u_r^p \geq \varepsilon, v_i^p \geq \varepsilon)$.

But model (3.3) is not a linear model. For modifying it to a linear model, we prove the following theorems.

Theorem 3.1. *model (3.3) is feasible.*

Proof. Consider the following model for $p = 1,\ldots,n$.

$$Max \ \frac{\sum_{r=1}^{S} u_r^p y_{rp}}{\sum_{i=1}^{m} v_i^p x_{ip}} \tag{3.10}$$
$$S.t \ \frac{\sum_{r=1}^{S} u_r^p y_{rj}}{\sum_{i=1}^{m} v_i^p x_{ij}} \leq 1$$
$$j=1,\ldots,n; \quad p=1,\ldots,n$$
$$u_r^p \geq \varepsilon \quad r=1,\ldots,s; \quad p=1,\ldots,n$$
$$v_i^p \geq \varepsilon \quad i=1,\ldots,m; \quad p=1,\ldots,n$$

The obtained solutions by solving the above model for $p=1,\ldots,n$ could be a part of feasible solution of model (3.3). The optimal solutions of above model along with

$$\hat{x}_{mj} = x_{mj} \quad j=1,\ldots,n$$
$$\hat{y}_{rj} = y_{rj} \quad j=1,\ldots,n$$
$$d_p = 0 \quad p=1,\ldots,n$$

is a feasible solution of model (3.3) as follows:

$$(u_1^{1*},\ldots,u_s^{1*},\ldots, u_1^{n*},\ldots,u_s^{n*},v_1^{1*},\ldots,v_m^{1*},$$
$$\ldots,v_1^{n*},\ldots,v_m^{n*},x_{m1},\ldots,x_{mn},y_{11},\ldots, y_{1n},$$
$$\ldots,y_{s1},\ldots,y_{sn},\ldots,d_1=0 \ ,\ldots, \ d_n=0 \ )$$

□

Theorem 3.2. *An optimal solution of model (3.3) can be found that the following condition is satisfied.*

$$\sum_{i=1}^{m-1} v_i^{p*} x_{ip} + v_m^{p*} \hat{x}_{mp}^* = 1 \qquad p=1,\ldots,n$$

Proof. suppose the optimal solution for the model (3.3) is obtained as follows:

$$(u_1^{1*},\ldots,u_s^{1*},\ldots, u_1^{n*},\ldots,u_s^{n*},v_1^{1*},\ldots,v_m^{1*},\ldots,$$
$$v_1^{n*},\ldots,v_m^{n*},\hat{x}_{m1}^* \ ,\ldots, \ \hat{x}_{mn}^* \ ,\hat{y}_{11}^*,\ldots, \ \hat{y}_{1n}^*\ldots,$$

$\hat{y}^*_{s1}, \ldots, \hat{y}^*_{sn}, \ldots, d^*_1, \ldots, d^*_n)$

With regarding to the above optimal solution, we assume

$$\sum_{i=1}^{m-1} v_i^{p*} x_{ip} + v_m^{p*} \hat{x}^*_{mp} = k_p \quad p = 1, \ldots, n$$

We show that the following answer is an optimal solution of model (3.3).

$$(\frac{u_1^{1*}}{k_1}, \ldots, \frac{u_s^{1*}}{k_1}, \ldots, \frac{u_1^{n*}}{k_n}, \ldots, \frac{u_s^{n*}}{k_n}, \frac{v_1^{1*}}{k_1}, \ldots, \frac{v_m^{1*}}{k_1}, \ldots, \frac{v_1^{n*}}{k_n}, \ldots, \frac{v_m^{n*}}{k_n}, \hat{x}^*_{m1}, \ldots, \hat{x}^*_{mn}, \hat{y}^*_{11}, \ldots, \hat{y}^*_{1n} \ldots, \hat{y}^*{}_{s1}, \ldots, \hat{y}^*_{sn}, \ldots, d^*_1, \ldots, d^*_n)$$

It is evident that above solution is satisfied in all constraints of model (3.3) and the objective function value doesn't change. Therefore the above answer is an optimum solution of model (3.3) so,

$$\sum_{i=1}^{m-1} v_i^{p*} x_{ip} + v_m^{p*} \hat{x}^*_{mp} = 1 \quad p = 1, \ldots, n$$

□

With regards to theorem 3.2, model (3.3) can be written as following:

$$Max \sum_{p=1}^{n} \alpha_p d_p \tag{3.11}$$

$$\begin{aligned}
S.t \quad & \sum_{r=1}^{S} u_r^p \hat{y}_{rj} - \sum_{i=1}^{m-1} v_i^p x_{ij} - v_m^p \hat{x}_{mj} \leq 0 \\
& \quad j = 1, \ldots, n; \quad p = 1, \ldots, n \\
& \sum_{i=1}^{m-1} v_i^p x_{ip} + v_m^p \hat{x}_{mp} = 1 \quad p = 1, \ldots, n \\
& \sum_{r=1}^{S} u_r^p \hat{y}_{rp} - d_p = \theta_p^* \quad p = 1, \ldots, n \\
& \sum_{j=1}^{n} \hat{x}_{mj} \leq \sum_{j=1}^{n} x_{mj} \\
& \sum_{j=1}^{n} \hat{y}_{rj} \geq \sum_{j=1}^{n} y_{rj} \quad r = 1, \ldots, s \\
& (1 - \gamma_j)\, x_{mj} \leq \hat{x}_{mj} \quad j = 1, \ldots, n \\
& \hat{y}_{rj} \leq (1 + w_j)\, y_{rj} \\
& \quad j = 1, \ldots, n; \quad r = 1, \ldots, s \\
& u_r^p \geq \varepsilon \quad r = 1, \ldots, s; \quad p = 1, \ldots, n \\
& v_i^p \geq \varepsilon \quad i = 1, \ldots, m; \quad p = 1, \ldots, n \\
& d_p \geq 0 \quad p = 1, \ldots, n \\
& \hat{x}_{mj} \geq \varepsilon \quad j = 1, \ldots, n \\
& \hat{y}_{rj} \geq \varepsilon \quad r = 1, \ldots, s; \quad j = 1, \ldots, n
\end{aligned}$$

In this model α_p *is* the penalty coefficient of deviation variable d_p. The model (3.11) is a nonlinear model that can be transferred to the following linear model by defining the following variables.

$$\hat{u}_{rj}^p = u_r^p \hat{y}_{rj}$$

$$\hat{v}_{mj}^p = v_m^p \hat{x}_{mj}$$

So following linear model for reallocation of one source between DMUs is proposed which satisfy all of the mentioned assumptions.

$$Max \sum_{p=1}^{n} \alpha_p d_p \tag{3.12}$$

$$\begin{aligned}
S.t \quad & \sum_{r=1}^{S} \hat{u}_{rj}^p - \sum_{i=1}^{m-1} v_i^p x_{ij} - \hat{v}_{mj}^p \leq 0 \\
& \quad j = 1, \ldots, n; \quad p = 1, \ldots, n \\
& \sum_{i=1}^{m-1} v_i^p x_{ip} + \hat{v}_{mp}^p = 1 \quad p = 1, \ldots, n \\
& \sum_{r=1}^{S} \hat{u}_{rp}^p - d_p = \theta_p^* \quad p = 1, \ldots, n \\
& \sum_{j=1}^{n} \hat{v}_{mj}^p \leq v_m^p \sum_{j=1}^{n} x_{mj} \quad p \in \{1, \ldots, n\} \\
& \sum_{j=1}^{n} \hat{u}_{rj}^p \geq u_r^p \sum_{j=1}^{n} y_{rj} \quad p \in \{1, \ldots, n\}; \\
& \quad r = 1, \ldots, s \\
& (1-\gamma_j) v_m^p\, x_{mj} \leq \hat{v}_{mj}^p \quad j = 1, \ldots, n \\
& \hat{u}_{rj}^p \leq (1 + w_j)\, u_r^p y_{rj} \quad j = 1, \ldots, n; \\
& \quad r = 1, \ldots, s \\
& u_r^p \geq \varepsilon \quad r = 1, \ldots, s; \quad p = 1, \ldots, n \\
& v_i^p \geq \varepsilon \quad i = 1, \ldots, m; \quad p = 1, \ldots, n \\
& d_p \geq 0 \quad p = 1, \ldots, n \\
& \hat{u}_{rj}^p \geq \varepsilon \quad p = 1, \ldots, n; \quad j = 1, \ldots, n; \\
& \quad r = 1, \ldots, s \\
& \hat{v}_{mj}^p \geq \varepsilon \quad p = 1, \ldots, n; \quad j = 1, \ldots, n
\end{aligned}$$

In next section this model is illustrated by a numerical example and it is used for a real data set in Tehran Stock Exchange.

Table 1: Data of numerical example.

DMU	DMU_A	DMU_B	DMU_C	DMU_D	DMU_E
Input	1	2	4	3	2
output	1	2	4	1.5	1

Table 2: Effciency score.

DMU	DMU_A	DMU_B	DMU_C	DMU_E	DMU_E
θ^*	1	1	1	0.5	0.5

Table 3: Results of solving model (3.12) (input).

x_A	x_B	x_C	x_D	x_E
1.0589	1.0004	2.0002	1.5884	1.0589

Table 4: Results of solving model (3.12) (output).

y_A	y_B	y_C	y_D	y_E
1.4999	1.4165	2.8330	2.2498	1.4999

Table 5: Efficiency score with inputs and outputs in Tables 3 and 4.

DMU	DMU_A	DMU_B	DMU_C	DMU_E	DMU_E
θ^*	0.99988	1	1	0.99985	0.99988

Table 6: Results of solving model (3.12) (input)

x_A	x_B	x_C	x_D	x_E
0.5004	3.7908	2.9750	2.8431	1.8954

Table 7: Results of solving model (3.12) (input)

y_A	y_B	y_C	y_D	y_E
0.3956	2.9994	2.3536	2.2495	1.4997

Table 8: Efficiency score with inputs and outputs in Tables 6 and 7

DMU	DMU_A	DMU_B	DMU_C	DMU_E	DMU_E
θ^*	1	0.99910	0.99924	0.99913	0.99913

4 Numerical Example

Now the presented model in this paper is used for the data of Table 1 related to five DMUs with one input and output.

Evaluating these DMUs through model (2.2) has been revealed that DMUs A, B and C are efficient.

Suppose $p = A$ and $\gamma_j = w_j = 0.5$, then model

(3.12) is solved for the data of Table 1.

Now DMUs are evaluated by model (2.2) again. But inputs and outputs in Tables 3 and 4 are considered this time. The results are displayed in table 5. DEA solver is used for solving model (2.2).
Suppose $p = B$ and $\gamma_j = w_j = 0.5$ then model (3.12) is solved for the data of Table1.
The efficiency scores of DMUs with inputs and outputs in Tables 6 and 7 are displayed in table 8.
As can be seen in Tables 5 and 8, after the reallocation of inputs, the total unit efficiency has reached from 4 to 4.99961 and 4.9966 respectively, which is very desirable in centralized environment.

5 Applied example and statistical analysis

5.1 Empirical example

In this example, ten companies which are members of Tehran stock exchange with two inputs (long term investment and current assets) and one output (operating profit) are considered (Table 9).
The results of Table 10 are obtained through using the software DEA-Solver and the reallocation results of Table 11 are derived by using the presented model in this paper (model (3.12)). Pay attention just, the second input (current assets) is reallocated between the units. Model (3.12) is solved for $p = DMU_1$ and $\gamma_j = w_j = 0.5$.
Then the CCR efficiency of all units are evaluated again and the related results are presented in Table 12.
Comparing Table 10 and Table 12, it can be seen that all units have better efficiency after reallocating the second input (current assets). For example, the efficiency of DMU_1 from 0.3 to 1, the efficiency of DMU_4 from 0.1 to 0.3, the efficiency of DMU_5 from 0.6 to 1, the efficiency of DMU_6 from 0.4 to 1 are improved.
It is clear that the number of efficient units have been increased from one unit (DMU_2) to five units (DMU_1, DMU_2,DMU_5,DMU_6,DMU_8) . It means that by reallocating the current assets of all units, not only they maintain their efficiency but will have a better status than before. As you can see, the sum of efficiency has been changed from 2.891 to 6.058, which indicates the whole system become more efficient than before.

5.2 Statistically

For long term investment, current assets and operating profit variables are calculated Mean, Median, Standard deviation, Skewness, Kurtosis, and Minimum, Maximum and also Kolmogorov - Smirnov statistic and its probability which is presented in Table 13 respectively.

The probability of Kolmogorov – Smirnov statistic shows in the table above that the distribution of all three variables is normal which is accepted at a significant level of 0/05.
All of these parameters are recalculated again for Current assets and Operating profit variables after reallocation which is presented in Table 14. It is evident from the results of above table that the distribution of new Current assets and new Operating profit variables is normal which is accepted at a significant level of 0.05

6 Conclusion

In this paper a model is proposed that by using it, can reallocate one of the resources so that the summation of efficiency scores of all units is maximized. The best advantage of this model is its linearity which makes it easier to solve using existing software. Furthermore in most of existing resource allocation models all of the DMUs become efficient after reallocation, that is unreasonable because a DMU with low efficiency should not become efficient in one stage. In some other existing models of resource allocation, the efficiency of each unit remains constant, which is also unreasonable as it can pursue a variety of reallocation objectives. In this article, neither of these two goals is intended, but we seek to maximize the overall efficiency of all units. The reason is that in a centralized environment, the manager seeks to increase the efficiency of the whole system, not the units individually. In this paper a model is presented that by using it, can reallo-

Table 9: Data of empirical example

DMU	Long term investment	Current assets	Operating Profit
1	87125	231454	170242
2	43741	204136	498277
3	174256	652967	42494
4	289653	359103	99447
5	1196235	31056	51756
6	2213564	1160100	1159736
7	187369	216807	39335
8	89362	652165	208393
9	286323	2389150	174528
10	2461436	531941	37722

Table 10: Efficiency score

DMU	1	2	3	4	5
Efficiency	0.301	1	0.026	0.113	0.682
DMU	6	7	8	9	10
Efficiency	0.409	0.074	0.204	0.053	0.029

Table 11: Results of solving model (3.12)

DMU	Current assets	Operating profit
1	115727	255363
2	102068	129327
3	326483	63741
4	179551	149170
5	15528	77634
6	580050	1739604
7	108403	59002
8	2102927	312589
9	1194575	261792
10	265970	56583

Table 12: CCR effciency of units after reallocation

DMU	1	2	3	4	5	6	7	8	9	10
Efficiency	1	1	0.124	0.342	1	1	0.221	1	0.304	0.067

cate a source between all units so that the summation of DMU's efficiency is maximized. Furthermore in our model a lower bound for portion of each DMU for the input that is reallocate between them is assumed. This lower bound can be defined by the manager. This constraint doesn't leave the DMUs utilize mentioned input arbitrarily. Also, a change in the amount of reallocated source may result in a change in the outputs of the units. This is also included in the proposed

Table 13: Statistics related to data of Table 9

	Long term investment	Current assets	Operating profit
Mean	702906.40	642887.90	248193.00
Median	236846.00	445522.00	134844.50
Std. Deviation	924817.00	693432.26	349202.93
Skewness	1.374	2.074	2.394
Kurtosis	0/289	4.738	5.965
Minimum	43741.0	31056.0	37722.0
Maximum	2461436.0	2389150.0	1159736.0
Kolmogorov Smirnov Z	1.178	0/930	1.092
Asymp. Sig. (2tailed)	0/125	0/352	0/184

Table 14: Statistics relating to data after reallocation

	Current assets	Operating profit
Mean	499128.200	310480.500
Median	222760.500	139248.500
Std. Deviation	661635.389	510930.289
Skewness	1.976	2.966
Kurtosis	3.620	9.073
Minimum	15528.00	56583.00
Maximum	2102927.00	1739604.00
Kolmogorov Smirnov Z	0/958	1.260
Asymp. Sig. (2-tailed)	0/318	0/084

model. Finally the proposed model has been utilized in two examples. The first example is numerical and the second one is related to Tehran stock exchange. In both examples, after the reallocation of the resource in question, the total efficiency of the units has increased significantly. In the second example, common statistical parameters for input and output values before and after reallocation are also calculated.

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