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Research Article

Classifying Flexible Measures in Two-Stage Network DEA

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Abstract

Standard Data Envelopment Analysis (DEA) supposes that the performance measure status from the point of view of input or output is known. Nevertheless, in some situations, determining the status of a performance measure in two-stage network is not easy. Measures with unknown status of input/output are called flexible measures. In all of the previous studies did not point to classify flexible measures in two-stage network DEA. In this paper we propose FNDEA models based on the multiplier model under constant returns to scale (CRS) and variable returns to scale (VRS) in general two-stage network structures. Our models classify flexible measures, in which each one of the flexible measure is treated as either input or output to maximize the overall network efficiency of the DMU under evaluation. The current paper develops an additive efficiency decomposition approach wherein the overall efficiency is expressed as a (weighted) sum of the efficiencies of the individual stages. This approach can be applied under both CRS and (VRS) assumptions. Numerical examples are used to illustrate the procedures.

Keywords : Data envelopment analysis; Flexible measures; CRS and VRS assumptions; General two-stage network.

1 Introduction

Data envelopment analysis (DEA) is a non-parametric technique to measure the relative efficiency of a set of similar units referred to as decision making units (DMUs). In the standard DEA, it is assumed that the input and output status of each performance measure related to the DMUs has been known. However, in some situations the role of a variable may be flexible. Cook and Zhu [5] considered flexible variables and proposed a mixed integer linear programming (MILP) problem for classifying these variables. Also, they introduced the aggregate model from the perspective of the manager of the collection of DMUs. Toloo [16] claimed in practice their individual method may produce incorrect efficiency scores due to a computational problem as a result of introducing a large positive number to the model and introduces a revised model that does not need such a large positive number. Amirteimoori and Emrouznejad [1] proposed a mixed integer linear programming model by fo-

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cusing on the impact of flexible measures on the definition of the production set and the assessment of technical efficiency. Toloo [17] considered alternative solutions for classifying input and output in data envelopment analysis. Kordrostami and Noveiri [11] proposed a model to evaluate the efficiency of decision-making units where flexible and negative data exist. Amirteimoori et al. [2] suggested a flexible slack-based measure of efficiency to maximize the performance. The performance measurement in the presence of interval and flexible factors have also been addressed by Kordrostami and Noveiri [12]. Tohidi and Matroud [15] proposed a new non-oriented model that not only selected the status of each flexible measure as an input or output but also determined returns to scale status. Toloo et al. [18] proposed a non-radial directional distance method to classify flexible factors, including an application of the banking industry. Recently, Kordrostami et al. [13] proposed the integer-valued FSBM approach for evaluating the relative efficiency of DMUs where flexible and integer measures are present.

In recent years, a number of DEA studies have focused on DMUs with internal network structures. Among a wide variety of internal structures studied, one basic and popular internal structure is called a two-stage network structure. Kao and Hwang [10] modeled the overall efficiency of two-stage process as the product of the efficiencies of two individual stages. A closer examination by Chen et al. [4] revealed that Kao and Hwang [10]s two-stage DEA model assumed constant returns to scale (CRS) and was not applicable to variable returns to scale (VRS) assumption. They thus introduced an additive approach for aggregating the efficiencies in two-stage process. Specifically, they modeled the overall efficiency of two-stage process as a weighted sum of the efficiencies of the two individual stages. Cook et al. [6] extended the approach to a general network structures. The additive efficiency decomposition approach was developed wherein the overall efficiency was expressed as a (weighted) average of individual stage efficiencies. Li et al. [14] proposed models to evaluate the performance of general two-stage network structures. Despotis et al. [7] showed that the weighting technique used in Chen et al. [4] was biased toward the second stage. They presented a new approach to estimate unique and unbiased efficiency scores for the individual stages. Their results were made implicitly reference about the monotonic property of decomposition weights for the two-stage DEA model without exogenous inputs and outputs. Ang and Chen [3] developed this property to a general two-stage and a multi-stage model. They found that the decomposition weights for the input-oriented multi-stage models were non-increasing in the sequence of stages and the monotonic property of decomposition weights can interfere with the estimated stage and overall efficiency scores. Then they proposed to use constant weights to calculate additive overall efficiency. They showed the differences between the efficiency scores of additive model with constant weights and Chen et al. [4] model by the differences between the product of stage efficiency scores and weights. Gue et al. [9] examined additive efficiency decomposition where the overall efficiency was defined as a weighted average of stage efficiencies and the weights were used to reflect relative importance of individual stages. Fang [8] developed a methodology for assessing the overall and stage efficiencies by considering the different and DMU-specific degree of priority given to the stages.

Flexible measures are encountered in many real world situations in two-stage network structures. For example, in a conventional study of efficiency of bank branch operations, inputs to the first stage are resources such as various staff types, and outputs to the second stage are the standard counter transactions such as deposits and withdrawals. In order to explain the possible application of our proposed model, assume a factor such as the number of worthwhile customers that can be considered as a flexible measure to the first stage to evaluate bank branch operations. Cook and Zhu [5] declared "from one prospective, such a measuring plays the role of proxy for future investment, hence can reasonably be classified as an output. On the other hand, it can

legitimately be considered as an environmental input that aids the branch in generating its existing investment portfolio". In this case, a factor such as deposits can be assumed as a flexible measure to the second stage. According to Cook and Zhu [5], it is a source of revenue for the branch and can thus also be regarded as an output. It should be considered here that arguments have been made claiming that staff time expended in processing customers who are making deposits or opening deposit accounts could also be used advantageously to sell more profitable products to customers at the same time; this factor can be supposed as an input.
The presentation of models with flexible measures in two-stage network structures is essential and beneficial because there are many situations in the real world such as that described above. So here we propose FNDEA models in which each one of the flexible measures is treated as either input or output to maximize the overall network efficiency of the DMU under evaluation in two-stage network structures. The current paper develops an additive efficiency decomposition approach wherein the overall efficiency is expressed as a (weighted) sum of the efficiencies of the individual stages. This approach can be applied under both CRS and variable returns to scale (VRS) assumptions. The status of one flexible measure in two CRS and VRS environments in general two-stage network structures may have different results. In other words, a flexible measure is considered as input in CRS environment, whereas it is selected as output in VRS environment.
The paper is organized as follows. In Section 2 we review the previous studies on flexible measures. Section 3 provides FNDEA models under the CRS and VRS assumptions to evaluate the performance of DMU_o and to classify exible measures in general two-stage network structure. In Section 4, numerical examples are used for clarication. Finally, conclusions are given in Section 5.

2 Flexible measures

Cook and Zhu [5] proposed the following classifier MILP program to determine the status of L flexible measures:

$$\max \sum_{r=1}^{s} u_r y_{ro} + \sum_{l=1}^{L} \delta_l w_{lo}$$

s.t:

$$\sum_{r=1}^{s} u_r y_{rj} + 2\sum_{l=1}^{L} \delta_l w_{lj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{l=1}^{L} \gamma_l w_{lj} \leq 0,$$

$$j = 1, \ldots, n,$$

$$\sum_{i=1}^{m} v_i x_{i0} + \sum_{l=1}^{L} \gamma_l w_{lo} - \sum_{l=1}^{L} \delta_l w_{lo} = 1, \tag{2.1}$$

$$0 \leq \delta_1 \leq M d_l, \forall l$$

$$\delta_l \leq \gamma_l \leq \delta_l + M\left(1 - d_l\right), \forall l$$

$$d_l \in \{0, 1\}, \gamma_l, \delta_l \geq 0, \forall l$$

$$u_r \geq 0, v_i \geq 0, \forall r, i.$$

For each L, a binary variable $d_l \in \{0, 1\}$ is introduced to identify that factor l is an input,$d_l = 0$, or an output $d_l = 1$ and M is a large positive number.

3 The proposed FNDEA models

In this section new models under the CRS and VRS assumption are suggested to evaluate the performance of DMU_o and to classify flexible measures in general two-stage network structures. We consider a general two-stage network structure shown in Figure 1 . Each $DMU_j, (j = 1, \ldots, n)$ has m inputs $x_{ij}, (i = 1, \ldots, m)$ to the first stage and H outputs $y'_{hj}, (h = 1, \ldots, H)$ that leave the system. In addition to these H outputs, the first stage has K outputs $z_{kj}, (k = 1, \ldots, K)$ called intermediate measures or links that become inputs to the second stage. The second stage has its own inputs $x'_{tj}, (t = 1, \ldots, T)$. The outputs from the second stage are $y_{rj} (r = 1, \ldots, s)$. Suppose also that there exist L_1 flexible measures to the first

Table 1: Data in general two-stage network with flexible measures.

	X1	X2	X3	Z1	Z2	$Y'1$	Y'2	X'1	X'2	Y1	Y2	w^1	w^2
DMU01	22.3	13.2	54.6	110.1	66.1	21.8	44.6	18	31	13.3	12.5	22.02	17.73
DMU02	68.3	8.3	15.8	75.4	116.4	19.8	12	19.6	25.8	2.4	18.2	69.03	45.17
DMU03	52	19.2	31.2	94.3	59.9	47.3	47.4	11.5	22.5	2.3	36	48.53	21.21
DMU04	31.8	12	40.3	66.4	127.2	10.5	35.8	16.8	37.1	37	19.5	31.08	71.07
DMU05	95.3	12	29	108.9	52.3	15	22.5	14	27.2	15.8	36.7	30.45	28.56
DMU06	52.8	6.1	22.6	102.4	78.8	69.6	27	14.8	44.9	12.6	20.4	15.68	30.01
DMU07	50.5	9.3	48.7	124.6	120.6	52.2	49.8	5.9	38.5	9.5	20.5	34.49	11.58
DMU08	80.1	17.4	58.4	64.5	131.2	37.7	14.6	10.3	65.6	16.7	39.9	69.94	10.59
DMU09	53.9	14	36.9	129.8	122.1	60.9	24.1	11.9	49.5	16.8	15	14.7	53.98
DMU10	20.9	9.5	48.8	66.4	132.5	12.2	68.7	10.1	54.5	10	28.3	45.01	12.47
DMU11	82.5	7.1	16.8	71.9	138.9	47.7	60.7	5.6	19.1	19.7	33.6	79.87	38.93
DMU12	27	10.6	25.6	51.9	84.4	47.3	63.3	11	39.6	12.2	43.7	50.51	18.24
DMU13	49.6	10.7	20.6	125.5	97.3	15.3	32.6	17.7	38.9	18.9	24.7	79.38	31.99
DMU14	55.7	19.4	46.6	91.5	117.3	79	60.3	11.8	26.4	7.5	38.7	63.36	13.26
DMU15	55.1	18.2	52.5	90.1	61	12.2	24.9	17	33.5	17.2	43.9	19.15	33.7
DMU16	66.3	8	34.9	131.1	63.7	57	30.7	10.7	12.5	11.2	15.5	54.78	22.75
DMU17	93.3	6.3	43.5	53.5	133.9	38.6	32.1	13.4	45	19.7	15.4	21.17	15.2
DMU18	10.8	11.9	31.5	118.7	89.4	34.9	23.6	11	67.3	8	20.5	27.51	49.25
DMU19	98.5	6.8	21.3	75.4	133	28	28.9	16.1	26.7	9.5	20.3	56.83	13.9
DMU20	27.8	17.1	24.9	81	52.2	30.6	14.3	16.9	35.3	17.4	15.4	40.47	30.85
DMU21	42	7.2	59.7	98.4	147.5	29.2	39.4	14.8	42.3	10.7	44.5	35.18	56.28
DMU22	98.7	8.5	51	132.8	60.6	27.3	69.3	19.8	61.9	19.9	33.3	34.6	62.91
DMU23	53.5	15.6	25.7	93.5	121.6	31.3	34.6	19.7	56.5	13	47.6	8.2	53.92
DMU24	25.1	16.7	56.8	81.6	145.6	62.1	74.8	11.7	17.4	77.6	29.9	20.51	29.87
DMU25	96.3	15.3	45.1	120.5	133.6	25.7	56.8	19.7	16.9	14.9	38.5	51.24	16.05
DMU26	97.9	6.8	53.1	103.8	89.8	45.7	49.6	17.7	56.3	44.9	12.5	68.16	17.18
DMU27	37.4	15	15.5	63.1	128.2	53.1	22	5.5	57.7	5.8	11.8	26.16	54.89
DMU28	70	12.8	21.5	126.1	97.2	28.3	44.3	11.4	56.7	4.9	47.2	56.6	48.17
DMU29	24	5.8	33.8	91.2	82.6	73.7	76.2	19.4	42.4	7.8	10.3	64.31	12.21
DMU30	48.6	18.6	55.9	126	73.8	15.4	57.6	17.1	76.2	13.2	25.6	31.26	65.47

stage $w^1_{l_1j}, (l_1 = 1, \ldots, L_1)$ and L_2 flexible measures to the second stage $w^2_{l_2j}, (l_2 = 1, \ldots, L_2)$ whose input/output statuses are not known, some DMUs may use these measures as inputs and other DMUs may use them as outputs. Let $\gamma^1_{l_1}$ be the weight for each measure l_1 to the first stage and $\gamma^2_{l_2}$ be the weight for each measure l_2 to the second stage.

3.1 Under the CRS assumption

We propose the following mathematical programming model for evaluating the overall efficiency and determining the status of exible measures in general two-stage network structure under the CRS assumption.

$$\theta_o^* = \max \Bigg[w_1 \frac{\sum_{k=1}^{K} \eta_k z_{ko} + \sum_{h=1}^{H} u'_h y'_{ho} + \sum_{l_1=1}^{L_1} d^1_{l_1} \gamma^1_{l_1} w^1_{l_10}}{\sum_{i=1}^{m} v_i x_{io} + \sum_{l_1=1}^{L_1} \left(1 - d^1_{l_1}\right) \gamma^1_{l_1} w^1_{l_10}} + w_2 \left(\sum_{r=1}^{s} u_r y_{ro} + \sum_{l_2=1}^{L_2} d^2_{l_2} \gamma^2_{l_2} w^2_{l_2o} \right) \Bigg/ \left(\sum_{k=1}^{K} \eta_k z_{ko} + \sum_{t=1}^{T} v'_t x'_{to} + \sum_{l_2=1}^{L_2} \left(1 - d^2_{l_2}\right) \gamma^2_{l_2} w^2_{l_20} + \sum_{l_1=1}^{L_1} d^1_{l_1} \gamma^1_{l_1} w^1_{l_1o} \right) \Bigg] \quad (3.2)$$

Table 2: The results obtained from the models (3.4), (3.5) and (3.6) under CRS.

DMU	θ_o^*	θ_o^{1*}	θ_o^{2*}	d^{1*}	d^{2*}
1	0.7241	1	0.2311	0	0
2	0.9617	0.9617	0.745	1	0
3	0.9678	0.8086	0.9679	0	1
4	0.997	0.9462	1	0	1
5	0.8325	0.714	0.8782	1	0
6	0.9999	1	0.5028	0	1
7	0.8322	0.883	0.7714	1	0
8	0.9996	0.5803	0.9999	1	0
9	0.9636	1	0.8444	0	1
10	0.9999	1	0.4743	1	0
11	1	1	1	0	1
12	1	1	1	1	0
13	0.9999	1	0.5081	1	0
14	0.9998	0.7321	1	0	1
15	0.8324	0.5787	0.9999	1	0
16	0.8928	1	0.8927	0	1
17	0.8419	1	0.4289	1	0
18	0.9999	1	0.8584	0	1
19	0.7429	0.743	0.4018	1	0
20	0.7555	0.7393	0.7974	0	1
21	0.9931	1	0.9851	0	1
22	1	1	1	0	0
23	1	1	0.935	1	0
24	1	1	1	1	0
25	0.9999	0.7673	0.9999	0	1
26	0.9998	0.9646	0.9999	0	1
27	1	1	1	1	0
28	0.9987	1	0.7654	0	1
29	0.9998	1	0.2453	0	1
30	0.8131	0.8746	0.7272	0	0

s.t:

$$\frac{\sum_{k=1}^{K} \eta_k z_{kj} + \sum_{h=1}^{H} u'_h y'_{hj} + \sum_{l_1=1}^{L_1} d^1_{l_1} \gamma^1_{l_1} w^1_{l_1 j}}{\sum_{i=1}^{m} v_i x_{ij} + \sum_{l_1=1}^{L_1} \left(1 - d^1_{l_1}\right) \gamma^1_{l_1} w^1_{l_1 j}} \leq 1,$$

$$j = 1, \ldots, n,$$

$$\left(\sum_{r=1}^{s} u_r y_{rj} + \sum_{l_2=1}^{L_2} d^2_{l_2} \gamma^2_{l_2} w^2_{l_2 j} \right) \Bigg/ \left(\sum_{k=1}^{K} \eta_k z_{kj} + \sum_{t=1}^{T} v'_t x'_{ty} + \sum_{l_2=1}^{L_2} \left(1 - d^2_{l_2}\right) \gamma^2_{l_2} w^2_{l_2^2} + \sum_{l_1=1}^{L_1} d^1_{l_1} \gamma^1_{l_1} w^1_{l_1,j} \right) \leq 1,$$

$$j = 1, \ldots, n$$

$$d^1_{l_1} \in \{0,1\}, d^2_{l_2} \in \{0,1\}, \gamma^1_{l_1} \geq \varepsilon, \gamma^2_{l_2} \geq \varepsilon, \forall l_1, l_2$$

$$\eta_k \geq \varepsilon, v_i \geq \varepsilon, u_r \geq \varepsilon,$$

$$k = 1, \ldots, K; i = 1, \ldots, m; r = 1, \ldots, s;$$

$$v'_t \geq \varepsilon, u'_h \geq \varepsilon, t = 1, \ldots, T; h = 1, \ldots, H.$$

For each L_1 and L_2, binary variables $d^1_{l_1} \in \{0,1\}$ and $d^2_{l_2} \in \{0,1\}$ are introduced to identify that factor l_1 and l_2 are inputs to the first and second stage, $d^1_{l_1} = 0$ and $d^2_{l_2} = 0$ or outputs to the first and second stage $d^1_{l_1} = 1$ and $d^2_{l_2} = 1$. If $d^1_{l_1} = 1$ then the factor l_1 is considered as an output to the first stage and as an input to the second stage so the factor l_1 is an intermediate measure. If $d^2_{l_2} = 0$ then the factor l_2 is an input to the second stage.

Table 3: The results obtained from the models (3.8),(3.9) and (3.10) under VRS

DMU	θ_o^*	θ_o^{1*}	θ_o^{2*}	d^{1*}	d^{2*}
1	1	1	1	0	0
2	0.9999	1	0.7965	1	1
3	0.9871	0.8218	0.9872	0	0
4	0.9999	0.9663	0.9999	0	1
5	0.8515	0.5636	0.9117	0	0
6	0.9999	1	0.7359	1	1
7	0.8799	0.7654	0.9876	0	0
8	0.9999	0.7613	0.9999	1	0
9	0.9808	1	0.903	0	1
10	1	1	1	0	0
11	1	1	1	1	1
12	1	1	1	1	1
13	0.9999	1	0.6903	1	1
14	1	1	1	0	0
15	0.9998	0.7499	0.9999	0	0
16	0.8765	0.8987	0.5678	1	1
17	0.9876	1	0.98	0	0
18	1	1	1	1	1
19	0.9956	0.8767	0.6756	0	0
20	0.9662	0.9748	0.8697	1	0
21	1	1	1	1	1
22	1	1	1	1	1
23	1	1	1	0	0
24	1	1	1	1	1
25	0.9745	0.9856	0.8854	1	1
26	0.9878	0.6754	0.9570	1	0
27	1	1	1	1	1
28	1	1	1	1	1
29	0.8996	0.7658	0.8999	1	0
30	0.8869	1	0.8869	0	1

Model (3.2) is clearly nonlinear. It can, however, be linearized by way of the change of variables $d^1_{l_1}\gamma^1_{l_1} = \delta^1_{l_1}$ and $d^2_{l_2}\gamma^2_{l_2} = \delta^2_{l_2}$ for each l_1 and l_2 and imposing the following constraints.

$$0 \leq \delta^1_{l_1} \leq M d^1_{l_1},$$

$$\delta^1_{l_1} \leq \gamma^1_{l_1} \leq \delta^1_{l_1} + M\left(1 - d^1_{l_1}\right)$$

$$l_1 = 1, \ldots, L_1$$

$$0 \leq \delta^2_{l_2} \leq M d^2_{l_2},$$

$$\delta^2_{l_2} \leq \gamma^2_{l_2} \leq \delta^2_{l_2} + M\left(1 - d^2_{l_2}\right)$$

$$l_2 = 1, \ldots, L_2$$

Note that if $d^1_{l_1} = 1$ and $d^2_{l_2} = 1$ then $\gamma^1_{l_1} = \delta^1_{l_1}$ and $\gamma^2_{l_2} = \delta^2_{l_2}$ and if $d^1_{l_1} = 0$ and $d^2_{l_2} = 0$ then $\delta^1_{l_1} = 0$ and $\delta^2_{l_2} = 0$. In these constraints M is a large positive number. In this model, w_1 and w_2 are weights satisfying $w_1 + w_2 = 1$. We let

$$w = \sum_{i=1}^{m} v_i x_{io} + \sum_{l_1=1}^{L_1} \gamma^1_{l_1} w^1_{l_1 0} -$$

$$\sum_{l_1=1}^{L_1} \delta^1_{l_1} w^1_{l_1 0} + \sum_{k=1}^{K} \eta_k z_{ko} + \sum_{t=1}^{T} v'_t x'_{to} +$$

$$\sum_{l_2=1}^{L_2} \gamma^2_{l_2} w^2_{l_2 0} - \sum_{l_2=1}^{L_2} \delta^2_{l_2} w^2_{l_2 0} + \sum_{l_1=1}^{L_1} \delta^1_{l_1} w^1_{l_1 o},$$

$$w_1 = \frac{\sum_{i=1}^{m} v_i x_{io} + \sum_{l_1=1}^{L_1} \gamma^1_{l_1} w^1_{l_1 0} - \sum_{l_1=1}^{L_1} \delta^1_{l_1} w^1_{l_1 o}}{W},$$

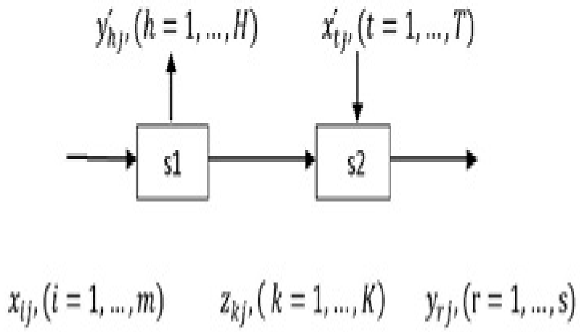

Figure 1: General two-stage network structure

and

$$w_2 = \left(\sum_{k=1}^{K} \eta_k z_{ko} + \sum_{t=1}^{T} v'_t x'_{to} + \sum_{l_2=1}^{L_2} \gamma^2_{l_2} w^2_{l_2 0} - \sum_{l_2=1}^{L_2} \delta^2_{l_2} w^2_{l_2 0} + \sum_{l_1=1}^{L_1} \delta^1_{l_1} w^1_{l_1 o} \right) \Big/ (w). \quad (3.3)$$

We, therefore, have the following mixed integer linear program Model (3.4):

$$\theta^*_o = \max \sum_{k=1}^{K} \eta_k z_{ko} + \sum_{h=1}^{H} u'_h y'_{ho} + \sum_{l_1=1}^{L_1} \delta^1_{l_1} w^1_{l_1 0} + \sum_{r=1}^{s} u_r y_{ro} + \sum_{l_2=1}^{L_2} \delta^2_{l_2} w^2_{l_2 o}$$

s.t.

$$\sum_{i=1}^{m} v_i x_{io} + \sum_{l_1=1}^{L_1} \gamma^1_{l_1} w^1_{l_1 o} - \sum_{l_1=1}^{L_1} \delta^1_{l_1} w^1_{l_1 o} + \sum_{k=1}^{K} \eta_k z_{ko} + \sum_{t=1}^{T} v'_t x'_{to} + \sum_{l_2=1}^{L_2} \gamma^2_{l_2} w^2_{l_2 o} - \sum_{l_2=1}^{L_2} \delta^2_{l_2} w^2_{l_2 o} + \sum_{l_1=1}^{L_1} \delta^1_{l_1} w^1_{l_1 o} = 1 \quad (3.4)$$

$$\sum_{k=1}^{K} \eta_k z_{kj} + \sum_{h=1}^{H} u'_h y'_{hj} + 2 \sum_{l_1=1}^{L_1} \delta^1_{l_1} w^1_{l_1 j} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{l_1=1}^{L_1} \gamma^1_{l_1} w^1_{l_1 j} \le 0, \quad j = 1, \ldots, n,$$

$$\sum_{r=1}^{s} u_r y_{rj} + 2 \sum_{l_2=1}^{L_2} \delta^2_{l_2} w^2_{l_2 j} - \sum_{k=1}^{K} \eta_k z_{kj} - \sum_{t=1}^{T} v'_t x'_{t_j} - \sum_{l_2=1}^{L_2} \gamma^2_{l_2} w^2_{l_2 j} - \sum_{l_1=1}^{L_1} \delta^1_{l_1} w^1_{l_1 j} \le 0,$$

$$j = 1, \ldots, n, \quad 0 \le \delta^1_{l_1} \le M d^1_{l_1},$$

$$\delta^1_{l_1} \le \gamma^1_{l_1} \le \delta^1_{l_1} + M\left(1 - d^1_{l_1}\right) \quad l_1 = 1, \ldots, L_1$$

$$0 \le \delta^2_{l_2} \le M d^2_{l_2} \quad , \quad \delta^2_{l_2} \le \gamma^2_{l_2} \le \delta^2_{l_2} + M\left(1 - d^2_{l_2}\right)$$

$$l_2 = 1, \ldots, L_2, d^1_{l_1} \in \{0,1\}, d^2_{l_2} \in \{0,1\}, \gamma^1_{l_1} \ge \varepsilon,$$

$$\delta^1_{l_1} \ge \varepsilon, \gamma^2_{l_2} \ge \varepsilon, \delta^2_{l_2} \ge \varepsilon, \forall l_1, l_2, \eta_k \ge \varepsilon, v_i \ge \varepsilon,$$

$$u_r \ge \varepsilon, \quad k = 1, \ldots, K; i = 1, \ldots, m; r = 1, \ldots, s;$$

$$v'_t \ge \varepsilon, u'_h \ge \varepsilon, t = 1, \ldots, T; h = 1, \ldots, H$$

Definition 3.1. *DMU_o is overall efficient if and only if $\theta^*_o = 1$. θ^{1*}_o is determined by solving the following LP model (3.5).*

$$\theta^{1*}_o = \max \sum_{k=1}^{K} \eta_k z_{ko} + \sum_{h=1}^{H} u'_h y'_{ho} + \sum_{l_1=1}^{L_1} \delta^1_{l_1} w^1_{l_1 o}$$

s.t:

$$\sum_{i=1}^{m} v_i x_{io} + \sum_{l_1=1}^{L_1} \gamma^1_{l_1} w^1_{l_1 o} - \sum_{l_1=1}^{L_1} \delta^1_{l_1} w^1_{l_1 o} = 1 \quad (3.5)$$

$$\theta^*_o = (1 - \theta^*_o) \sum_{k=1}^{K} \eta_k z_{ko} + \sum_{h=1}^{H} u'_h y'_{ho} + \sum_{l_1=1}^{L_1} \delta^1_{l_1} w^1_{l_1 o} + \sum_{r=1}^{s} u_r y_{ro} + (1 + \theta^*_o) \sum_{l_2=1}^{L_2} \delta^2_{l_2} w^2_{l_2 o} - \theta^*_o \sum_{l_2=1}^{L_2} \gamma^2_{l_2} w^2_{l_2 0} - \theta^*_o \sum_{t=1}^{T} v'_t x'_{to} - \theta^*_o \sum_{l_1=1}^{L_1} \delta^1_{l_1} w^1_{l_1 o}$$

$$\sum_{k=1}^{K} \eta_k z_{kj} + \sum_{h=1}^{H} u'_h y'_{hj} + 2 \sum_{l_1=1}^{L_1} \delta^1_{l_1} w^1_{l_1 j} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{l_1=1}^{L_1} \gamma^1_{l_1} w^1_{l_1 j} \le 0, j = 1, \ldots, n$$

$$\sum_{r=1}^{s} u_r y_{rj} + 2 \sum_{l_2=1}^{L_2} \delta^2_{l_2} w^2_{l_2 j} - \sum_{k=1}^{K} \eta_k z_{kj}$$

$$-\sum_{t=1}^{T} v_t' x_{t_j}' - \sum_{l_2=1}^{L_2} \gamma_{l_2}^2 w_{l_2 j}^2 - \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 j}^1 \leq 0,$$
$$j = 1, \ldots, n, \quad 0 \leq \delta_{l_1}^1 \leq M d_{l_1}^1,$$
$$\delta_{l_1}^1 \leq \gamma_{l_1}^1 \leq \delta_{l_1}^1 + M\left(1 - d_{l_1}^1\right), l_1 = 1, \ldots, L_1$$
$$0 \leq \delta_{l_2}^2 \leq M d_{l_2}^2, \delta_{l_2}^2 \leq \gamma_{l_2}^2 \leq \delta_{l_2}^2 + M\left(1 - d_{l_2}^2\right)$$
$$l_2 = 1, \ldots, L_2, d_{l_1}^1 \in \{0,1\}, d_{l_2}^2 \in \{0,1\}$$
$$\gamma_{l_1}^1 \geq 0, \delta_{l_1}^1 \geq 0, \gamma_{l_2}^2 \geq 0, \delta_{l_2}^2 \geq 0 \quad \forall l_1, l_2$$
$$\eta_k \geq \varepsilon, v_i \geq \varepsilon, u_r \geq \varepsilon, \quad k = 1, \ldots, K;$$
$$i = 1, \ldots, m; r = 1, \ldots, s; v_t' \geq \varepsilon,$$
$$u_h' \geq \varepsilon, t = 1, \ldots, T; h = 1, \ldots, H.$$

Definition 3.2. *DMU_o is efficient to the first stage if and only if $\theta_o^{1*} = 1$*

θ_o^{2*} is computed by

$$\theta_o^{2*} = \left(\theta_o^* - w_1^* \theta_o^{1*}\right) / w_2^*,$$

where w_1^* and w_2^* are the weights obtained from model (3.4) by way of (3.3). θ_o^{2*} can also be determined by solving the following LP model (3.6)

$$\theta_o^{2*} = \max \sum_{r=1}^{s} u_r y_{ro} + \sum_{l_2=1}^{L_2} \delta_{l_2}^2 w_{l_2 o}^2$$

s.t.

$$\sum_{k=1}^{K} \eta_k z_{ko} + \sum_{t=1}^{T} v_t' x_{to}' + \sum_{l_2=1}^{L_2} \gamma_{l_2}^2 w_{l_2 o}^2$$
$$- \sum_{l_2=1}^{L_2} \delta_{l_2}^2 w_{l_2 o}^2 + \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 o}^1 = 1 \qquad (3.6)$$
$$\theta_o^* = \sum_{k=1}^{K} \eta_k z_{ko} + \sum_{h=1}^{H} u_h' y_{ho}' +$$
$$(1 + \theta_o^*) \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 o}^1 + \sum_{r=1}^{s} u_r y_{ro} +$$
$$\sum_{l_2=1}^{L_2} \delta_{l_2}^2 w_{l_2 o}^2 - \theta_o^* \sum_{i=1}^{m} v_i x_{io} - \theta_o^* \sum_{l_1=1}^{L_1} \gamma_{l_1}^1 w_{l_1 o}^1$$
$$\sum_{k=1}^{K} \eta_k z_{kj} + \sum_{h=1}^{H} u_h' y_{hj}' + 2 \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 j}^1 -$$
$$\sum_{i=1}^{m} v_i x_{ij} - \sum_{l_1=1}^{L_1} \gamma_{l_1}^1 w_{l_1 j}^1 \leq 0, j = 1, \ldots, n$$
$$\sum_{r=1}^{s} u_r y_{rj} + 2 \sum_{l_2=1}^{L_2} \delta_{l_2}^2 w_{l_2 j}^2 - \sum_{k=1}^{K} \eta_k z_{kj} -$$
$$\sum_{t=1}^{T} v_t' x_{t_j}' - \sum_{l_2=1}^{L_2} \gamma_{l_2}^2 w_{l_2 j}^2 - \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 j}^1 \leq 0,$$
$$j = 1, \ldots, n, 0 \leq \delta_{l_1}^1 \leq M d_{l_1}^1,$$
$$\delta_{l_1}^1 \leq \gamma_{l_1}^1 \leq \delta_{l_1}^1 + M\left(1 - d_{l_1}^1\right)$$
$$l_1 = 1, \ldots, L_1, \quad 0 \leq \delta_{l_2}^2 \leq M d_{l_2}^2,$$
$$\delta_{l_2}^2 \leq \gamma_{l_2}^2 \leq \delta_{l_2}^2 + M\left(1 - d_{l_2}^2\right)$$
$$l_2 = 1, \ldots, L_2 d_{l_1}^1 \in \{0,1\}, d_{l_2}^2 \in \{0,1\}$$
$$\gamma_{l_1}^1 \geq \varepsilon, \delta_{l_1}^1 \geq \varepsilon, \gamma_{l_2}^2 \geq \varepsilon, \delta_{l_2}^2 \geq \varepsilon \quad \forall l_1, l_2$$
$$\eta_k \geq \varepsilon, v_i \geq \varepsilon, u_r \geq \varepsilon, \quad k = 1, \ldots, K;$$
$$i = 1, \ldots, m; r = 1, \ldots, s, v_t' \geq \varepsilon,$$
$$u_h' \geq \varepsilon, t = 1, \ldots, T; h = 1, \ldots, H$$

Definition 3.3. *DMU_o is efficient to the second stage if and only if $\theta_o^{2*} = 1$*

θ_o^{1*} can then be computed by
$\theta_o^{1*} = (\theta_o^* - w_2^* \theta_o^{2*}) / w_1^*$

3.2 Under the VRS assumption

We propose the following mathematical programming model for evaluating the overall efficiency and determining the status of flexible measures in general two-stage network structure under VRS assumption.

$$\theta_o^* = \max \left[w_1 \left(\sum_{k=1}^{K} \eta_k z_{ko} + \sum_{h=1}^{H} u_h' y_{ho}' \right.\right.$$
$$\left. + \sum_{l_1=1}^{L_1} d_{l_1}^1 \gamma_{l_1}^1 w_{l_1 0}^1 + u^A \right) \Big/ \left(\sum_{i=1}^{m} v_i x_{io} \right.$$
$$\left. + \sum_{l_1=1}^{L_1} \left(1 - d_{l_1}^1\right) \gamma_{l_1}^1 w_{l_{1o}}^1 \right)$$
$$+ w_2 \left(\sum_{r=1}^{s} u_r y_{ro} + \sum_{l_2=1}^{L_2} d_{l_2}^2 \gamma_{l_2}^2 w_{l_2 o}^2 + u^B \right) \Big/$$

$$\left(\sum_{k=1}^{K}\eta_k z_{ko}+\sum_{t=1}^{T}v'_t x'_{to}\sum_{l_2=1}^{L_2}\left(1-d^2_{l_2}\right)\gamma^2_{l_2}w^2_{l_{2o}} + \sum_{l_1=1}^{L_1} d^1_{l_1}\gamma^1_{l_1}w^1_{l_1o}\right)\Bigg] \quad (3.7)$$

S.t

$$\frac{\sum_{k=1}^{K}\eta_k z_{kj}+\sum_{h=1}^{H}u'_h y'_{hj}+\sum_{l_1=1}^{L_1}d^1_{l_1}\gamma^1_{l_1}w^1_{l_1j}+u^A}{\sum_{i=1}^{m}v_i x_{ij}+\sum_{l_1=1}^{L_1}\left(1-d^1_{l_1}\right)\gamma^1_{l_1}w^1_{l_1j}} \le 1, \quad j=1,\ldots,n$$

$$\left(\sum_{r=1}^{s}u_r y_{rj}+\sum_{l_2=1}^{L_2}d^2_{l_2}\gamma^2_{l_2}w^2_{l_2j}+u^B\right)\Bigg/ \left(\sum_{k=1}^{K}\eta_k z_{kj}+\sum_{t=1}^{T}v'_t x'_{tj}+\sum_{l_2=1}^{L_2}\left(1-d^2_{l_2}\right)\gamma^2_{l_2}w^2_{l_2j} + \sum_{l_1=1}^{L_1}d^1_{l_1}\gamma^1_{l_1}w^1_{l_1j}\right) \le 1, j=1,\ldots,n$$

$$d^1_{l_1}\in\{0,1\}, d^2_{l_2}\in\{0,1\}, \gamma^1_{l_1}\ge\varepsilon,$$
$$\gamma^2_{l_2}\ge\varepsilon \quad \forall l_1,l_2$$
$$\eta_k\ge\varepsilon, v_i\ge\varepsilon, u_r\ge\varepsilon, \quad k=1,\ldots,K;$$
$$i=1,\ldots,m; r=1,\ldots,s, v'_t\ge\varepsilon,$$
$$u'_h\ge\varepsilon, t=1,\ldots,T; h=1,\ldots,H$$

u^1, u^2 free in sign.

Model (3.7) is clearly nonlinear. It can, however, be linearized by way of the change of variables $d^1_{l_1}\gamma^1_{l_1}=\delta^1_{l_1}$ and $d^2_{l_2}\gamma^2_{l_2}=\delta^2_{l_2}$ for each l_1 and l_2 and imposing the following constraints.

$$0\le\delta^1_{l_1}\le Md^1_{l_1},$$
$$\delta^1_{l_1}\le\gamma^1_{l_1}\le\delta^1_{l_1}+M\left(1-d^1_{l_1}\right)$$
$$l_1=1,\ldots,L_1$$
$$0\le\delta^2_{l_2}\le Md^2_{l_2},$$
$$\delta^2_{l_2}\le\gamma^2_{l_2}\le\delta^2_{l_2}+M\left(1-d^2_{l_2}\right)$$
$$l_2=1,\ldots,L_2$$

Note that if $d^1_{l_1}=1$ and $d^2_{l_2}=1$ then $\gamma^1_{l_1}=\delta^1_{l_1}$ and $\gamma^2_{l_2}=\delta^2_{l_2}$ and if $d^1_{l_1}=0$ and $d^2_{l_2}=0$ then $\delta^1_{l_1}=0$ and $\delta^2_{l_2}=0$. In these constraints M is a large positive number.

By setting the weights in (3.3) and after doing Charnes and Cooper's transformation, Model (3.7) is transformed into the following mixed integer linear program Model (3.8).

$$\theta^*_o = \max\sum_{k=1}^{K}\eta_k z_{ko}+\sum_{h=1}^{H}u'_h y'_{ho}+\sum_{l_1=1}^{L_1}\delta^1_{l_1}w^1_{l_10} + u^A+\sum_{r=1}^{s}u_r y_{ro}+\sum_{l_2=1}^{L_2}\delta^2_{l_2}w^2_{l_20}+u^B \quad (3.8)$$

s.t.

$$\sum_{i=1}^{m}v_i x_{io}+\sum_{l_1=1}^{L_1}\gamma^1_{l_1}w^1_{l_10}-\sum_{l_1=1}^{L_1}\delta^1_{l_1}w^1_{l_10}+ \sum_{k=1}^{K}\eta_k z_{ko}+\sum_{t=1}^{T}v'_t x'_{to}+\sum_{l_2=1}^{L_2}\gamma^2_{l_2}w^2_{l_20} -\sum_{l_2=1}^{L_2}\delta^2_{l_2}w^2_{l_2o}+\sum_{l_1=1}^{L_1}\delta^1_{l_1}w^1_{l_1o}=1$$

$$\sum_{k=1}^{K}\eta_k z_{kj}+\sum_{h=1}^{H}u'_h y'_{hj}+2\sum_{l_1=1}^{L_1}\delta^1_{l_1}w^1_{l_1j}- \sum_{i=1}^{m}v_i x_{ij}-\sum_{l_1=1}^{L_1}\gamma^1_{l_1}w^1_{l_1j}+u^A\le 0, j=1,\ldots,n,$$

$$\sum_{r=1}^{s}u_r y_{rj}+2\sum_{l_2=1}^{L_2}\delta^2_{l_2}w^2_{l_2j}-\sum_{k=1}^{K}\eta_k z_{kj}- \sum_{t=1}^{T}v'_t x_{t_j}-\sum_{l_2=1}^{L_2}\gamma^2_{l_2}w^2_{l_2j}-\sum_{l_1=1}^{L_1}\delta^1_{l_1}w^1_{l_1j}+u^B\le 0,$$

$$j=1,\ldots,n, 0\le\delta^1_{l_1}\le Md^1_{l_1},$$
$$\delta^1_{l_1}\le\gamma^1_{l_1}\le\delta^1_{l_1}+M\left(1-d^1_{l_1}\right) \quad l_1=1,\ldots,L_1$$
$$0\le\delta^2_{l_2}\le Md^2_{l_2},$$
$$\delta^2_{l_2}\le\gamma^2_{l_2}\le\delta^2_{l_2}+M\left(1-d^2_{l_2}\right) \quad l_2=1,\ldots,L_2,$$
$$d^1_{l_1}\in\{0,1\}, d^2_{l_2}\in\{0,1\} \quad \gamma^1_{l_1}\ge\varepsilon,$$

$$\delta_{l_1}^1 \geq \varepsilon, \gamma_{l_2}^2 \geq \varepsilon, \delta_{l_2}^2 \geq \varepsilon \quad \forall l_1, l_2$$
$$\eta_k \geq \varepsilon, v_i \geq \varepsilon, u_r \geq \varepsilon, \quad k = 1, \ldots, K;$$
$$i = 1, \ldots, m; r = 1, \ldots, s$$
$$v_t' \geq \varepsilon, u_h' \geq \varepsilon, \quad t = 1, \ldots, T; h = 1, \ldots, H;$$

u^1, u^2 free in sign.

Definition 3.4. *DMU_o is overall efficient if and only if $\theta_o^* = 1$*

θ_o^{1*} can be determined by solving the following LP Model (3.9):

$$\theta_o^{1*} = \max \sum_{k=1}^{K} \eta_k z_{ko} + \sum_{h=1}^{H} u_h' y_{ho}' + \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 0}^1 + u^A$$

s.t.

$$\sum_{i=1}^{m} v_i x_{io} + \sum_{l_1=1}^{L_1} \gamma_{l_1}^1 w_{l_1 o}^1 - \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 o}^1 = 1 \quad (3.9)$$

$$\theta_o^* = (1-\theta_o^*) \sum_{k=1}^{K} \eta_k z_{ko} + \sum_{h=1}^{H} u_h' y_{ho}' + \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 0}^1$$
$$+ \sum_{r=1}^{s} u_r y_{ro} + (1+\theta_o^*) \sum_{l_2=1}^{L_2} \delta_{l_2}^2 w_{l_2 o}^2 - \theta_o^* \sum_{l_2=1}^{L_2} \gamma_{l_2}^2 w_{l_2 o}^2$$
$$-\theta_o^* \sum_{t=1}^{T} v_t' x_{to}' - \theta_o^* \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 o}^1 + u^A + u^B$$

$$\sum_{k=1}^{K} \eta_k z_{kj} + \sum_{h=1}^{H} u_h' y_{hj}' + 2 \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 j}^1 -$$
$$\sum_{i=1}^{m} v_i x_{ij} - \sum_{l_1=1}^{L_1} \gamma_{l_1}^1 w_{l_1 j}^1 + u^A \leq 0, \quad j = 1, \ldots, n$$

$$\sum_{r=1}^{s} u_r y_{rj} + 2 \sum_{l_2=1}^{L_2} \delta_{l_2}^2 w_{l_2 j}^2 - \sum_{k=1}^{K} \eta_k z_{kj} - \sum_{t=1}^{T} v_t' x_{t_j}' -$$
$$\sum_{l_2=1}^{L_2} \gamma_{l_2}^2 w_{l_2 j}^2 - \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 j}^1 + u^B \leq 0, j = 1, \ldots, n$$

$$0 \leq \delta_{l_1}^1 \leq M d_{l_1}^1, \delta_{l_1}^1 \leq \gamma_{l_1}^1 \leq \delta_{l_1}^1 + M\left(1 - d_{l_1}^1\right),$$
$$l_1 = 1, \ldots, L_1, 0 \leq \delta_{l_2}^2 \leq M d_{l_2}^2,$$
$$\delta_{l_2}^2 \leq \gamma_{l_2}^2 \leq \delta_{l_2}^2 + M\left(1 - d_{l_2}^2\right) \quad l_2 = 1, \ldots, L_2,$$
$$d_{l_1}^1 \in \{0,1\}, d_{l_2}^2 \in \{0,1\} \quad \gamma_{l_1}^1 \geq \varepsilon, \delta_{l_1}^1 \geq \varepsilon,$$
$$\gamma_{l_2}^2 \geq \varepsilon, \delta_{l_2}^2 \geq \varepsilon \quad \forall l_1, l_2 \eta_k \geq \varepsilon, v_i \geq \varepsilon, u_r \geq \varepsilon,$$
$$k = 1, \ldots, K; i = 1, \ldots, m; r = 1, \ldots, s;$$
$$v_t' \geq \varepsilon, u_h' \geq \varepsilon, \quad t = 1, \ldots, T; h = 1, \ldots, H;$$

u^1, u^2 free in sign.

Definition 3.5. *$DMU_o$is efficient to the first stage if and only if $\theta_o^* = 1$.*

θ_o^{2*} can then be computed by $\theta_o^{2*} = \left(\theta_o^* - w_1^* \theta_o^{1*}\right)/w_2^*$, where w_1^* and w_2^* are the weights obtained from model (3.8) by way of (3.3).

θ_o^{2*} can also be determined by solving the following LP Model (3.10) :

$$\theta_o^{2*} = \max \sum_{r=1}^{s} u_r y_{ro} + \sum_{l_2=1}^{L_2} \delta_{l_2}^2 w_{l_2 o}^2 + u^B$$

s.t.

$$\sum_{k=1}^{K} \eta_k z_{ko} + \sum_{t=1}^{T} v_t' x_{to}' + \sum_{l_2=1}^{L_2} \gamma_{l_2}^2 w_{l_2 o}^2 -$$
$$\sum_{l_2=1}^{L_2} \delta_{l_2}^2 w_{l_2 o}^2 + \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 o}^1 = 1 \quad (3.10)$$

$$\theta_o^* = \sum_{k=1}^{K} \eta_k Z_{ko} + \sum_{h=1}^{H} u_h' y_{ho}' + \sum_{r=1}^{s} u_r y_{ro} +$$
$$(1+\theta_o^*) \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 0}^1 + \sum_{l_2=1}^{L_2} \delta_{l_2}^2 w_{l_2 o}^2 -$$
$$\theta_o^* \sum_{i=1}^{m} v_i x_{io} - \theta_o^* \sum_{l_1=1}^{L_1} \gamma_{l_1}^1 w_{l_1 o}^1 + u^A + u^B$$

$$\sum_{k=1}^{K} \eta_k z_{kj} + \sum_{h=1}^{H} u_h' y_{h_j}' + 2 \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 j}^1 - \sum_{i=1}^{m} v_i x_{ij}$$
$$- \sum_{l_1=1}^{L_1} \gamma_{l_1}^1 w_{l_1 j}^1 + u^A \leq 0, j = 1, \ldots, n$$

$$\sum_{r=1}^{s} u_r y_{rj} + 2 \sum_{l_2=1}^{L_2} \delta_{l_2}^2 w_{l_2 j}^2 - \sum_{k=1}^{K} \eta_k z_{kj} - \sum_{t=1}^{T} v_t' x_{t_j}' -$$
$$\sum_{l_2=1}^{L_2} \gamma_{l_2}^2 w_{l_2 j}^2 - \sum_{l_1=1}^{L_1} \delta_{l_1}^1 w_{l_1 j}^1 + u^B \leq 0, j = 1, \ldots, n$$

$$0 \le \delta^1_{l_1} \le Md^1_{l_1}, \delta^1_{l_1} \le \gamma^1_{l_1} \le \delta^1_{l_1} + M\left(1 - d^1_{l_1}\right)$$
$$l_1 = 1, \ldots, L_1, 0 \le \delta^2_{l_2} \le Md^2_{l_2},$$
$$\delta^2_{l_2} \le \gamma^2_{l_2} \le \delta^2_{l_2} + M\left(1 - d^2_{l_2}\right) \quad l_2 = 1, \ldots, L_2,$$
$$d^1_{l_1} \in \{0,1\}, d^2_{l_2} \in \{0,1\}, \gamma^1_{l_1} \ge \varepsilon, \delta^1_{l_1} \ge \varepsilon,$$
$$\gamma^2_{l_2} \ge \varepsilon, \delta^2_{l_2} \ge \varepsilon \quad \forall l_1, l_2$$
$$\eta_k \ge \varepsilon, v_i \ge \varepsilon, u_r \ge \varepsilon, \quad k = 1, \ldots, K;$$
$$i = 1, \ldots, m; \quad r = 1, \ldots, s;$$
$$v'_t \ge \varepsilon, u'_h \ge \varepsilon, \quad t = 1, \ldots, T; h = 1, \ldots, H;$$

u^1, u^2 free in sign.

Definition 3.6. *DMU_o is efficient to the second stage if and only if $\theta^{2*}_o = 1$*

θ^{1*}_o can then be computed by

$$\theta^{1*}_o = {}^{(\theta^*_o - w^*_2 \theta^{2*}_o)}/w^*_1$$

4 Numerical examples

In this section, we apply the proposed models in general two-stage network structure under CRS and VRS assumptions for evaluating the overall efficiency and classifying flexible measure to the data sets shown in Table 1. These data are random. There are 30 DMUs, three inputs to the first stage $(X1, X2, X3)$, two outputs to the first stage $(Y'1, Y'2)$, that leave the system, two intermediate measures $(Z1, Z2)$, two final outputs from the second stage $(Y1, Y2)$. The second stage has its own inputs $(X'1, X'2)$. There is one flexible measure w^1 to the first stage and one flexible measure w^2 to the second stage. The results of the models (3.4), (3.5) and (3.6) in general two-stage network structure under CRS for classifying flexible measures are shown in Table 2. As seen in Table 2, the DMU $11, 12, 22, 23, 24$ and 27 are overall efficient and the other DMUs are non-efficient. The DMU $1, 6, 9, 10, 11, 12, 13, 16, 17, 18, 21, 22, 23, 24,$ 27,28 and 29 are efficient to the first stage and the DMU $4, 11, 12, 14, 22, 24$ and 27 are efficient to the second stage. In these models, 13 DMUs select the flexible measure to the first stage as an output measure $\left(d^{1*}_{l_1} = 1\right)$ and 17 DMUs select it as an input measure $\left(d^{1*}_{l_1} = 0\right)$ and 14 DMUs select the flexible measure to the second stage as an output measure $\left(d^{2*}_{l_2} = 1\right)$ and 16 DMUs select it as an input measure $\left(d^{2*}_{l_2} = 0\right)$. Most DMUs select the flexible measure w^1 to the first stage as an input measure and select the flexible measure w^2 to the second stage as an input measure in CRS environment. Hence the network system selects the flexible measure w^1 to the first stage as an input measure and selects the flexible measure w^2 to the second stage as an input measure in CRS environment.

The results of the models (3.8), (3.9) and (3.10) in general two-stage network structure under VRS for classifying flexible measures are shown in Table 3. As seen in Table 3, the DMU $1, 10, 11, 12, 14, 18, 21,$ $22, 23, 24, 27$ and 28 are overall efficient and the other DMUs are non-efficient. The DMU $1, 2, 6, 9, 10,$ $11, 12, 13, 14, 17, 18, 21, 22, 23, 24, 27, 28$ and 30 are efficient to the first stage and the DMU $1, 10, 11,$ $12, 14, 18, 21, 22, 23, 24, 27$ and 28 are efficient to the second stage. In these models, 17 DMUs select the flexible measure w^1 to the first stage as an output measure $\left(d^{1*}_{l_1} = 1\right)$ and 13 DMUs select it as an input measure $\left(d^{1*}_{l_1} = 0\right)$ and 16 DMUs select the flexible measure w^2 to the second stage as an output measure $\left(d^{2*}_{l_2} = 1\right)$ and 14 DMUs select it as an input measure $\left(d^{2*}_{l_2} = 0\right)$. Most DMUs select the flexible measure w^1 to the first stage as an output measure and select the flexible measure w^2 to the second stage as an output measure in VRS environment. Hence the network system selects the flexible measure w^1 as an output measure to the first stage and as an input measure to the second stage so the flexible measure w^1 is an intermediate measure. But the network system selects the flexible measure w^2 to the second stage as an output measure.

The status of one flexible measure in two CRS and VRS environments in general two-stage network may have different results. For examples the DMU $6, 11, 16, 18, 20, 21, 22, 25, 26, 28$ and 29 select the flexible measure w^1 to the first stage as an output measure in VRS environment but, in CRS environment select the flexible measure w^1 to the first stage as an input measure. Moreover, the DMU $2, 12, 13, 22, 24, 27$ and 30 select the flexible measure w^2 to the second stage as an output measure in VRS environment but, in CRS environment select the flexible measure w^2 to the second stage as an input measure. On the other hand the network system selects the flexible measures w^1, w^2 to the first and second stage as input measures in CRS environment but in VRS environment the network system selects the flexible measure w^1 as an intermediate measure and selects the flexible measure w^2 to the second stage as an output measure.

5 Conclusion

In conventional DEA models it is assumed that each measure has to be identified whether it is an input or output. Nevertheless, in some cases flexible measures can exist which can play the role of either an input or output in two-stage network. We propose FNDEA models to evaluate the performance of and to classify exible measures in which each one of the flexible measures is treated as either input or output to maximize the network overall efficiency of the DMU under evaluation in general two-stage network structure. The current paper develops an additive efficiency decomposition approach wherein the overall efficiency is expressed as a (weighted) sum of the efficiencies of the individual stages. This approach can be applied under both constant returns to scale (CRS) and variable returns to scale (VRS) assumptions. The status of one flexible measure in two CRS and VRS environments may have different results. In other words, a flexible measure is considered as input in CRS environment, whereas it is selected as output in VRS environment. We believe that our proposed models can be modified into multi-stage models and can be developed for fuzzy flexible measures and interval flexible measures in two-stage network structures. Future researchers can extend these improvements.

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