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Research Article

A New Decomposition of Cost Efficiency Based on the Price and Cost-Based Production Possibility Sets in Non-Competitive Space in DEA

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Abstract

Identification of various sources of inefficiency plays an important role in the performance analysis aimed at developing plans for the improvement of decision making. In this regard, not only technical, cost, and allocative efficiency can be estimated by information on inputs and outputs and their prices, but losses due to the lack of profit and revenue and optimal cost can also be calculated based on the relevant inefficiency. The present paper aimed at providing new estimation of cost efficiency and sources of losses in the total efficiency in a non-competitive environment where there is the possibility of change in prices of inputs and outputs from one DMU to another. In line with studies (K. Tone, A Strange Case of the Cost and Allocative Efficiencies in DEA, Journal of the Operational Research Society 53 (2002) 1225-1231) and (K. Tone et al., Ecomposition of Cost Efficiency and its Application to Japanese-Us Electric Utility Comparisons, Socio-Economic Planning Sciences 47 (2007) 91-106), the present study sought to introduce new sources of inefficiency and related losses by presenting new price-based and cost-based production possibility set.

Keywords : Data Envelopment Analysis; Cost efficiency; Different Prices; Non-competitive Environment.

1 Introduction

DAta Envelopment Analysis (DEA) is a mathematical technique for estimating various types of efficiency in cases where units being evaluated use multiple inputs to generate multiple outputs. In this regard, the introduction and study of efficiency depends on the data of evaluating unit. If there is only data on inputs and outputs of evaluating units, we can talk about technical and scale efficiency and like them; but if there is price data for inputs and outputs, it is possible to analyze and evaluate performance from perspective of estimated cost, revenue and profit efficiency and its analyses such as technical and allocative efficiency.

Since the introduction of efficiency concepts in the paper by [16] as well as the introduction of

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efficiency measure by [20] based on the DEA technique, there have been efforts to analyze units under the evaluation in different situations depending on the availability of data. By the help of various mathematical models, DEA can evaluate various types of efficiency such as the cost efficiency (CE). When Two Decision Making Units (DMUs) have equal inputs and outputs, and the price vector of a DMU is a multiplication of another DMU's price vector, the conventional CE will be equal and this is a defect for the CE. [25] and later [30] found this defect and tried to resolve it by discussing different prices. [12] proposed a new Production Possibility Set (PPS) to solve this problem. Later, [13] presented a CE analysis of its application compared with Japan and America's electrical tools.

Various measures have been taken for the CE and its analysis in a competitive or non-competitive environment. For instance, [21] evaluated the CE in the presence of shadow prices. [24] studied the CE under the non-parametric DEA in the US steel industry; and [1] investigated three aspects of pricing, orientation, and envelopment level method as components of efficiency in each model of DEA. [11] provided the slacks-based measure of efficiency in DEA. [26] measured the economic efficiency with incomplete price information and its application in European commercial banks. [27] provided the nonparametric efficiency analysis under the price uncertainty; and [28] measured the economic efficiency with the incomplete price information. [6] measured the economic efficiency for input spending when DMUs face different input prices. [4] examined the cost-efficiency amount in the presence of uncertain prices and their application in bank branches. [2] provided an improvement to the CE interval by a DEA-based approach.[5] provided a generalization of Farrell's CE measure applicable to non-fully competitive settings. [23] examined the input price variation across locations and a generalized measure of CE. [7] examined the CE with triangular fuzzy number input prices and the application of DEA. [3] provided the CE measures in the DEA with data uncertainty. [6] provided a non-parametric measure of an economics scale in non-competitive environments with uncertain prices. [17] used DEA-R models in the revenue and CE. [18] examined the application of DEA to evaluate the efficiency level of the operational cost of Brazilian electricity distribution utilities. [9] measured the CE in the presence of quasi-fixed inputs using the dynamic DEA and its application in the port infrastructure. [15] measured the CE in the DEA under the law of one price. [14] provided the centralized resource allocation based on the efficiency analysis for step-by-step improvement paths. Afterwards, [10] provided a fully fuzzy DEA approach for cost and revenue efficiency measurement in the presence of undesirable outputs and its application in the Indian Banking. [19] also used nonparametric techniques to measure the CE of postal delivery branches. [22] investigated the cost risk taking in DEA model with random inputs and outputs and studied cost-efficiency range changes in the DEA. Among these studies, [12] and [13] provided interesting suggestions for calculating efficiency when input prices were different from a DMU to another. Therefore, [12] multiplied prices of each input of DMUs by their input vectors and constructed new points, and then built a PPS based on these points, and then calculated the efficiency of each one of corresponding units in a new space. They named the radial efficiency value as the price efficiency. Subsequently, the defined the allocative inefficiency by finding a point with the minimum sum of inputs in this set producing at least the same output value of the evaluated unit. In another study ([13]), which compared the CE of electric power generation companies in the United States and Japan, they introduced various losses due to various types of inefficiency including technical and price inefficiency in line with the previous study by building a new PPS by a similar method to the previous study, but by application of projection points of technical and CE, and then presented the superiority of their proposed method to calculate the CE compared to the traditional method by providing diagrams ([13]).

There is not any generalization of mentioned methods in both studies by [12] and [13] and other studies on the CE. The present paper aimed to generalize a method by [12] and [13], so that we can use modified units and prices to build a new PPS instead of considering modified units and observed prices to build it. Since applied points to build PPS in the present will dominates applied points used by [13], the proposed PPS of the present study will cover their PPS. Therefore, it is expected that values of its efficiency and analysis and losses in total costs will be different due to various factors of inefficiency, and this difference is due to the modification in the price sets.

The structure of paper is as follows. The second section presents some initial concepts and definitions in literature about the CE, its factors and the loss due to some of various factors. The third section explains the proposed method by an example after a brief explanation of the previous method by [12] and [13]. To this end, we first build a PPS and express the efficiency and loss of inefficiency. A practical example is presented in the Section 3 and the final section presents the conclusion for research and suggestions for future studies.

2 Background

Like other studies within the framework of DEA, assume that there are n DMUs that use m inputs for production of s output. The PPS is defined as the set of all (X, Y) in which the output vector Y can be produced by the input vector X. Accepting axioms of the inclusion of observations, convexity, feasibility, and returns to scale in DEA, the PPS is converted as follows:

$$P = \{(x, y) | x \geq X\lambda, y \leq Y\lambda, \lambda \geq 0\}$$

In the case that the constraint $1\lambda = 1$ is added to the above-mentioned PPS, a PPS is generated with varied returns to scale. Based on the position of DMUs relative to the frontier of this PPS, evaluating DMUs are classified into efficient and inefficient groups. DMUs on the efficient frontier are technically efficient and have an efficiency score of one; otherwise, they are technically inefficient with score of efficiency less than one. The input orientation of CCR model in the envelopment form for evaluating DMUo is as follows:

$$\begin{array}{lll} E_o = \min & \theta & \\ \quad s.t & \sum_{j=1}^{n} \lambda_j x_{ij} \geq \theta x_{io} & i = (1, \cdots, m) \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \leq y_{ro} & r = (1, \cdots, s) \\ & \lambda_j \geq 0 & j = (1, \cdots, n) \end{array} \tag{2.1}$$

θ_o^* is the amount of Technical Efficiency (TE) of DMUo. A point with coordinates of $(x_o^*, y_o^*) = (\sum_{j=1}^{n} \lambda_j^* x_j, \sum_{j=1}^{n} \lambda_j^* y_j)$ is the input oriented projection of DMUo on the efficiency frontier.

Assume that we have price information c_{ij} forall i, j and p_{rj} forall r, j data for inputs and outputs, respectively in general, since the market is not entirely competitive, prices vary from one unit to another. Suppose that we want to estimate the CE. In the traditional DEA, we first use the following model to find a frontier point of the PPS with at least the same output value of DMUo, and the least cost with the unit price vector C_o:

$$\begin{array}{ll} \min & C_0 = c_o x \\ s.t. & \sum_{j=1}^{n} \lambda_j x_j \leq x \\ & \sum_{j=1}^{n} \lambda_j y_j \geq y_{ro} \\ & x, \lambda \geq 0 \end{array} \tag{2.2}$$

The CE of DMUo is then defined as $\frac{c_o x^*}{c_o x_o}$ where in that $x^*$is obtained by solving model (2.2). Obviously, there is a difference between cost of observed unit and its projection point based on the radial model of CCR, and an obtained minimum point from Model (2.2). Fig. 1 shows a state where there are two inputs and a constant output. $DMUo = (X_o, Y_o)$ is inefficient due to appositive distance from the frontier; and according to the comparison with its radial projection point on the frontier, $(\theta^* X_o, Y_o) = (X_o^*, Y_o^*)$, its efficiency is θ^*. Noticing the observed price vector C_o for DMUo, the observed cost is $Z_1 = C_o X_o$. The cost of inputs for the projection point is $Z_2 = C_o X_O^*$.

Since $X_O^* \leq X_o$, we have $Z_2 = C_o \theta^* X_o \leq C_o X_o = Z_1$ and $\theta^* = \frac{Z_2}{Z_1}$.

$L_1 = Z_1 - Z_2$ is the excess cost that is resulted from the technical inefficiency. The linear cost function $Z = C_o X$ will have its minimum value equal to $Z_3 = C_o X^*$ in the point (X^*, Y_o). The allocative efficiency is defined as $\varphi^* = \frac{Z_3}{Z_2}$ indicating the correct input allocation for producing the best combination of inputs with the lowest cost. Due to this allocation, the amount of imposed loss cost to DMUo is less than the optimal value and is equal to $L_2 = Z_2 - Z_3$. CE of DMUo is defined as $CE_o = \frac{Z_3}{Z_1} = \theta^* \phi^*$. In other words, the CE is defined as the multiplication of technical and allocative efficiency, thus the inefficiency of evaluating unit can be related to both of these sources, namely technical and allocative inefficiency that make distance of L_1 and L_2 from an optimal cost for DMUo. Therefore, $Z_1 = L_1 + L_2 + Z_3$ indicating that an observed cost is equal to the minimum cost plus two losses due to costs of technical and allocative inefficiency.

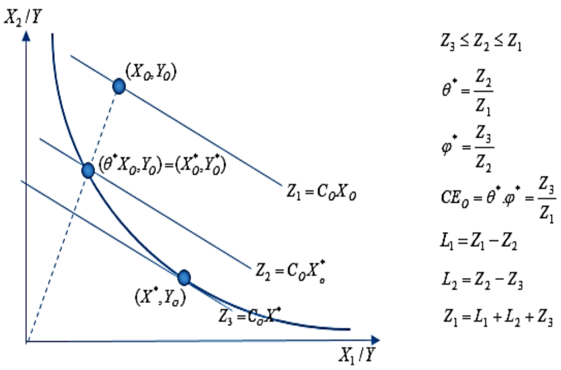


Figure 1: Technical, cost and allocative efficiencies and losses due to technical and allocative inefficiency

3 Cost Efficiency and Its Factors in a Non-Competitive Space

In this section, the proposed method by [13] is briefly mentioned in this section. In all the steps, it is assumed that input price data of w_{io} is available for input i of DMUo. As mentioned by [13], it is possible to develop a similar method for calculating the revenue and profit efficiency in the presence of the output price data.

Step 1) The radial TE in the input orientation for DMUo and its projection point (x^*_{io}, y_{ro}) is calculated using model (2.1). The corresponding technically efficient total cost for DMUo is $C^*_o = \sum w_{io} x^*_{io}$. Obviously, $C^*_o \leq C_o$in which $C_o = \sum w_{io} x_{io}$. The loss in input cost due to the technical inefficiency is $L^*_o = C_o - C^*_o \geq 0$.

Step 2) Considering the observed prices c_{ij} for DMUj and its projection point on the efficient frontier of the PPS P, i.e. (x^*_{ij}, y_{rj}), the PPS $P_c = \{(\bar{x}, y) | \bar{x} \geq \bar{X}\lambda, y \leq Y\lambda, \lambda \geq 0\}$ can be constructed. Adding the slacks to the constraints of model (2.1), the TE of $(\bar{x}_{io}, y_{ro}) = (c_{io} x^*_{io}, y_{ro})$ relative to the frontier of PPS P_c is calculated using (2.1). The projection is:

$(\bar{x}^*_o = \rho^* \bar{x}_o - t^{-*}, Y^*_o = Y_o + t^{+*})$. The cost of this new projection point is $C^{**}_o = \sum \bar{x}^*_{io} \leq C^*_o$. The price efficiency is defined as:

$$E^*_o = \text{Price efficiency } = \frac{C^{**}_o}{C^*_o} \leq 1$$

The loss due to the price inefficiency is: $L^{**}_o = C^*_o - C^{**}_o \geq 0$.

Step 3) To calculate the cost and allocative efficiency the following model is solved:

$$\begin{aligned} C^{***}_o = \min_{\tilde{x},\mu} \quad & e\tilde{x} \\ s.t \quad & e\tilde{x} \geq \tilde{X}\mu \\ & y_o \leq Y\mu \\ & \mu \geq 0 \end{aligned} \tag{3.3}$$

The obtained point $(\bar{x}^{**}, y_o)$ by model (3.3) has the lowest cost in the PPS $P_c$with at least the output y_o and calculating the allocative efficiency using the following equation

$$E^{**}_o = \text{Allocative efficiency } = \frac{C^{***}_o}{C^{**}_o} \leq 1$$

Loss due to the suboptimal cost mix (allocative inefficiency) is: $L^{***}_o = C^{**}_o - C^{***}_o \geq 0$.

Considering the definitions of L^*_o, L^{**}_o, L^{***}_o, C_o, C^*_o, C^{**}_o, and C^{***}_o, we have:

$$C^{***}_o = C_o \times E_o \times E^*_o \times E^{**}_o$$

3.1 The Proposed Method

To explain the proposed method and its motivation, consider a simple hypothetical example in which seven DMUs uses two inputs to generate an output. Table 1 presents their data and corresponding prices of each factor. It is assumed for simplicity that all units have the same output 1 and the output price 1. The efficiency scores and the projection points obtained by model (2.1) can be seen in columns 5 to 7 of Table 1. The PPS including points of A to G is a technology-based production possibility set (T-PPS) and shown in Fig. 2-a. Projection point sare shown by prime. Fig. 2-b shows the input price vector. Using technical projection points in Fig. 2-a, observed costs of Fig. 2-b, and axioms of DEA, [13] built a cost-based PPS (C-PPS). Fig. 2-c shows this set. In

Table 1: Inputs, outputs, and prices data of 7 hypothetical DMUs and their TE and technological projections

DMU	Input1	Input2	Output	TE	Input 1 projection	Input 2 projection	Input 1 Price	Input 2 Price	Output Price
A	2.5	0.5	1	1	2.5	0.5	1.5	1	1
B	1	2	1	1	1	2	2.5	1	1
C	0.5	4	1	1	0.5	4	0.5	1	1
D	1.5	4.5	1	0.57	0.857	2.571	2	2	1
E	2	2	1	0.75	1.5	1.5	1	2.5	1
F	5	2.5	1	0.4	2	1	1	1.5	1
G	5	0.5	1	1	5	0.5	1	0.5	1

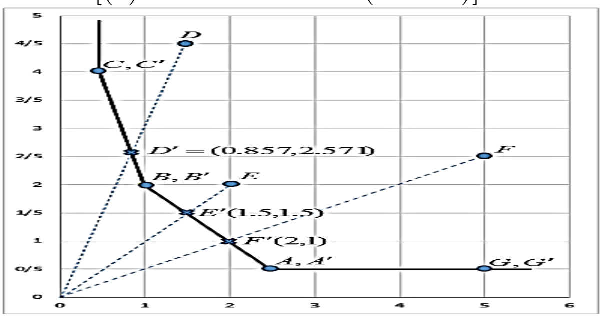
this Fig., there are 2 groups of points. First, the group related to actual observed prices, which are derived from the component multiplication of the observed inputs by their prices which have index 1. Points with index 2 are obtained by using technical projection points and observed prices. In this PPS, the radial efficiency of DMUs is calculated, and the projection points on the frontier called the price efficient. Consider Unit B. This unit is on the frontier of T-PPS, and thus its projection (B') matches with it. Using this value and the price vector C_B, them atached points B_1 and B_2 can be built. These points are not on the frontier in the C-PPS; and the obtained radial efficiency is equal to 0.75 relative to their projection on the frontier, F_2. In other words, there is a 25% potential improvement in the efficiency of $B_2(F_2= \rho B_2= \rho C_B B'= (\rho C_B)B)$. Since the C-PPS is built based on the cost data including technical and price, any improvement can be attributed to improvements in inputs and outputs, their prices, or a combination of them. Due to the TE of B, assume that all of these improvements are due to the possibility of improvement in prices. Accordingly, there can be 25% of improvement in C_B. This means that $\hat{C}_B= 0.75\, C_B= (1.875, 0.75)$ should be used in Fig. 2-a instead of using the price vector C_B. Assume the technical inefficient unit D. D' is its technical projection. Using C_D price vector for D and D' leads to $D_1= (3, 9)$ and $D_2= (1.714, 5.143)$ in C-PPS (Fig. 2-c). Unlike point D_1, since D_1 corresponds to the technical efficient point D' in T-PPS, it has no any technical inefficiency; and this is a difference between D_2 and D_1.D_2 has the radial efficiency score of 0.695 and inefficiency score of 0.305 towards the C-PPS frontier. $\hat{D}\,(1.192, 3.576)$ is the projection of D_2 on frontier1-C. In other words, inputs of D_2 should be improved to 0.305.

Since we have assumed that all technical inefficiencies related to the quantity of inputs have already been eliminated, this inefficiency in D_2 can only be attributed to its prices. Accordingly, its price vector C_D has an efficiency of 0.695, which can leads to the improved point $\hat{C}_D= 0.695\, C_D= (1.39, 1.39)$ in Fig. 2-b. According to the preceding process, it comes to mind that not only improved points should be considered in T-PPS set, but better price vectors than observed prices should be considered for achieving points with better performance in the C-PPS. In order to generalized methods by [12] and [13], the present paper suggest considering a set in the space of price vectors with not only observed price vectors, but also the possibility of further improvement in prices, and consequently, corresponding cost vectors of units in new C-PPS. Therefore, the Price-based PPS (P-PPS) is defined as follows.

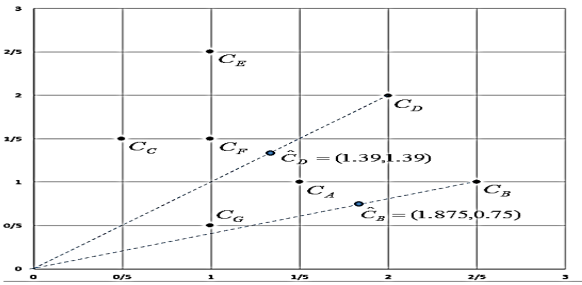
$$P - PPS = \{(C, P)|C \text{ can produce} P\}$$

In other words, the price vector C, which is related to an input vector, can generate an output-related price vector P. It seems that it is possible to accept a series of principles for building such a set. Obviously, observed price vectors can be attributed to this set (observation inclusion). Furthermore, if a fixed price for outputs can be made by a specific price, then the same input can be made at a higher price (input price possibility). It is the same as the output price; hence, if an output can be sold at a price, it can be also sold at a lower price (output price possibility).

[(a) DMUs in PPS P(T-PPS)]



[(b) Price vectors of DMUs]



[(c) Cost-based PPS (C-PPS)]

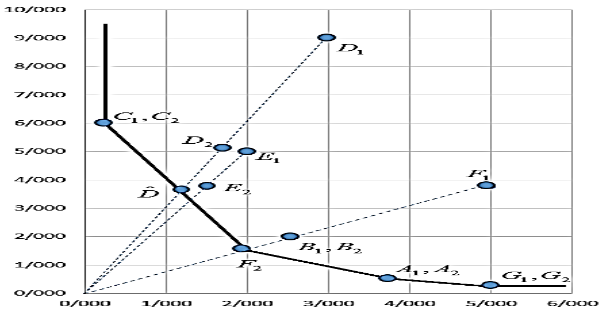


Figure 2

Accepting the axiom of convexity also seems logical because if two price vectors are acceptable for inputs, their intermediate prices are also acceptable. Different P-PPSs can be built by accepting different hypotheses about returns to scale. By accepting all of these axioms, like the ordinary DEA, P-PPS$_C$ is defined as follows:

$$\mathrm{P-PPS}_c = \{(c,p) | c \geq C\mu, p \leq P\mu \geq 0\}$$

Where, C is the matrix of observed input prices and P is the matrix of observed output prices relating to evaluating DMUs. Superscript c means accepting the axiom of constant returns to scale. Fig. 3 shows the P-PPS of the observed price data of Table 1 accompanied by their projection points on the efficiency frontier of this PPS. Accordingly, price vectors of DMUs C and G are efficient; and the other price vectors are inefficient and can be improved. For technical efficient unit A, using the price vector $C_{A'}(0.937, 0.625)$ instead of observed price vector $C_A(1.5, 1)$, the CE can be improve. Consider units C and D. It was possible to increase the CE by applying vectors $\hat{C}_B$ and $\hat{C}_D$ through presented method in previous studies. As shown in Fig. 3, there was a possibility of further improvement along these two vectors. Those points were $C_{B'}$ and $C_{D'}$ on P-PPS frontier. This means an improvement in cost vectors can be values of $1-0.5 = 0.5$ and $1-0.417 = 0.683$, in which 0.5 and 0.417 are radial efficiency of price vectors C_B and C_D in P-PPS, instead of $1 - 0.75 = 0.25$ and $1 - 0.695 = 0.305$. Columns 4 to 6 of Table 2 show radial efficiency values and the projection points of the observed price vectors for DMUs.

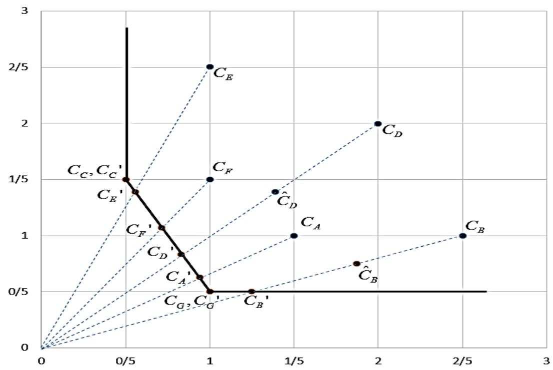


Figure 3: The P-PPS, observed prices, and their projections on the P-PPS frontier

Two new cost vectors with coordinates of D_3= (1.250, 3.750) and D_4= (0.714, 2.142), which do not exist in the introduced cost-based PPS by [13], can be built using projection vector of $C_{D'}$ and vector of observed inputs D and its technical projection (D'). See Fig. 4 for further explanation. In this Fig., there are four units correspond to any DMU, based on observed units and their projections on the T-PPS frontier, and based on observed prices and their images on the frontier P-PPS. Units, which are obtained using input vectors and observed values, are shown by index 1. The points, which are obtained from technical projection points of input vectors and observed prices, are shown by index 2; the points, which are obtained from observed input vectors and projection prices, are shown by the index 3;

Table 2: Example's data in P-PPS

	Price 1	Price2	Efficiency		Price1-Pro	Price2-Pro
C_A	1.5	1	0.625	$C_{A'}$	0.9375	0.625
C_B	2.5	1	0.500	$C_{B'}$	1.25	0.5
C_C	0.5	1.5	1.000	$C_{C'}$	0.5	1.5
C_D	2	2	0.417	$C_{D'}$	0.833333	0.833333
C_E	1	2.5	**0.556**	$C_{E'}$	0.555556	1.388889
C_F	1	1.5	0.714	$C_{F'}$	0.714286	1.071429
C_G	1	0.5	1.000	$C_{G'}$	1	0.5

and the points, which are obtained from technical projection of input vectors in T-PPS and technical projection of price vectors in P-PPS, are shown by index 4. If we consider a PPS for each of these four groups, then we will have four PPS; and since they are made on the basis of cost vectors, they can be shown by C- PPS$_i$ for $i = 1, \cdots, 4$. $\alpha\, G_1A_1B_1C_1\beta$ is the frontier of C-PPS$_1$; $\alpha\, G_2A_2F_2C_2\beta$ is the frontier of C-PPS$_2$; $\alpha\, G_3A_3B_3C_3\beta$ is the frontier of C-PPS$_3$; and $\alpha\, G_4A_4B_4D_4C_4\beta$ is the frontier of C-PPS$_4$. We now D' dominates D, since D' is he projection of D on the T-PPS frontier. If we consider the C_D price vector for both points, clearly C_DD' dominates C_DD, which means that the cost unit D$_2$dominates over $D1$. The same result is obtained for using $C_{D'}$ vector instead of C_D, i.e. D_4 dominates D_3. On the other hand, we now $C_{D'}$ dominates C_D, since $C_{D'}$ is he projection of C_D on the P-PPS frontier. Considering D vector for both, clearly $C_{D'}D$ dominates C_DD, which means that the cost unit D_3 dominates over $D1$. The same result is obtained for using D' vector instead of D, i.e. D_4 dominates D_2. Therefore, both of D_2 and D_3 dominate D_1; and D_4 dominate both of D_2 and D_3 vectors.

In general, it is not possible to express an explicit relationship between units with indexes 2 and 3. For example, while in Figure 4, D_3 dominates D_2, however there is an inverse relationship for F_2 and F_3. This is also true for the rest of points. As a result, not only C-PPS$_2$ and C-PPS$_3$ cover C-PPS$_1$, but also points related to C-PPS$_2$ and C-PPS$_3$ dominate their corresponding points in C-PPS$_1$. Furthermore, not only C-PPS$_4$ cover C-PPS$_1$, C-PPS$_2$ and C-PPS$_3$, but also points relating to C-PPS$_4$ dominate their corresponding points in other sets. It should be noted that there is not any relationship between C-PPS$_2$ and C-PPS$_3$. For instance, F_2 point from C-PPS$_2$ dominates F_3 point in C-PPS$_3$. This is not the same as D_2 and D_3. Now, we pay attention to the total cost of each point in sets above obtaining from the sum of components of these vectors. Given relations about dominance of these sets, we have:

$$1D_4 = 2.857143 < 1D_3 = 5$$
$$5 < 1D_2 = 6.857143 < 1D_1 = 12$$

D_4 point dominants $\hat{D}$ point and $1\hat{D} = 1(1.192, 3.576) = 4.768$. If D_2 is compared to frontier used by [13] or C-PPS$_2$, its efficiency will be 0.7, but if it is compared to C-PPS$_4$, its efficiency will be 0.42. This amount of change in the radial efficiency is due to the modification of cost-based PPS in a research by [13] and the proposed method in the present paper. Table 3 presents the coordinates of all points in Fig. 4. The outputs are ignored since the amount of output is 1 in all cases. Table 4 presents the efficiency of generated points by observed inputs and prices, or A_1

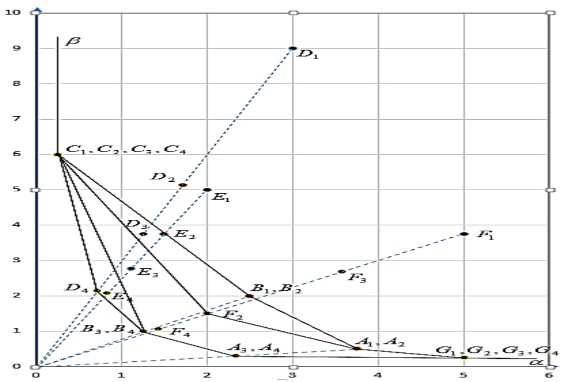


Figure 4: Production possibility sets C-PPS$_1$, C-PPS$_2$,C-PPS$_3$, C-PPS$_4$

Table 3: Coordinates of points in Fig. 4

	$C1X1$	$C2X2$	C_o^1		$C1X1'$	$C2X2'$	C_o^2
$A1$	3.75	0.5	4.25	$A2$	3.75	0.5	4.25
$B1$	2.5	2	4.5	$B2$	2.5	2	4.5
$C1$	0.25	6	6.25	$C2$	0.25	6	6.25
$D1$	3	9	12	$D2$	1.71	5.14	6.85
$E1$	**2**	**5**	7	$E2$	1.5	3.75	5.25
$F1$	5	3.75	8.75	$F2$	2	1.5	3.5
$G1$	5	0.25	5.25	$G2$	5	0.25	5.25
	$C1'X1$	$C2'X2$	C_o^3		$C1'X1'$	$C2'X2'$	C_o^4
$A3$	2.34	0.31	2.65	$A4$	2.34	0.31	2.65
$B3$	1.25	1	2.25	$B4$	1.25	1	2.25
$C3$	0.25	6	6.25	$C4$	0.25	6	6.25
$D3$	1.5	3.75	5.25	$D4$	0.71	2.14	2.85
$E3$	1.11	2.77	3.88	$E4$	0.83	2.08	2.91
$F3$	3.57	2.67	6.249	$F4$	1.42	1.07	2.49
$G3$	5	0.25	5.25	$G4$	5	0.25	5.25

Table 4: TE and projection points of A_1 to G_1 towards frontiers C-PPS$_1$, C-PPS$_2$, C-PPS$_3$ and C-PPS$_4$

	θ_o^1	x_1^1	x_2^1	θ_o^2	x_1^2	x_2^2	θ_o^3	x_1^3	x_2^3	θ_o^4	x_1^4	x_2^4
A_1	1	3.75	0.5	1	3.75	0.5	0.63	2.35	0.31	0.63	2.35	0.31
B_1	1	2.50	2	0.79	1.97	1.58	0.5	1.25	1	0.5	1.25	1
C_1	1	0.25	6	1	0.25	6	1	0.25	6	1	0.25	6
D_1	0.45	1.35	4.05	0.4	1.19	3.58	0.3	0.91	2.72	0.24	0.71	2.14
E_1	0.75	1.51	3.77	0.65	1.31	3.27	0.48	0.97	2.42	0.4	0.79	1.98
F_1	0.51	2.56	1.92	0.4	2	1.5	0.26	1.3	0.97	0.26	1.3	0.97
G_1	1	5	0.25	1	5	0.25	1	5	0.25	1	5	0.25

Table 5: TE and projection points of A_1 to G_1 towards frontiers C-PPS$_1$, C-PPS$_2$, C-PPS$_3$ and C-PPS$_4$

	θ_o^2	x_1^2	x_2^2	θ_o^4	x_1^4	x_2^4
A_2	1	3.75	0.50	0.63	2.35	0.31
B_2	0.79	1.97	1.58	0.5	1.25	1
C_2	1	0.25	6	1	0.25	6
D_2	0.7	1.19	3.58	0.42	0.71	2.14
E_2	0.87	1.31	3.27	0.53	0.79	1.98
F_2	1	2	1.5	0.65	1.3	0.97
G	1	5	0.25	1	5	0.25

to G_1 and their projection points towards four frontiers. Table 5 shows the difference between the proposed method and Tone et al.'s method [13]. Differences in existing efficiency in columns 2 and 5 show differences in efficiency of both methods. This amount is equal to $0.63/1 = 0.63$ for A_2 and $0.5/0.79 = 0.63$ for B_2. Table 6 also presents TE and the projections of A_3 to G_3 relating to C-PPS$_3$ and C-PPS$_4$ frontiers. Table 7 presents the efficiency of each point resulting from projection prices and inputs in the P-PPS and technology-based PPS (T-PPS). The TE level is below 1 indicating that despite the correction of prices and applied inputs of observed units in P-PPS and T-PPS, it is possible to improve prices by considering both of these factors as the cost-based factors, like E_4 and F_4. Table 7 presents the efficiency of

Table 6: TE and the projections A_3 to G_3 towards C-PPS$_3$ and C-PPS$_4$ frontiers

	θ_o^3	x_1^3	x_2^3	θ_o^4	x_1^4	x_2^4
A_3	1	2.34	0.31	1	2.34	0.31
B_3	1	1.25	1	1	1.25	1
C_3	1	0.25	6	1	0.25	6
D_3	0.86	1.08	1.85	0.76	0.95	1.63
E_3	0.95	1.05	1.98	0.82	0.91	1.71
F_3	0.54	1.92	0.58	0.54	1.92	0.58
G_3	1	5	0.25	1	5	0.25

Table 7: TE and projection points A_4 to G_4 towards the C-PPS$_4$ frontier

	θ_o^4	x_1^4	x_2^4
A_4	1	2.34	0.31
B_4	1	1.25	1
C_4	1	0.25	6
D_4	1	0.71	2.14
E_4	0.95	0.79	1.98
F_4	0.91	1.3	0.97
G_4	1	5	0.25

each point resulting from projection prices and inputs in the P-PPS and technology-based PPS (T-PPS). The TE level is below 1 indicating that despite the correction of prices and applied inputs of observed units in P-PPS and T-PPS, it is possible to improve prices by considering both of these factors as the cost-based factors, like E_4 and F_4. The line $k = x_1 + x_2$ is used to get a point with the lowest total cost. $A_1 = (3.75, 0.5)$ with the total cost of 4.25 in C-PPS$_1$, $F_2 = (2, 1.5)$ with the total cost of 3.5 in C-PPS$_2$, $B_3 = (1.25, 1)$ with the total cost of 2.25 in C-PPS$_3$, and $B_4 = (1.25, 1)$ with the total cost of 2.25 in C-PPS$_4$ are the points with a minimum total cost. Therefore, we can obtain the CE of each point of the quadruple cost-based PPSs using each of these points. For example, the cost-efficiency of D_1 towards C-PPS$_1$ frontier is equal to $CE^1_{D_1} = 4.25/3 + 9 = 0.35$. The CE of D_2 towards C-PPS$_2$ frontier is equal to $CE^2_{D_2} = 3.5/1.714 + 5.142 = 0.51$; CE of D_3 towards C-PPS$_3$ frontier is equal to $CE^3_{D_3} = 2.25/1.5 + 3.75 = 0.6$; and CE of D_4 towards C-PPS$_4$ frontier is equal to $CE^4_{D_4} = 2.25/0.714 + 2.142 = 0.787$. The CE of each internal point can be compared to external frontiers, if needed. For instance, the CE of D_1 towards C-PPS$_2$, C-PPS$_3$ and C-PPS$_4$ frontiers is $CE^2_{D_1} = 3.5/12 = 0.29$, $CE^3_{D_1} = 2.25/12 = 0.187$ and $CE^4_{D_1} = 2.25/12 = 0.187$, respectively. Table 8 presents data of the CE for each unit towards its frontiers as well as external frontiers. Efficiency of units A_2 to G_2 is only calculated based on C-PPS$_2$ and C-PPS$_4$ frontier because as mentioned earlier, there is no a definitive relationship between these two sets. Columns 7 and 8 present the CE of introduced units by [13] towards C-PPS$_2$ as well as our introduced C-PPS$_4$. Existing differences represent a new source of inefficiency due to the consideration of P-PPS and C-PPS$_4$.

Considering Table 9 as a summary of Tables 4 to 7 as well as Table 8, we can measure allocative efficiency values based on corresponding frontiers of each unit and external frontiers. Table 10 presents results. Therefore, an analysis of the CE for each unit can be presented based on technical and allocative efficiency towards each four frontiers. It is shown that the use of the PPS, which is built by corrected prices and units, creates a new source in the radial efficiency as well as the total cost and allocative efficiency. According to the above-mentioned cases, we can formally propose a method for calculating the CE and its compo-

Table 8: Cost efficiency

	CE^{1*}	CE^2	CE^3	CE^4		CE^2	CE^4		CE^3	CE^4		CE^4
$A1$	1.00	0.82	0.53	0.53	$A2$	0.82	0.53	$A3$	0.85	0.85	$A4$	0.85
$B1$	0.94	0.78	0.50	0.50	$B2$	0.78	0.50	$B3$	1.00	1.00	$B4$	1.00
$C1$	0.68	0.56	0.36	0.36	$C2$	0.56	0.36	$C3$	0.36	0.36	$C4$	0.36
$D1$	0.35	0.29	0.19	0.19	$D2$	0.51	0.33	$D3$	0.45	0.45	$D4$	0.79
$E1$	0.61	0.50	0.32	0.32	$E2$	0.67	0.43	$E3$	0.58	0.58	$E4$	0.77
$F1$	0.49	0.40	0.26	0.26	$F2$	1.00	0.64	$F3$	0.36	0.36	$F4$	0.90
$G1$	0.81	0.67	0.43	0.43	$G2$	0.67	0.43	$G3$	0.43	0.43	$G4$	0.43

Table 9: Technical efficiency

	TE^{1*}	TE^2	TE^3	TE^4		TE^2	TE^4		TE^3	TE^4		TE^4
$A1$	1	1	0.63	0.63	$A2$	1	0.63	$A3$	1	1	$A4$	1
$B1$	1	0.79	0.5	0.5	$B2$	0.79	0.5	$B3$	1	1	$B4$	1
$C1$	1	1	1	1	$C2$	1	1	$C3$	1	1	$C4$	1
$D1$	0.45	0.4	0.3	0.24	$D2$	0.7	0.42	$D3$	0.86	0.76	$D4$	1
$E1$	0.75	0.65	0.48	0.4	$E2$	0.87	0.53	$E3$	0.95	0.82	$E4$	**0.95**
$F1$	0.51	0.4	0.26	0.26	$F2$	1	0.65	$F3$	0.54	0.54	$F4$	0.91
$G1$	1	1	1	1	$G2$	1	1	$G3$	1	1	$G4$	1

*Superscript of C-PPS number.

Table 10: Allocative efficiency

	AE^1	AE^2	AE^3	AE^4		AE^2	AE^4		AE^3	AE^4		AE^4
$A1$	1.00	0.82	0.84	0.84	$A2$	0.82	0.84	$A3$	0.85	0.85	$A4$	0.85
$B1$	0.94	0.98	1.00	1.00	$B2$	0.98	1.00	$B3$	1.00	1.00	$B4$	1.00
$C1$	0.68	0.56	0.36	0.36	$C2$	0.56	0.36	$C3$	0.36	0.36	$C4$	0.36
$D1$	0.79	0.73	0.63	0.78	$D2$	0.73	0.78	$D3$	0.52	0.59	$D4$	0.79
$E1$	0.81	0.77	0.67	0.80	$E2$	0.77	0.81	$E3$	0.61	0.71	$E4$	**0.81**
$F1$	0.95	1.00	0.99	0.99	$F2$	1.00	0.99	$F3$	0.67	0.67	$F4$	0.99
$G1$	0.81	0.67	0.43	0.43	$G2$	0.67	0.43	$G3$	0.43	0.43	$G4$	0.43

*Superscript of C-PPS number.

nents as well as cost-loss factors.

Assuming vectors of observed inputs and outputs (X_{ij}, Y_{rj}) and observed prices of inputs and outputs (C_{ij}, P_{rj}) for observed DMUj, consider the following steps for measuring the new CE of under evaluation unit (X_{io}, Y_{ro}) as well as the observed price vector (C_{io}, P_{ro}) with observed cost $C_o^1 = \sum c_{io}x_{io}$. Superscript 1 presents the first cost associated with the under evaluation unit or the observed cost:

Step 1: Calculating radial TE in the input oriented for evaluating unit DMU_o towards the frontier of production set $T(\theta_o^T)$ using model (2.1) and obtaining the projection point $(X_o^{\prime T} = \theta_O^T X_o - S^{-T}, Y_o^{\prime T} = Y_o + S^{+T})$ where θ_o^T, S^{-T} and S^{+T} are the optimal values of model (2.1) (Considering the slacks S^{-T} and S^{+T} for its constraints).

The corresponding technology-based technically efficient total input cost for DMU_o:

$$C_o^2 = \sum c_{io}x_{io}^{\prime T}$$

Obviously, $C_o^2 \leq C_o^1$.

The loss in input cost due to the technology-based technical inefficiency is $L_o^{1-2} = C_o^1 - C_o^2 \geq 0$.

Step 2: Calculating the radial TE in the input oriented for observed prices of DMU_o i.e. (C_o, P_o) towards the frontier of P-PPS (θ_o^P) using model (2.1) and obtaining the projection point of $(C_o^{\prime P} = \theta_O^P C_o - S^{-P}, P_o^{\prime P} = P_o + S^{+P})$ where θ_o^P,

S^{-P} and S^{+P} are the optimal values of (2.1). The corresponding price-based technically efficient total input cost for DMU$_o$ is: $C_o^3 = \sum c_{io}^{'T} x_{io}$. Obviously, $C_o^3 \leq C_o^1$. The loss in input cost due to the price-based technical inefficiency is $L_o^{1-3} = C_o^1 - C_o^3 \geq 0$.

Step 3: Building cost points using projection units in T-PPS and P-PPS, i.e. $(X_o^{'T}, Y_o^{'T})$ and $(C_o^{'P}, P_o^{'P})$ points as follows:

$$\begin{pmatrix} x_{io}^4 \\ y_{ro}^4 \end{pmatrix} = \begin{pmatrix} c_{io}^{'p} x_{io}^{'T}, & i = 1, \cdots, m \\ p_{ro}^{'p} y_{ro}^{'T}, & r = 1, \cdots, s \end{pmatrix}$$

And the formation of a C-PPS through these points as follows:

$$\text{C-PPS}_4 = \{(\bar{x}, \bar{y}) | \bar{x} \geq X^{'T}\lambda, \bar{y} \leq Y^{'T}\lambda, \lambda \geq 0\}$$

The corresponding mutually technology and price-based technically efficient total input cost is

$$C_o^4 = \sum c_{io}^{'P} x_{io}^{'T} = \sum x_{io}^4$$

Obviously, $C_o^4 \leq C_o^1$ $C_o^4 \leq C_o^2$ $C_o^4 \leq C_o^3$.

The loss in input cost due to the mutually technology and price-based technical inefficiency is

$$L_o^{1,4} = C_o^1 - C_o^4 \geq 0.$$

Step 4: Calculating the radial TE in the input oriented for cost points of the Step 3, i.e. (X_o^4, Y_o^4) towards the frontier of C-PPS$_4$ using model (2.1) and obtaining the projection point

$$(X_o^{4-tech} = \theta_O^{C-4} X_o^4 - S^{-C}, Y_o^{4-tech} = Y_o^4 + S^{+C}).$$

The superscript "tech" represents the technical projection point. The corresponding cost-based technically efficient total input cost for DMU$_o$ (Radial efficient cost): $C_o^{4-\text{tech}} = \sum x_{io}^{4-\text{tech}}$. Obviously, $C_o^{4-\text{tech}} \leq C_o^4$. The loss in input cost due to the cost-based technical inefficiency is $L_o^{4,4-\text{tech}} = C_o^4 - C_o^{4-\text{tech}} \geq 0$.

Step 5: Finding the lowest total input in the C-PPS$_4$ with at least the same amount of output corresponding to (X_o^4, Y_o^4) according to the following model:

$$\begin{aligned} C_o^{4-\text{overal}} = \min \quad & \sum x_i^{4-\text{overal}} \\ s.t. \quad & e x^{4-\text{overal}} \geq e X^4 \mu \\ & y_o^4 \leq Y^4 \mu \\ & \mu \geq 0 \end{aligned}$$

$(x^{4-\text{overal}*}, y_o^4)$ has the lowest cost in the C-PPS$_4$with at least the same amount of output. It is obvious that $C_o^{4-\text{overal}} \leq C_o^{4-\text{tech}}$. Allocative efficiency of (X_o^4, Y_o^4) in C-PPS$_4$ is calculated as follows.

$$AE_o^4 = \frac{C_o^{4-\text{overal}}}{C_o^{4-\text{tech}}} \leq 1$$

The loss due to the suboptimal cost mix (Allocative inefficiency) is: $L_o^{4-\text{allocative}} = C_o^{4-\text{tech}} - C_o^{4-overal} \geq 0$.

CE of (X_o^4, Y_o^4) in C-PPS$_4$is calculated as follows.

$$CE_o^4 = \frac{C_o^{4-\text{overal}}}{C_o^4}.$$

And the observed CE of DMU$_o$ in the C-PPS$_4$ is

$$CE_o = \frac{C_o^{4-overal}}{C_o^1}.$$

Given the definitions of $L_o^{1,4}$, $L_o^{4,4-\text{tech}}$, $L_o^{4-\text{allocative}}$, C_o^1, C_o^4, $C_o^{4-\text{tech}}$ and $C_o^{4-\text{overal}}$ we have:

$$C_o^1 = L_o^{1,4} + L_o^{4,4-\text{tech}} + L_o^{4-\text{allocative}} + C_o^{4-\text{overal}} \tag{3.4}$$

and

$$\begin{aligned} CE_0 = \frac{C_o^{4-\text{overal}}}{C_o^1} &= \frac{C_o^{4-\text{overal}}}{C_o^4} \times \frac{C_o^4}{C_0^1} \\ &= \frac{C_o^{4-\text{overal}}}{C_0^{4-\text{tech}}} \times \frac{C_o^{4-\text{tech}}}{C_0^4} \times \frac{C_o^4}{C_0^1} \end{aligned}$$

On the other hand:

$$\begin{aligned} \frac{C_o^{4-tech}}{C_0^4} &= \frac{\sum x_{io}^{4-tech}}{C_0^4} \\ &= \frac{\sum q_O^{C-4} x_{io}^4 - s_i^{-C}}{C_0^4} \\ &= q_O^{C-4} - \frac{\sum s_i^{-C}}{C_0^4} \end{aligned} \tag{3.5}$$

and

$$\begin{aligned} \frac{C_o^4}{C_o^1} &= \frac{\sum c_{io}^{'} x_{io}^{'}}{C_o^1} \\ &= \frac{\sum (q_o^P c_{io} - s_i^{-P})(q_o^T x_{io} - s_{io}^{-T})}{C_o^1} \\ &= q_o^P q_o^T \\ &\quad + \frac{\sum s_i^{-P} s_{io}^{-T} - q_o^P \sum c_{io} s_i^{-T} - q_o^T \sum x_{io} s_i^{-p}}{C_o^1} \end{aligned} \tag{3.6}$$

If slack values are zero in (3.5) and (3.6), that is, the radial projection points associated with (X_o, Y_o), (C_o, P_o) and (X_o^4, Y_o^4) on the T-PPS, P-PPS and C-PPS$_4$ are not on the weak frontier, then $\frac{C_o^{4-tech}}{C_0^4} = \theta_O^{C-4}$ and $\frac{C_o^4}{C_o^1} = \theta_o^P \theta_o^T$ and we have:

$$C_o^{4-\text{overal}} = C_0^1 \times AE_o^4 \times q_o^{C-4} \times q_o^P \times q_o^T \quad (3.7)$$

Obviously, when (X_o, Y_o) is overally efficient if all TE in the T-PPS, P-PPS and C-PPS$_4$and allocative efficiency in C-PPS$_4$ is equal to 1.For instance, for unit E in Fig. 5 we have:

$$C_E^{4-\text{overal}} = C_E^1 \times AE_E^4 \times q_E^{C-4} \times q_E^P \times q_E^T$$
$$2.25 = 7 \times 0.81 \times 0.95 \times 0.556 \times 0.75$$

Similarly, based on Tables 1 to 10, we can write the analyses of other units. Table 11 presents the data of analysis (3.7) for all units. The cost-efficiency values of units (CE_o) is presented in the column 4. The presented analysis by [13] can be obtained by making changes in the equation (3.7). To this end, just change the left side of equation to $C_o^{2-\text{overal}}$ that is equal to (3.6) for a numeric example. On the right side, θ_o^P phrase is also deleted; and AE_o^4 and θ_O^{C-4} are AE_o^2 and θ_O^{C-2} values of changes. Table 12 shows this analysis. The equation (3.4) is written as follows for unit E.

$$\begin{aligned} C_E^1 &= L_E^{1,4} + L_E^{4,4-\text{tech}} \\ &\quad + L_E^{4-\text{allocative}} + C_E^{4-\text{overal}} \\ &= (7 - 2.916) + (2.916 - 2.77) \\ &\quad +(2.77 - 2.25) + 2.25 \\ &= 4.084 + 0.146 + 0.52 + 2.25 \end{aligned}$$

Obviously, the highest lost cost for this unit is 4.084 corresponding to the inefficiency of technology and the inappropriate choice of prices. The lost cost associated with the technical inefficiency towards the C-PPS$_4$ is equal to 0.146; and the allocative inefficiency towards the C-PPS$_4$is equal to 0.52. Table 13 presents values for this type of separation for other units. It should be noted here $L_o^{1,4}$ is due to the simultaneous elimination of technical inefficiencies θ_o^T and θ_o^P in the T-PPS and P-PPS.

Consider Figure 1 for a better understanding of effective factors in the overall efficiency (cost) of units. In this figure, bars for each of units from left to right are CE_o^1, AE_o^4, θ_O^{C-4}, θ_o^T and θ_o^P in the same order of columns 4 to 7 of Table 11 respectively. Unit A has the highest cost efficiency.

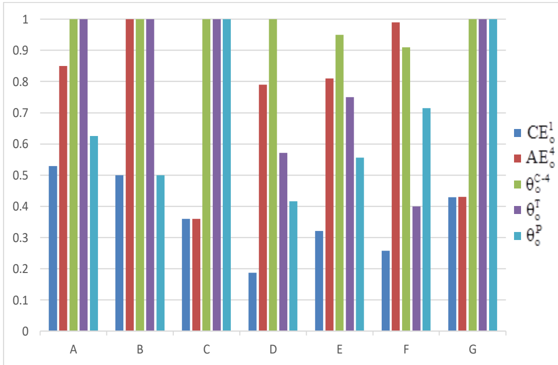


Figure 5: Effective factors in the CE

This unit has a technical inefficiency in choosing prices, i.e. in P-PPS, and after radial correction of these price, it is technically efficient in C-PPs. However, it has the allocative inefficiency in costs. Unit C also is the technically efficient in both of inputs and outputs quantities and prices. Its corresponding cost unit is also technically efficient in the C-PPS$_4$, but it is allocative inefficient.

About the unit F it should also be noted that despite the removal of technical inefficiencies in T-PPS and P-PPS, again the resultant point F4 is technically inefficient C-PPS$_4$. It also has allocative inefficiencies. Therefore, it is not merely to be efficient in the first two, and even to eliminate the inefficiencies, which is why there is no reason to achieve total CE.

Figure 6 also shows the share of each loss of $L_o^{1,4}$, $L_O^{4.4-\text{tech}}$ and $L_O^{4-\text{allocative}}$ in losing the lowest cost for each unit.

4 Conclusion

Given the radial projection points of input orientation in price and technology PPSs, the present paper constructed points with pricing components based on which a cost PPS was generated. The selection of target points on the efficiency frontier is based on decision makers and policy makers' perspectives, and thus target points may change in some circumstances. On this basis, projection points and thus price points can change,

Table 11: Total cost analysis based on its factors

	$C_o^{4-overal}$	C_o^1	CE_o	AE_o^4	θ_O^{C-4}	θ_o^T	θ_o^P
A	2.25	4.25	0.529412	0.85	1	1	0.625
B	2.25	4.5	0.5	1	1	1	0.5
C	2.25	6.25	0.36	0.36	1	1	1
D	2.25	12	0.1875	0.79	1	0.571429	0.416667
E	2.25	7	0.321429	0.81	0.95	0.75	0.555556
F	2.25	8.75	0.257143	0.99	0.91	0.4	0.714286
G	2.25	5.25	0.428571	0.43	1	1	1

Table 12: Analysis of cost Efficiency by factors based on the possibility set of [13]

	$C_o^{2-\text{overal}}$	C_o^1	AE_o^2	θ_O^{C-2}	θ_o^T
A	3.5	4.25	0.82	1	1
B	3.5	4.5	0.98	0.79	1
C	3.5	6.25	0.56	1	1
D	3.5	12	0.73	0.7	0.571429
E	3.5	7	0.77	0.87	0.75
F	3.5	8.75	1	1	0.4
G	3.5	5.25	0.67	1	1

Table 13: Analysis of observed cost for evaluating units based on its factors

	C_o^1	$L_o^{1,4}$	$L_o^{4.4-\text{tech}}$	$L_o^{4-\text{allocative}}$	$C_o^{4-\text{overal}}$
A	4.25	1.5945	0	0.4	2.25
B	4.5	2.25	0	0	2.25
C	6.25	0	0	4	2.25
D	12	9.144	0	0.6	2.25
E	7	4.084	0.14	0.52	2.25
F	8.75	6.251	0.22	0.02	2.25
G	5.25	0	0	3	2.25

and larger cost PPSs can be built by accumulating such points in different orientations. Non-radial models without any orientation such as SBM and modified Russell models can be used for the non-dependence on the nature of input or output.

Interestingly, unlike TE evaluations, in which at least one unit is always efficient, these states may not be feasible for the CE in the present paper because the unit may be inefficient in a factor as properly shown in the proposed method. Therefore, there is no reason to have at least one DMU with a CE of 100%.

The present paper focused on calculating the CE. It is easy to develop an approach to the profit and revenue efficiency. The hypo thesis about the return to scale for PPSs was also the hypothesis about the constant return to scale. We can also consider effects of scale inefficiency in the absence of a minimum cost considering variable returns to scale.

The future studies can develop the proposed method when the data of DMUs or prices are inaccurate and they can also consider network structures in DEA and unit performance evaluation with respect to time factor. It is also suggested developing the proposed method for the situation where prices are not constant and input quantities and their prices can be changed simultaneously, and estimating the malmquist productivity index of cost, revenue and profit in a non-competitive space.

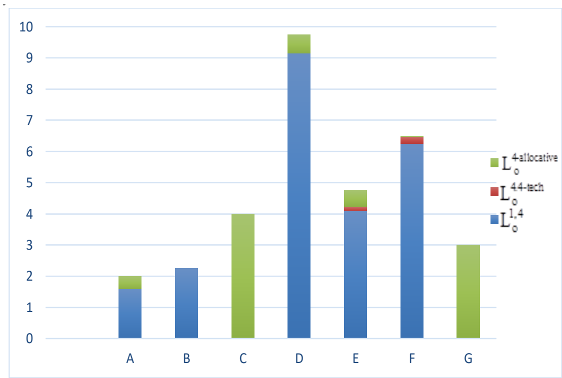


Figure 6: The share of each loss of $L_o^{1,4}$, $L_o^{4.4-\text{tech}}$ and $L_o^{4-\text{allocative}}$ in losing of the lowest cost for each DMU

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