

Estimating Capacity Utilization in Two-Stage Production Systems: A Data Envelopment Analysis Approach

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Abstract

The current study investigates performance in two-stage production systems through the measurement of capacity utilization (CU). In short-run, the degree of capacity utilization depends on the ability of decision making units to utilize fixed production factors. To develop this indicator in two-stage production processes, these factors are classified into fixed and variable inputs. Then the modified SBM network DEA model is proposed to estimate the capacity utilization and further study the effect of intermediate products on CU. Ultimately the numerical example is presented to point out the applicability and effectiveness of current model.

Keywords : Network Data Envelopment Analysis; Capacity Utilization; Intermediate product; Two-stage production system.

1 Introduction

Data Envelopment Analysis (DEA) is an effective technique to measure the relative performance of peer decision making units (DMUs) which use multiple inputs to produce multiple outputs. This non-parametric approach is introduced by Charnes et al. [4] for the first time. They developed the Farrell [16] outlook and proposed a mathematical programming approach to evaluate the efficiency of any system, such as commercial, manufacturing, educational, serving, etc.

In classical DEA models, DMUs are considered as black-box that transform inputs to outputs. In many situations the production process in each DMU is composed of two or several processes, therefore using these models, internal structures are neglecting. Thus, these models need to be improved in order to do the assessment more precisely. Considering internal linking activities of the DMUs, network DEA models have been proposed [13]. Unlike the DEA models, network DEA models reflect the internal structure of units in evaluating their performance. In recent years, various models have been extended to assess the efficiency of network production systems. For example, Kazemi Matin and Azizi [22] introduced a general new model for evaluating the efficiency score of network production processes with arbitrary internal structures. Despotis et al. [11] also presented a general network DEA method

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to measure the performance of series multi-stage processes. So offering a novel approach based on multi-objective programming, in a very unique and unbiased way attempted to estimate the stage efficiency scores. Kao [19] made a review of studies done on network DEA. He classified network DEA models based on network structures. According to Kao article [19], a large number of studies applied on network were based on two-stage processes. The two-stage production processes are the simplest network systems which the outputs of the first stage (intermediate products) are employed as the inputs of the second stage. Seiford and Zhu [30] presented a two-stage network structure to examine the profitability and marketability of 55 US commercial banks. Using a traditional DEA model and neglecting serial relationship between the two stages, they obtained the efficiency of whole system. Liang et al [26] used game theory concepts and developed the DEA methods to examine the efficiency of two-stage network systems. Their model and Despotis et al. [11]'s approach are recently used by Li et al [24] to generate a Parato solution and identify the leader stage in network DEA. This method indicates that the optimal solution for the Chen et al. proposed model is also a leader-follower solution. Kao and Hwang [20] suggested a different method for measuring performance of two-stage processes and defined the overall efficiency of the system as the product of the efficiencies of the two stages. Also, Chen et al. [5] proposed another model to evaluate these systems similar to Kao and Hwang's, but in an additive format. Their additive model can be used both in constant returns to scale (CRS) and variable returns to scale (VRS). Both models ([5, 20]) are based on the reasonable assumption that the weights used for the intermediate products are the same. Despotis et al. [10] argued that the optimal solution of additive decomposition approach (Chen et al.'s model [5]) and multiplicative decomposition approach (Kao and Hwang's model [20]), which are the popular models in two-stage network DEA for the measuring the overall efficiency and sub-system efficiencies, may be non-unique and biased. So, they offered a novel approach to estimate unique and unbiased efficiency scores for the two stages. Their modeling paradigm is

based on the selection of an output orientation for first stage and an input orientation for the second stage. Guo et al. [17] investigated both the additive efficiency decomposition in two-stage network DEA and the factors influencing the variations in overall efficiency to demonstrate the association between variation in the overall efficiency with constant stage efficiencies. They proposed a new of overall efficiency to reflect entirely on the stage efficiencies. Cook et al. [8] reviewed proposed two-stage DEA methods, and showed that all the existing approaches can be classified into four categories: the standard DEA approach, the efficiency decomposition approach, the network DEA approach and the game theoretic approach. Kao and Liu [21] extended the two-stage DEA efficiency decomposition method of Kao and Hwang [20] to solve fuzzy two-stage DEA problems. They employed a two-level optimization approach to overcome the problem of intermediate products. Akther et al [1] also investigated the performance of 21 banks in Bangladesh and applied a two-stage network approach to maximize desirable outputs and minimize bad outputs. Zhou et al. [37] extended the conventional DEA models on two-stage processes with stochastic data and proposed a stochastic centralized two-stage network DEA model. This technique is used to evaluate the performance of 16 commercial banks in China. Unlike researches done in the banking industry, Boloori and Pourmahmoud [2] used a more general method to evaluate this industry and improved it into a more precise structure within three processes in each bank branch. They obtained efficiency targets and developed an envelopment form of the network SBM model. Their introduced model is a developed version of Tone and Tsutsui's model.

Chen et al. [6] used a SBM-based model to develop the work of Tone and Tsutsui [32]. They extended two models (envelopment-based and multiplier-based) to obtain the frontier points (projections) for inefficient DMUs in two-stage systems. They also showed that the overall inefficiency of the system is equivalent to the sum of inefficiencies of the two processes. Li et al. [25] used and extended the approach of Liang et al. [26] to analyze the efficiency of two-stage processes in which the second stage had its own inputs in ad-

dition to the outputs from the first stage. Liu et al. [27] studied the two-stage processes with undesirable input-intermediate-outputs. They applied the free-disposal axiom to build production possibility set and the corresponding two-stage network DEA models with undesirable factors. Abundant studies on two-stage production systems indicate that this system possess significant and particular place in analyzing new modelling ideas in network production systems.

The most important topics in the management of science is the determination of the capacity and capacity utilization (CU) in production systems. Capacity is described as the ability of a firm to produce a potential output [33]. There are two measures of capacity: technical and economic. Johansen [18] and Morrison [28] provided capacity and CU's theoretical and economic framework. Johansen ([18], p.68) presented the production function to define the technical measure of single output capacity as, "the maximum amount that can be produced per unit of time exiting plant and equipment, provided the availability of variable factors of production is not restricted". Morrison [28] provided the economic measure of capacity in which the optimal output measure is the tangency between the short-run and long-run average cost curve. The major differentiation between technical and economic measures is that the technical measure of capacity output does not require information regarding input prices. The present study adopts the technical concept of capacity output to estimate the capacity utilization. Capacity utilization represents the industry performance indicator to describe the relationship between actual output (what is actually produced) and potential output (what could be produce). Generally the capacity utilization refers to proportion of potential capacity which is used and typically estimated as the ratio of actual output to capacity output. In recent years, a number of researches have been focused on Johansen's definition and used the DEA methodology to measure capacity output and CU's manufacturing firms. This approach was initially introduced by fare [12]. Their presented approach could be considered the weaker version of the Johansen's because outputs are bounded by fixed inputs of production. Later, this technique was

modified and developed by Fare et.al ([14, 15]). They proposed that an output-oriented measure of technical efficiency could be applied to calculate the capacity output and CU. Also, Fare et al. [14] argued that presented measure of CU by Fare et al. [12] may be biased downward. Kirkley and Squires [23] used the offered method by Fare et.al [15] to assess capacity in fisheries. Also, Vestergaard et al. [33] presented an analysis and estimation of capacity and CU in the multi-species Danish gill-net fishery that was based on DEA. To propose a non-radial CU measure, Cooper et al. [9] introduced and expanded SBM model to estimate these concepts. Sahoo and Tone [29] applied the proposed technique by Cooper et al. [9] to study CU in Indian banks. Zhang et al [36] suggested a dynamic SBM-DEA model to introduce a dynamic CU measure and estimate the CU of China's industrial sector. Yu et al [35] also estimated the physical capacity utilization and cost gap between actual and global long-run minimum costs using an input oriented SBM-DEA model. The main difference between their work and other studies was that they took the situation that firms operate in markets which are not fully competitive into consideration; this more closely matches real life. Their method is illustrated on a real case study of 13 Low-cost carriers around the world for the year 2010. Recently, Yang and Fukuyama [34] developed a novel generalized CU indicator and defined it as the difference between two directional distance functions. Their introduced indicator measures the extent to which the current variable inputs are utilized.

All the recent studies have extensively used the single-stage DEA models (traditional models) to estimate capacity utilization (CU) of production systems. In fact, they considered production systems as single stage process (black- box) while we develop this concept in two-stage production systems and investigate effect of intermediate products of systems of CU Following a technological notion and developing non-parametric Cooper's [9] technique.

The rest of the paper is organized as follows. The required background is provided in Section 2. Estimating CU in two-stage production systems is presented in Section 3. A numerical example illustrates the effectiveness of the proposed model

discussed in Section 4. Finally, conclusions are presented in Section 5.

2 Background

This section is devoted to brief introduction of Tone and Tsutsuis’s model [18] and concepts and models used to estimate CU. It should be noted that there exist two fundamental approaches in DEA to evaluate the performance of DMUs with different characteristics; radial and non-radial. The radial DEA models are based on the proportional change of input or output resources and usually ignore the existence of slacks in the efficiency scores whereas the non-radial models consider the slack of each input or output and the alteration of inputs and outputs are not proportional; in other words in non-radial DEA models the input (output) resources allowed the reduction (increase) at different rates. The most fundamental non-radial model was extended by Tone [31] which called slacks-based measure (SBM). The SBM model directly deals with input excess and output shortfall. In recent years, it has been generally applied to estimate efficiency of production systems.

2.1 The slacks-based measure model

Consider a set of n DMUs denoted by $DMU_j (j = 1, \dots, n)$, that consume m inputs to produce s outputs. The observed input and output vectors of DMU_j be denoted by $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})$ and $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj})$, respectively. Also, it is assumed that all inputs and outputs are positive. One of the non-radial DEA model which estimates the efficiency score of DMU_o , under the VRS assumption, is the SBM model which was

represented by Tone [31] as follow:

$$\begin{aligned} \varphi^* = \min & \frac{(1 - \frac{1}{m} \sum_{i=1}^m \frac{s_i^-}{x_{io}})}{(1 + \frac{1}{s} \sum_{r=1}^s \frac{s_r^+}{y_{ro}})} & (2.1) \\ \text{s.t.} & \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = x_{io} \quad i = 1, \dots, m \\ & \sum_{r=1}^s \lambda_j y_{rj} - s_r^+ = y_{ro} \quad r = 1, \dots, s \\ & \sum_{j=1}^n \lambda_j = 1 \\ & \lambda_j \geq 0 \quad j = 1, \dots, n \end{aligned}$$

Here λ_j is the intensity variable, and $s_r^+, (r = 1, 2, \dots, s)$ and $s_i^-, (i = 1, 2, \dots, m)$ denote excesses in input resources and shortfalls in output products, respectively. Note that model (2.1) is a nonlinear program, which can convert into an equivalent linear form [3]. It is significant to note that among input resources for production, some are fixed and unable to change during a production period e.g., plant and equipment, while some other inputs could be exactly controlled by the manufacturing firms in the short-run; e.g., number of employees, working hours and days. Former inputs are called fixed (x^F) and later inputs variable (x^V). Needless to say that all inputs can be altered in the long-run. According to aforementioned subjects, the inputs are classified into k fixed inputs that cannot alter in short-run and $(m - k)$ variable inputs. The output-oriented of the above SBM model (SBM-O) is presented as follows:

$$\begin{aligned} \varphi_o^* = \max & \left(1 + \frac{1}{s} \sum_{r=1}^s \frac{s_r^+}{y_{ro}}\right) & (2.2) \\ \text{s.t.} & \sum_{j=1}^n \lambda_j x_{ij}^F \leq x_{io}^F, i = 1, \dots, k \\ & \sum_{j=1}^n \lambda_j x_{ij}^V \leq x_{io}^V, i = k + 1, \dots, m \\ & \sum_{r=1}^s \lambda_j y_{rj} - s_r^+ = y_{ro}, r = 1, \dots, s \\ & \sum_{j=1}^n \lambda_j = 1 \\ & \lambda_j \geq 0, j = 1, \dots, n \end{aligned}$$

The standard SBM models is applicable for manufacturing systems as black boxes where some inputs are used to produce some outputs. Hence, they are not useful to measure the performance of two-stage production processes due to overestimating the overall efficiency scores, [32].

2.2 Two-stage production systems

Consider a basic two-stage structure as shown in Fig. 1. In this structure all outputs from the first stage are seen as intermediate products which constitute the inputs to the second stage. Also, z_{dj} is indicated as d^{th} intermediate product, $d = 1, \dots, D$, of DMU_j . Furthermore, all data are positive, i.e. X_i, Z_d and $Y_r > 0$ for all possible $i = 1, \dots, m; d = 1, \dots, D$ and $r = 1, \dots, s$. Tone and Tsutsui [32] argued that high rate of

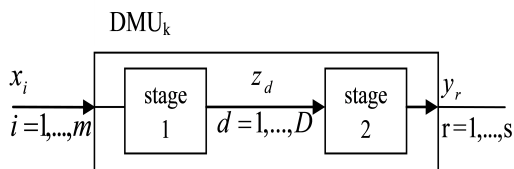


Figure 1: Two stage system

attention should be taken while applying radial network DEA models in evaluating the performance of two-stage systems, because these measures (radial efficiency measures) assume that all inputs or outputs change proportionally. Then, they extended the SBM model to assess the overall efficiency score of network systems. Note that the obtained efficiency score of their proposed model is more precisely than the other conventional models such as SBM model. The output-oriented of this model in two-stage systems is pre-

sented as follow:

$$\begin{aligned}
 \rho^* = \max & \left(1 + \frac{1}{r} \sum_{r=1}^s \frac{s_r^+}{y_{ro}} \right) & (2.3) \\
 \text{s.t.} & \sum_{j=1}^n \lambda_j x_{ij}^F \leq x_{io}^F, \quad i = 1, \dots, k \\
 & \sum_{j=1}^n \lambda_j x_{ij}^V \leq x_{io}^V, \quad i = k + 1, \dots, m \\
 & \sum_{j=1}^n (\lambda_j - \mu_j) z_{qj} = 0, \quad q = 1, \dots, p \\
 & \sum_{r=1}^n \mu_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1, \dots, s \\
 & \sum_{j=1}^n \lambda_j = 1 \\
 & \sum_{j=1}^n \mu_j = 1 \\
 & \lambda_j, \mu_j, s_r^+ \geq 0, \quad j = 1, \dots, n; \quad r = 1, \dots, s
 \end{aligned}$$

Where λ_j and μ_j are the intensity weights of each are stage, and $s_r^+ (r = 1, \dots, s)$ denotes output slacks. Suppose that ρ^* is the optimal value of model (2.3). $\rho^* > 1$ means that the DMU Can extend some output without giving damage other outputs, employing the all current inputs (fixed and variable). We will benefit from this formulation to introduce and measure capacity utilization in DEA framework.

2.3 Capacity utilization in DEA framework

The technical measure of capacity is a short-run concept suggested by Johansen [18] and indicated the ability of a firm to produce a potential output. On this basis, inputs be classified as fixed (x^f) and variable (x^v); i.e. $x = (x^f, x^v)$, for each DMU . Now, to introduce non-radial measure of capacity output, Cooper et al. [9] used the output-oriented SBM model and then assumed $DMUs$ access to many variable inputs needed for full capacity (consistent with Johansen’s definition). So, corresponding restrictions of variable inputs are omitted and following model repre-

sented as:

$$\begin{aligned} \varphi_o^F = \max & \left(1 + \frac{1}{s} \sum_{r=1}^s \frac{s_r^+}{y_{ro}} \right) & (2.4) \\ \text{s.t.} & \sum_{j=1}^n \lambda_j x_{ij}^F \leq x_{io}^F, \quad i = 1, \dots, k \\ & \sum_{r=1}^s \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1, \dots, s \\ & \sum_{j=1}^n \lambda_j = 1 \\ & \lambda_j \geq 0, \quad j = 1, \dots, n \end{aligned}$$

Assume that φ_o^* and φ_o^{*F} are optimal values obtained by models (2.2) and (2.4) respectively, then CU measure is defined and calculated by following equation [9]:

$$CU = \frac{\varphi_o^*}{\varphi_o^{*F}}$$

The value of capacity utilization, which can be no greater than 1, implies whether a DMU has the potential for greater production with the existing fixed inputs. Recent relation shows that cu measures the gap between actual and capacity output. This gap is created particularly by inefficient utilization of the fixed inputs. However, when technical inefficiency exists, part of the output gap is produced through inefficient utilization of variable inputs. Fare et al [14] proposed the above relation, under the CRS assumption, applying the radial (CCR-O) model. They called this measure, the plant capacity utilization measure of the DMU_o which was evaluated. In fact Fare et al. [14] argue that a more proper CU measure is the technically efficient output level ratio to the capacity output level.

3 DEA estimation of capacity utilization in two-stage production systems

In this section, the proposition is that all of DMUs have the two-stage structure and develop the concept of CU for these systems. To determine each DMU's capacity output with the existing fixed inputs whose variable inputs are not

restricted, the following model is proposed:

$$\begin{aligned} \rho_F^* = \max & \left(1 + \frac{1}{r} \sum_{r=1}^s \frac{s_r^+}{y_{ro}} \right) & (3.5) \\ \text{s.t.} & \sum_{j=1}^n \lambda_j x_{ij}^F \leq x_{io}^F, \quad i = 1, \dots, k \\ & \sum_{j=1}^n (\lambda_j - \mu_j) z_{qj} = 0, \quad q = 1, \dots, p \\ & \sum_{r=1}^s \mu_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1, \dots, s \\ & \sum_{j=1}^n \lambda_j = 1 \\ & \sum_{j=1}^n \mu_j = 1 \\ & \lambda_j, \mu_j, s_r^+ \geq 0, \\ & j = 1, \dots, n; \quad r = 1, \dots, s \end{aligned}$$

The only main difference between two models (2.3) and (3.5) is the treatment of variable inputs. In model (3.5), it is assumed that the DMU has availability to numerous variable inputs required for full capacity, therefore; their corresponding restrictions are omitted from model.

We assume that ρ^* and ρ_F^* are the optimal value of models (2.3) and (3.5) respectively, by definition, the capacity utilization in two-stage systems can be calculated as follow:

$$CU(y, z, x^F, x^V) = \frac{\rho^*}{\rho_F^*} (\leq 1) \quad (3.6)$$

This measure lacks of any technical inefficiency since latter appears in the numerator and denominator. In other words it is not downward biased. Also, $((1-CU)*100)$ could be interpreted as additional output percent which is produced in full capacity without variable inputs restrictions.

4 Numerical example

In this section, a real data set is applied to illustrate the results of the new proposed approach. We use our method to 27 firms in the banking industry in US originally studied in Chen and Zhu [7]. They consider a two-stage production process with three inputs (IT investment $-x_1$

Table 1: Data of 27 US banks.

	x_1 (\$ billions)	x_2 (\$ billions)	x_3 (thousands)	z (\$ billions)	y_1 (\$ billions)	y_2 (\$ billions)
DMU_1	0.150	0.713	13.3	14.478	0.232	0.986
DMU_2	0.170	1.071	16.9	19.502	0.340	0.986
DMU_3	0.235	1.224	24.0	20.952	0.363	0.986
DMU_4	0.211	0.363	15.6	13.902	0.211	0.982
DMU_5	0.133	0.409	18.485	15.206	0.237	0.984
DMU_6	0.497	5.846	56.42	81.186	1.103	0.955
DMU_7	0.060	0.918	56.42	81.186	1.103	0.986
DMU_8	0.071	1.235	12.0	11.441	0.199	0.985
DMU_9	0.500	18.120	89.51	124.072	1.858	0.972
DMU_{10}	0.120	1.821	19.8	17.425	0.274	0.983
DMU_{11}	0.120	1.915	19.8	17.425	0.274	0.983
DMU_{12}	0.050	6.918	13.1	14.342	0.177	0.985
DMU_{13}	0.370	4.432	12.5	32.491	0.648	0.945
DMU_{14}	0.440	4.504	41.9	47.653	0.639	0.979
DMU_{15}	0.431	1.241	41.1	52.630	0.741	0.981
DMU_{16}	0.110	5.892	14.4	17.493	0.243	0.988
DMU_{17}	0.053	0.973	7.6	9.512	0.067	0.980
DMU_{18}	0.345	0.444	15.5	42.469	1.002	0.948
DMU_{19}	0.128	0.508	12.6	18.987	0.243	0.985
DMU_{20}	0.055	0.370	5.6	7.546	0.153	0.987
DMU_{21}	0.057	0.395	5.7	7.595	0.123	0.987
DMU_{22}	0.098	2.680	14.1	16.906	0.233	0.981
DMU_{23}	0.104	0.781	14.6	17.264	0.263	0.983
DMU_{24}	0.206	0.872	19.6	36.430	0.601	0.982
DMU_{25}	0.067	1.757	10.5	11.581	0.120	0.987
DMU_{26}	0.100	0.713	12.1	22.207	0.248	0.972
DMU_{27}	0.0106	0.713	12.7	20.670	0.253	0.988

, Fixed assets $-x_2$ and The number of employees $-x_3$, one intermediate measure (The deposits generated $-z$) and two final outputs (Profit $-y_1$ and Fraction of loans recovered $-y_2$) where x_3 indicates the only variable input and fixed elements are x_1 and x_2 . Table 1 exhibits inputs, intermediate measure and outputs of these firms. According to the method introduced in Section 3, we determine the CU scores in three steps: (a)

calculating the maximum output obtainable from all observed inputs; (b) determining each DMU's capacity only with observed fixed input, which allows variable inputs to be unlimited (accordant with Johansen's definition of capacity); and (c) taking the ratio of the first two steps to estimate a capacity utilization measure.

To achieve this goal, we first apply models (2.3) and (3.5) to determine the maximum output with

Table 2: CU estimation of 27 banks in two different cases: (Black box and Two-stage).

	(ρ^*) Two-Stage	(ρ^*) Black box	(ρ_F^*) Two-Stage	(ρ_F^*) Black box	$CU = (\rho^*/\rho_F^*)$ Two-Stage	$CU = (\rho^*/\rho_F^*)$ Black box
DMU_1	1.293321	1.174515	2.240475	2.038965	0.577253	0.576035
DMU_2	1.182898	1.078117	2.122059	2.122059	0.557429	0.508052
DMU_3	1.307245	1.219677	2.019284	2.019284	0.647380	0.604015
DMU_4	1.187470	1.000000	1.187470	1.000000	1.000000	1.000000
DMU_5	1.317610	1.003840	1.317610	1.003840	1.000000	1.000000
DMU_6	1.153813	1.081662	1.200930	1.112177	0.960766	0.972563
DMU_7	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
DMU_8	1.441813	1.273573	3.322411	3.286301	0.433966	0.387540
DMU_9	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
DMU_{10}	1.544226	1.442728	2.642039	2.571409	0.584483	0.561065
DMU_{11}	1.547021	1.445160	2.642039	2.571409	0.585541	0.562011
DMU_{12}	1.490083	1.326583	2.438829	2.196879	0.610983	0.603849
DMU_{13}	1.093462	1.076467	1.670299	1.496591	0.654650	0.719279
DMU_{14}	1.326380	1.272758	1.536536	1.485863	0.863227	0.856578
DMU_{15}	1.186873	1.140410	1.373352	1.351527	0.864216	0.843794
DMU_{16}	1.020576	1.000000	1.020576	1.000000	1.000000	1.000000
DMU_{17}	2.000919	1.000000	2.000919	1.000000	1.000000	1.000000
DMU_{18}	1.000000	1.000000	1.254413	1.143544	0.797186	0.874474
DMU_{19}	1.326964	1.172336	2.806542	2.774999	0.472811	0.422464
DMU_{20}	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
DMU_{21}	1.124510	1.000000	1.756893	1.321003	0.640056	0.757001
DMU_{22}	1.237353	1.000000	1.237353	1.000000	1.000000	1.000000
DMU_{23}	1.152525	1.000000	1.205580	1.000000	0.955992	1.000000
DMU_{24}	1.018877	1.000000	1.516305	1.482636	0.671947	0.674474
DMU_{25}	1.632793	1.496785	3.325000	2.232217	0.491066	0.670537
DMU_{26}	1.504095	1.142825	2.957946	2.583540	0.508493	0.442348
DMU_{27}	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
				Mean	0.752	0.779

all inputs (fixed and variable) and capacity output of each unit with only fixed inputs respectively. We assume that ρ^* and ρ_F^* are the optimal value of models (2.3) and (3.5) respectively. Then, using formula (3.6), we obtain each DMU's CU. Table 2 reports the estimated CU for each of these DMUs in two different positions (two-stage and black box).

It should be noted that in the black box position, the intermediate measures are disregarded. In this position, the DMUs are treated as one-stage production processes that transform inputs to outputs.

The comparison of estimated CU scores from the two different positions leads to following conclusions. According to Table 2, due to taking or

ignoring the intermediate measures, the CU value for special DMU is not necessarily the same in the both positions. For example consider DMU_{23} , regarding the assessment of this unit in black box position $CU=1$ i.e., even with applying the current fixed inputs it is unable to produce more output while if it is analyzed in two-stage form, its CU is less than 1 and has excess capacity in two-stage position. Namely this firms possesses a potential to produce more output with the current fixed inputs. So, what is clear is that taking the intermediate measures or their negligence affect the firm's ability to access certain outputs levels. In Table 2, the average score of CU is 0.752, indicating that 24.8% of the capacities of DMUs are not utilized.

The received results of Table 2 in two-stage position show that only nine of all DMUs would operate with full capacity. These DMUs (4, 5, 7, 9, 16, 17, 20, 22 and 27) fully utilize their fixed inputs. In other words only 33% of total units operate with full capacity and 67% of DMUs has excess capacity because their corresponding CU are estimated less than 1. Therefore, they could increase their outputs without alternation in level of fixed inputs. Note that insufficient employees results to low CU. According to Table 2, the lowest capacity utilization which equals 0.434 is associated to DMU_8 . (i.e. it only utilizes 43.4% of its capacity).

Based on obtained results of CU in this position, rather greater proportion of DMUs (51%) exhibit CU measures between 0.8-1. In the other words, 51% of total DMUs use higher than 80% of their capacity. To check the results of Table 2, we compute the CU score of DMU_{14} . Using models (2.3) and (3.5), in the evaluation of this unit, the following optimal values are obtained

$$\rho^* = 1.326380 \quad \text{and} \quad \rho_F^* = 1.536536$$

Now, applying relation (3.5), we calculate its corresponding CU as follows:

$$CU_{(DMU_{14})} = \frac{\rho^*}{\rho_F^*} = \frac{1.326380}{1.536536} = 0.863227 \approx 0.86 \text{ or } 86\%$$

Since $CU_{(DMU_{14})} < 1$ then it has excess capacity and could produce more than its current levels.

In other words, this measure illustrates that there is a possibility to improve its production by 14% without additional fixed inputs such as the hiring new employees. Similar analyses can be carried out for other firms (DMUs).

5 Conclusion

The classical DEA models regard the production systems as a whole in which the evaluation performance is exclusively effective in external inputs and final outputs. Thus, the obtained efficiency scores of these models are often inaccurate. Recently, network DEA models are introduced to accurately evaluate and study their internal structure. The present article has estimated capacity utilization measure in two-stage production system using non-radial network DEA model. Since in the short-run, the degree of capacity utilization depends on the ability of DMUs to utilize their fixed factors consequently the factors of production are categorized into fixed and variable inputs. In this regards, the SBM-based network DEA model is proposed to develop the Cooper et al.'s work. To show the effect of intermediate measures on CU scores, numerical example is considered in black box and two-stage positions and then model is used to compare the results with Cooper et al.'s approach (classical DEA model). The estimated CU scores for some of DMUs were not necessarily the same in the both positions because of taking or ignoring the intermediate measures. The results of numerical example indicate that more than half of the DMUs should improve their capacity utilization. Furthermore, it was noted that the DMUs which their corresponding CU is less than 1, do not fully utilize their inputs and also there is a possibility to improve the outputs without adding current fixed inputs. The obtained measure of proposed model not just demonstrates intermediate measures effect but reflects firms' thoughts on interpolation decision making. The suggestion provided for the prospective researcher is to develop the fuzzy and stochastic version of proposed model. Besides, application to negative data model are potential subjects for future research.

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