Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 11, No. 3, 2019 Article ID IJIM-1176, 9 pages
Research Article

The Comparison of Two Multiple Testing Methods for Outliers Detection in Nonparametric Profile Monitoring

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Received Date: 2018-03-12 Revised Date: 2018-07-15 Accepted Date: 2019-03-18

Abstract

Multiple testing is one of the methods to analyze such data. We apply multiple testing methods in profile monitoring. Profile monitoring is a relatively new method of quality control, which is best used when data processing is performed using curves or profiles within a given period. In this research, we introduce different multiple testing methods for outlier profile detection. We compare the performance of these methods by simulations.

Keywords : Local linear Kernel Smoothing; Multiple testing; Nonparametric Mixed-effects models; Parametric Bootstrap; Profile monitoring.

1 Introduction

Profile monitoring is one of the most important subjects in statistical process monitoring (SPM) that has been recently developed by researchers. A profile is created when an important performance quality indicator depends on one or more independent variables. Instead of observing a measurement in each product, a set of values is considered and by drawing them, a curve is obtained. In the other words, there is a response variable y and one or more explanatory variables $x_1, x_2, , x_n$, similar to conditions used in Regression Analysis. The fundamental idea for profile monitoring is modeling profiles by parametric, nonparametric or semi parametric methods. A specific model is later utilized to determine whether the variations exist in profiles. The profile monitoring methods can be performed in phase I or II. In Phase I, by historical data, information about variation of profiles over time, evaluating the stability of profiles and estimating in control parameters of model, are obtained. In Phase II, the task is to detect rapid shifts from parameters of model base upon designed in phase I. In phase I, a constant value is considered for the probability of type I error and the performance of proposed method is examined by its power function. Instead, in Phase II the ARL[1] control charts are used to measure the performance.

[17] and [21] have conducted studies in the field of simple linear profile monitoring. [22] and [25] studied VDP[2]s of the particle boards to show there are so many situations in which the pro-

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[1] Average run length
[2] Vertical density profile

file cannot be represented by linear model. Nonlinear profiles are to be used under such conditions. [24] suggested three general methods for nonlinear profile monitoring. [14] represented two types of chart scheme for phase I and II profile monitoring. One of them multiple T^2 Hotelling chart and the other combined EWMA[3] chart with variation chart. [14] used the combination three EWMA chats to detect changes in intercept slope and standard deviation simultaneously in Phase II. They also used Shewhart control charts for simple linear profile monitoring. [16] suggested a univariate control chart for monitoring standard deviation error and with F test for monitoring regression coefficient, in Phase I. Also [13] proposed Phase I monitoring of polynomial profiles. [9] compared the performance of control charts proposed by [5] and [14] for simple linear profiles monitoring in Phase II, and concluded that the proposed method by [14], has more desirable performance.

Parametric models have some underlying assumptions, in most studies concerning profile monitoring. In practice, however, these may not be reasonable assumptions. Therefore, nonparametric models which are not predetermined function can be partially solved the profile monitoring problems. [29] considered the nonparametric modeling of profiles for Phase II profile monitoring. [28] treated profiles as vectors in a high-dimensional space and applied a ^{2}-type control chart to identify outliers by comparing each profile to a central vector. Also, [23] developed a nonparametric L1 location-scale model to screen shapes of profiles in Phase II analysis. Likewise, [18] described within-profile correlation by using a nonparametric mixed-effect model. They proposed Phase II control chart for nonparametric profiles based on the local linear kernel smoothing of profiles and an EWMA weighting scheme. Recently, [1] introduced parametric, nonparametric and mixed model robust profile monitoring, based on a version of Hotellings T^2 statistics which is employed in Phase I analysis for identifying outlying profiles.

Furthermore, high dimensional data studies in which evaluating more than one question at the same time are abundant. Simply multiple testing is about a set of statistical hypotheses that are tested simultaneously. Clearly there needs to be analysis of complex sets of data in the form of thousands hypotheses tests simultaneously instead of analysis the large set of data individually. In terms of simultaneous testing, the probability of one or more false rejections is called Family-Wise Error Rate (FWER). When testing several hypotheses, the FWER, should not exceed a nominal level (e.g. at the α level) for all possible sets of true and false hypotheses. Hence, the following requirement:

$$\text{FWER} \leq \alpha$$

is needed. This notion is called strong error control. For controlling FWER, the first classic single-step procedure proposed by [4]. [20] proposed a method similar to the Bonferroni method. [11] and [12] then introduced other methods to improve Bonferroni procedure. In order to further improve the special case of stepdown procedure that proposed by Holms and find the optimal method among the classes of multiple testing problems step-down MaxT (Maximin) procedures were presented by [15]. [7] used nonparametric estimation with multiple testing for detecting outliers, also [2] applied linear and polynomial profile monitoring by Bonferroni correction in multiple testing. [8] defined a simple statistic to detect outliers and [19] proposed two new methods for detecting outlying profiles in phase I nonparametric profiles within step-down MaxT procedure in multiple testing. In this paper, we compare two methods in multiple testing, step-down MaxT and Bonferroni method for control FWER in phase I nonparametric profile monitoring.

This article is organized as follows: Section 2 provides a detailed description of the proposed methodology; Section 3 investigates the performance of our proposed method in comparison with each other via simulation studies. Section 4 presents concluding remarks and a discussion.

[3] Exponentially Weighted Moving Average

2 The Nonparametric Mixed Effect Profiles Monitoring Approach in Phase I

According to [18], nonparametric mixed-effects (NME) models consist of m profiles, and the i^{th} profile has been observed in n_i design points.

Let $(x_{i1}, y_{i1}), (x_{i2}, y_{i2}), \ldots, (x_{in_i}, y_{in_i})$ denote the pairs of sample points collected on the i^{th} profile. Then the following in control (IC) process is:

$$Y_{ij} = g(x_{ij}) + f_i(x_{ij}) + \epsilon_{ij} \quad j = 1, 2, \ldots, n_i \quad i = 1, 2, \ldots, m \tag{2.1}$$

Here, g is the IC population profile function, called the fixed-effect term, f_i is the random-effect term which explains the profile deviation of the i^{th} profile from the population average profile g and the errors ϵ_{ij} are i.i.d. random variables with mean 0 and variance σ^2 which are assumed to be independent from the random effects f_i. In addition, we assume that the random effects f_i are realizations of a process with the mean 0 and the common covariance function

$$\gamma(x_1, x_2) = E[f_i(x_1), f_i(x_2)], \quad i = 1, 2, \ldots, m. \tag{2.2}$$

Without loss of generality, we assume that the design points $x_{ij} \in [0, 1]$, for all i and j.

2.1 Proposed Methodology and Parameter Estimation Model

[3] indicated that outliers are observations appearing to deviate markedly from other members of the sample in which they occur. Similarity [27] defined outliers in a functional dataset as the observations whose means are signicantly different from the others. Our proposed for phase I profile monitoring based on [27] definition.

According to this definition, we can state the case of no outlying profile or curve in the samples, as the following null hypothesis

$$H : g_i(t) = g_0(t) \quad \forall t, i = 1, 2, \ldots, m \tag{2.3}$$

Against the existence of some outlying profile among data, that can be stated as the following alternative hypothesis:

$$K : \exists \mathcal{A}_m \subseteq \{1, \ldots, m\} \begin{cases} g_k(t) = g_0(t) \ \forall t \ \forall k \notin \mathcal{A}_m \\ g_k(t) \neq g_0(t) \ \exists t \ \forall k \in \mathcal{A}_m \end{cases} \tag{2.4}$$

In both H and K assumptions, $g_0(t)$ is the IC common fixed effect term that considers unknown. $g_k(t)$, for any $k \in A_m$, is k^{th} out of control (OC) outlying population profile function and A_m is the set of outlying profiles.

Under the null hypothesis H, there is no outlying profile, therefor under this assumption; the model (2.1) is reduced to:

$$Y_{ij} = g_0(x_{ij}) + f_i(x_{ij}) + \epsilon_{ij} \quad j = 1, 2, \ldots, n_i, \quad i = 1, 2, \ldots, m, \tag{2.5}$$

Against, under K the observation follow the model:

$$Y_{ij} = \begin{cases} g_i(x_{ij}) + f_i(x_{ij}) + \epsilon_{ij} & i \in \mathcal{A}_m \\ g_0(x_{ij}) + f_i(x_{ij}) + \epsilon_{ij} & i \notin \mathcal{A}_m \end{cases} \tag{2.6}$$

To fit the model (2.1), using local linear mixed effect smoothing method that proposed by [26]. Let $\mathbf{z}_{ij}^T = (1, x_{ij} - s), \boldsymbol{\beta}^T = (g_0(s), g_0'(s))$ and $\mathbf{b}_i^T = (f_i(s), f_i'(s))$ replace in (2.1) within the neighborhood of $s \in [0, 1]$ and g' and f_i' are the derivatives of the functions g and f_i, respectively. By minimizing the following penalized weighted sum of squares, estimate $\boldsymbol{\beta}, \mathbf{D}$ and σ^2

$$\sum_{i=1}^{m} \left\{ \frac{1}{\sigma^2} \sum_{i=1}^{n_i} [y_{ij} - z_{ij}^T(\boldsymbol{\beta} + \mathbf{b}_i)]^2 K_h(x_{ij} - s) + \mathbf{b}_i^T \mathbf{D}^{-1} \mathbf{b}_i + \ln(|\mathbf{D}|) + n_i \ln(\sigma^2) \right\}$$

Where K_h is a symmetric kernel density function, define $K_h(t) = h^{-1}K(t/h)$ for $t \in R$ where h is the bandwidth. It is assumed that the vectors of random effects b_i are independent and identically distributed (Normal) with a mean of 0 and 2×2 covariance matrix $D = E[b_i b_i^T]$. [19] presented detailed information and the solution of an iterative algorithm in detail. Then the estimates

Table 1: The proportion of true signal (shifts) for out-of-control profiles. Data are generated with $m = 50, n = 40$ and type I error $\alpha = 0.05$

Shifts	Methods	FCC	Specificity	Sensitivity	FNR
2* 0.05	MaxT	0.83576	0.91232	0.89861	0.276748
	Bonferroni	0.863255	0.887213	0.79344	0.332401
2* 0.1	MaxT	0.82598	0.91343	0.87918	0.264098
	Bonferroni	0.873256	0.88983	0.78974	0.312061
2* 0.15	MaxT	0.82657	0.912212	0.87918	0.274128
	Bonferroni	0.87567	0.88543	0.78974	0.322061
2* 0.2	MaxT	0.82877	0.91323	0.912218	0.25328
	Bonferroni	0.87998	0.88785	0.80000	0.342122
2* 0.25	MaxT	0.82904	0.91413	0.91305	0.2588
	Bonferroni	0.87891	0.88851	0.80072	0.303222
2* 0.3	MaxT	0.92384	0.96302	0.93259	0.29898
	Bonferroni	0.97844	0.89251	0.812881	0.30435

Table 2: Empirical FWER of the proposed Methods with $m = 50$ and $n = 20, 40$ at the nominal levels $\alpha = 0.05$

2*b	2* Methods	$m = 50$	
		$n = 20$	$n = 40$
2* 0.1	$MaxT$	0.0465	0.0430
	Bonferroni	0.0295	0.0265
2* 1	$MaxT$	0.0480	0.0450
	Bonferroni	0.0305	0.0312
2* 1.1	$MaxT$	0.0490	0.0470
	Bonferroni	0.0325	0.0322

Table 3: Decision and Actual set in multiple testing

Actual set Decision	Out of control process	In control process
Out of control process	A	B
In control process	C	D

obtain as:

$$\hat{g}_0(s) = \mathbf{e}_1^T \widehat{\boldsymbol{\beta}}(s)$$
$$\hat{f}_i(s) = \mathbf{e}_1^T \widehat{\mathbf{b}}_i(s)$$
$$\hat{\gamma}(s_1, s_2) = \frac{1}{m} \sum_{i=1}^{m} \hat{f}_i(s_1) \hat{f}_i(s_2)$$
$$\text{for any } s_1, s_2 \in [0, 1], \quad (2.7)$$

Where $\mathbf{e}_1 = (1, 0)^T$. [18] suggested the following nonparametric estimator

$$\hat{\sigma}^2 = \frac{1}{m} \sum_{i=1}^{m} \frac{1}{n_i} \sum_{j=1}^{n_i} [y_{ij} - \hat{g}_0(x_{ij}) - \hat{f}_i(x_{ij})]^2, \quad (2.8)$$

similar to that of [10], for σ^2 parameter.

As mentioned before, identifying an outlying profile is a test problem with the assumption that profiles are a realization of a process with average $g_0(x)$ or it is from a process with a different average $g_i(x)$. Since this test must be performed for all observations, it is appropriate to assume this assumption test as a multiple assumption testing

$$H_{oi} : g_i(x_{ij}) = g_0(x_{ij}), \ \forall j = 1, 2, \ldots, n_i$$
$$H_{1i} : g_i(x_{ij}) \neq g_0(x_{ij}), \ \exists j = 1, 2, \ldots, n_i \quad (2.9)$$

The H assumption is rejected if and only if the first assumptions of H_{oi}s is rejected. Let

$$\mathbf{d}_{ij} = Y_{ij} - \hat{g}_0(x_{ij}) - \hat{f}_i(x_{ij}),$$
$$i = 1, 2, \ldots, m, j = 1, 2, \ldots, n_i$$

Where $\hat{g}_0(x_{ij})$ and $\hat{f}_i(x_{ij})$ are obtained from (2.7). If there is no outlying profile in phase I, all assumptions $H_{01}, \ldots, H_{0m}$ are true, and therefore equation (2.5) must hold true where all values $|\mathbf{d}_{ij}|$ are expected to be small. Hence we define

$$T_i = \sum_{j=1}^{n_i} \mathbf{d}_{ij}^2 \quad i = 1, 2, \ldots, m \tag{2.10}$$

It is clear that the large values of such statistics can be evidence of the rejection of the assumption H_{oi}. Note T_i represents the deviation value of the i^{th} observed profile from the respective fitted profile. A test statistic is based on the order statistic T_i, i.e.

$$T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(n)} \tag{2.11}$$

2.2 Multiple Testing Approaches

Consider multiple hypothesis testing consisting of m null hypothesis H_{oi} versus H_{1i} according to (2.9). The most important Type I error rate is FWER[4] which indicates the probability of at least one Type I error. The most common approach for controlling FWER in multiple testing is Bonferroni method that controls FWER for the joint distribution of the test statistics corresponding to the null hypotheses. For controlling FWER in a nominal significant level of α single-step Bonferroni method, reject null hypothesis H_{oi} whenever the corresponding p-value is less or equal to the single-step $\frac{\alpha}{m}$ cut-off point (determine rejection regions for every null hypotheses). Note that if $\frac{m_0}{m}$ is very small, Bonferroni method will yield a conservative result.

In order to further improvement and find the more convenient way among one class of multiple testing problems, [15] presented step-down MaxT (Maximin) procedure. Suppose test statistics T_i ordered as $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(m)}$ and also null hypotheses corresponding to test statistics as $H_{0(1)}, H_{0(2)}, \ldots, H_{0(m)}$. Reject null hypothesis $H_{0(m)}$, corresponding to the m^{th} most signicant test statistic $T_{(m)}$, if $T_{(m)} \geq C_1$ and $T_{(m-1)} < C_2$. For a given $i = 1, 2, \ldots, m$ and $0 < \alpha < 1$, let C_i be the threshold determined by $P(T_{(i)} \geq C_i) = \alpha$.

Not rejecting any of the null hypotheses $H_{0(1)}, H_{0(2)}, \ldots, H_{0(m)}$ if $T_{(m)} < C_1$.

As pointed out in Lemma 9.2.1 by [15], this procedure controls the FWER at level α.

[4]Family Wise Error Rate

2.3 Determining cut-off points of MaxT method by Bootstrap Technique

In order to obtain threshold C_i, the test statistic distribution under null hypothesis is required, however, since the distribution of these tests are complicated, we can use parametric bootstrap procedure to estimate the threshold points corresponding to test statistics. Parametric bootstrap procedure in the following algorithm shows how to obtain these points.

Step1. Apply equations (2.7) and (2.8) to estimate, g_0, f_i and σ^2 by $\hat{g}_0, \hat{f}_i$ and $\hat{\sigma}^2$, respectively and generate B independent bootstrap samples of m profiles from model (2.5) under null hypothesis based on $\hat{g}_0, \hat{f}_i$ and $\hat{\sigma}^2$.

Step2. For $r^{th}(r = 1, 2, \ldots, B)$ bootstrap sample, let $\hat{g}_0^{(r)}$, $\hat{f}_i^{(r)}$ and $\hat{\sigma}^{2(r)}$ denote the respective bootstrap estimates that are obtained from equations (2.7) and (2.8).

Step3. For $r^{th}(r = 1, 2, \ldots, B)$ bootstrap sample, compute test statistics $T_1^{(r)}, T_2^{(r)}, \ldots, T_m^{(r)}$ from (2.10) and then obtain maximum statistic for each bootstrap sample $S_i^{(r)} = \max\left(T_1^{(r)}, T_2^{(r)}, \ldots, T_i^{(r)}\right)$ $i = 1, 2, \ldots, m$.

Step4. For any $i = 1, 2, \ldots, m$, approximate the threshold C_i by $\lfloor \alpha B \rfloor$ largest value among $S_i^{(1)}, S_i^{(2)}, \ldots, S_i^{(B)}$, where $\lfloor \alpha B \rfloor$ denotes the greatest integer less than or equal to α B (e.g. B=200 and $\alpha = 0.05$ equals to 10).

3 Simulation of Bonferroni and MaxT for controlling FWER in Phase I profile monitoring

In this section, in order to compare the performance of two procedures in multiple testing for controlling FWER in Phase I profile monitoring, we present amount of simulation study in different situations. Using different in-control or out-of-control designs, reporting empirical experimental and representing power test of proposed statistics indicate the superiority of MaxT than Bonferroni procedure in multiple testing. For this purpose, first consider nonparametric mixed effect model which pursues an in-control process and then compare type I error and finally obtain power test of two methods MaxT and Bonferroni in the case of model contaminated by out-of-control design to show the efficiency of two methods to detect outlying profile.

3.1 The Empirical Size of Tests

According to (2.5) consider:

$$Y_{ij} = g_0(x_{ij}) + f_i(x_{ij}) + \epsilon_{ij}$$
$$j = 1, 2, \ldots, n_i, \quad i = 1, 2, \ldots, m$$

For determining the false alert rate in Phase I profile monitoring, we consider the following one scenario (second degree) under in-control model that considered above for g_0:

$$g_0(x) = 1 + 2(x - 0.5)^2 \tag{3.12}$$

For simplicity, it can also be assumed that the number of sample points in each profile is equal (i.e. $n_i = n$). Furthermore, designed points $x_{i1}, x_{i2}, \ldots, x_{in_i}$, are the same for all profiles $x_1, x_2, \ldots, x_n$. For all the proposed schemes, the random effect is $f_i(x_j) = b\alpha_i x_j$ in which $\alpha_1, \alpha_2, \ldots, \alpha_m$ is a random sample with standard normal distribution and b is a constant value that can be selected from $\{0.1, 1, 1.8\}$. Then we have

$$Y_{ij} = 1 + 2(x - 0.5)^2 + b\alpha_i x_j + \epsilon_{ij}$$
$$j = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, m$$

In order to estimate the parameters of the model (2.5) by the local linear kernel smoothing method, we use the following Epanechnikov kernel function

$$K(x) = \frac{3}{4}(1 - x^2) \text{ for } -1 < x < 1. \tag{3.13}$$

This function has optimal properties expressed in detail [26].

For bandwidth selection, first we apply cross validation procedure for choosing the best bandwidth and then compare it by [26] proposed, that is a combination of leave-one-subject-out and leave-one-point-out cross-validation schemes. However, we found the empirical bandwidth suggested by [18] more suitable for our purpose in the current paper. The empirical bandwidth of [18] is given by

$$h_E = cn^{-\frac{1}{5}} \left(\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n} \right)^{\frac{1}{2}}, \tag{3.14}$$

where n is the number of fixed design points in (2.5). c is any value in the interval $[1, 2]$ and $\bar{x}$ is the mean of $x_1, x_2, \ldots, x_n$ (i.e. $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$). Additionally, we assume the fixed design points $x_1, x_2, \ldots, x_n$ are randomly chosen from the Uniform $(0, 1)$ distribution and consider the number of profiles and design points as $m = 50, n = 20$ and $n = 40$. Thus, proles represent from the in-control process. Based on the parametric Bootstrap method defined by algorithm *2.3*, the thresholds are obtained by Monte Carlo simulation study via performing $5,000$ replications. This method computes the FWER at the nominal level of $\alpha = 0.05$ for proposed scenario in Bonferroni and MaxT procedures. For empirical FWERs in Phase I of a control chart, the proportion of remote misconfigured profiles is shown in Table 2. Note that these values are very close to the significant nominal level, especially when n increases in two ways. However, in Bonferroni method usually the FWER is much less than nominal level due to its conservative properties.

3.2 Power Study

As mentioned in the single hypothesis test, the power function is a measure or a criterion for determining the ability of a test to truly reject a null hypothesis. However, in the multiple hypothesis tests, there is no single unit power measure; therefore, the power of a test is evaluated by several different measures or criteria. The two methods have the ability to detect better outliers by computing the following performance metrics: fraction correctly detected (FCD), false positive rate (FPR), and false negative rate (FNR). The definitions of these terms are given below. In Table 3, “A” represents the number of profiles from the out-of-control process that are correctly identified as from the out-of-control process and “D” represents the number of profiles from the in-control process that are correctly identified as the in control process after the Phase I analysis. The value “B” represents the number of profiles that are obtained from the out-of-control process but mistakenly identified as in-control process while “C” represents the number of profiles that are obtained from the in-control process but are identified as out-of-control process.

Considering table 3, the FCD can be defined as

$$FCD = \frac{(A + D)}{(A + B + C + D)} \tag{3.15}$$

The specificity, on the other hand, represents the ability to identify those profiles obtained from the in-control process correctly as profiles derived from the in-control process and is defined as

$$\text{Specificity} = \frac{D}{(C + D)} \tag{3.16}$$

The Sensitivity is the fraction of those profiles identified as from the out-of-control process that are actually from the out-control process. The FPR is the fraction of those profiles identified as out-of-control process that are actually in-control process. The Sensitivity and FNR values are defined as

$$\text{Sensitivity} = \frac{A}{(A + B)} \tag{3.17}$$

and

$$\text{FNR} = \frac{C}{(A + C)} \tag{3.18}$$

It is easy to show that all these metrics are bounded by 0 and 1 and that a method will perform well in Phase I analysis by achieving large values for FCD, Specificity and Sensitivity but small values for FPR. Because FCD, Sensitivity and Specificity are three power measures that are adopted in the present performance study of the proposed Phase I profile monitoring. FPR are error measure.

Simulations are based on different in-control and out-of-control models. In accordance with the two models under the control of the preceding section, as well as with (3.12) and (3.13), we consider the following model for out-of-control profiles:

$$g_i(x) = 1 + 2(x - 0.5)^2 + i, \quad i = 1, 2, \ldots, m. \tag{3.19}$$

Note that in the case of out of control, each outlying profile follows its specific model. Then we have

$$\begin{aligned} Y_{ij} &= \rho(1 + 2(x - 0.5)^2 + i + b\alpha_i x_j + \epsilon_{ij}) \\ &\quad + 1 + 2(x - 0.5)^2 + b\alpha_i x_j + \epsilon_{ij} \\ j &= 1, 2, \ldots, n, \quad i = 1, 2, \ldots, m \end{aligned} \tag{3.20}$$

Since the performance of the tests depends on the proportion of outlying profiles, in order to investigate this effect in simulation study, compare two methods (Bonferroni and MaxT) to applying different proportions of outlying profiles. Suppose ρ is the ratio of profiles. In the simulation study of the power under each of the values of $\rho(0.05, 0.1, 0.15, 0.2, 0.25, 0.3)$ the out of control model (3.20) was carried out. The specificity, the FCD and the Sensitivity to the proposed two methods under the model (3.20) for $m = 50, n = 40$ and $b = 1$ were calculated.

4 Conclusions

In this paper, we proposed 2 multiple testing methods for outlier detection in profile monitoring. These methods are based on the nonparametric mixedeffect model, specifically LLME approach. Indeed, these methods are adopted the phase II nonparametric profile monitoring proposed by [18] and according to [19] in phase I profile monitoring, with differences that in this article we compare Bonferroni method with MaxT proposed in [19]. We determined the p-values of these methods according to $T_i = \sum_{j=1}^{n_i} d_{ij}^2$s by a parametric bootstrap technique. Our simulation studies demonstrated that the MaxT method outperform, in three performance criteria, than Bonferroni method. We discuss Sidaks procedure with step-down MaxT later. We suggest other multiple testing procedures such as adaptive multiple testing and multiple testing under the dependency conditions be applied for outlier detection in profile monitoring, and a comprehensive study of their performance is to be conducted.

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