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Research Article

A New Voting Model For Groups With Members of Unequal Power and Proficiency

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Abstract

Cook et al. [*A data envelopment model for aggregating preference rankings, Management Science 36 (1990) 1302-1310*] used data envelopment analysis (DEA) as their starting point to propose a procedure for rank ordering the candidates in a preferential election. Notionally, each candidate is permitted to choose the most favorable weights to be applied to his/her standings (first place, second place, etc. votes) in the usual DEA manner with the additional assurance region (AR) restriction that the weight for a j place vote should be more than that for a j+1 place vote by some amount. However, DEA very often identifies more than one candidate in a voting system to be DEA efficient. So, a method to discriminate between these efficient candidates is needed. In addition to proposing a voting model for groups with members of unequal power and proficiency, in this paper we present some models for rank ordering efficient candidates, by extending the ideas of some authors. Then, we propose a new methodology to rank the ranking models for the performance indices of only DEA efficient candidates based on a classical voting model. Also, an approach for combining the results obtained from the ranking models is presented. Finally, we employ the new method to rank order five Iranian automobile manufacturing groups in terms of "job independence".

Keywords : Data Envelopment Analysis; Voting; Group decision support system; Ranking.

1 Introduction

An important issue in the decision-making framework is how to obtain a social ranking or a winning candidate from individuals' preferences on a set of candidates$\{A_1, A_2, , A_n\}$. In some voting systems, each voter selects s candidates and ranks them from most to least preferred (ranked voting systems). Among these systems, a well-known procedure to obtain a social ranking or a winning candidate is scoring rule, where fixed scores are assigned to the different ranks. In this way, $Z_j = \sum_{r=1}^{s} u_r y_{rj}$ is the score obtained by the candidateA_j, where y_{jr} is the number of rth place ranks that candidate A_j occupies and $(u_1, u_2, , u_s)$ is the scoring vector used. The plurality rule,where $u_1 = 1$ and $u_r = 0$ for all $r = 2, , s$, and the Borda rule, where $s = n$ and $u_r = n - r$ for all $r \in 1, 2, , s$, are the best known examples of scoring rules. It is worth noting that the Borda [4] rule has interesting properties in relation to other scoring rules (Brams and Fishburn, [5]). Let A denote a finite set of 'alternatives' (e.g., candidates, proposals, sports teams),

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and let V denote a finite set of individuals (e.g., voters, experts, sports writers).

A voter's input to Borda's Method (i.e., a 'ranked ballot') consists of an ordered list of a subset B of the alternatives in A, with either a symbol like " > " (is preferred to) or a symbol like "=" (is tied with) separating the alternatives in the list. Also, no alternative is allowed to be listed twice (or more) on any ranked ballot considered here.

Hence, all ranked ballots considered below are transitive, and so any matrix ballot that corresponds to a ranked ballot is also transitive. However, a matrix ballot that does not correspond to a ranked ballot need not be transitive. On a ranked ballot, only a subset, say B, of the alternatives in A need to be ranked, and any alternative that is not ranked (i.e., is in the set A-B) is considered inferior to all alternatives in B on that ballot. The alternatives are scored by awarding 'points' according to the following rules: For each ballot, Borda [4] suggested that each alternative ranked on that ballot be given one point for each alternative ranked below it on that ballot.

Also, if ties are allowed and alternative a is tied with exactly k other alternatives on a ballot, then a is given 1/2 of a point for each of those k alternatives, for a total of k/2 points for those tied alternatives on that ballot. The score for alternative a is found by summing (over all ballots) the number of points a earns, and the ranking is determined by these scores.

Indeed Borda's Method does enjoy (at least) four properties that many other voting methods do not. These four properties have been called (see, e.g., Young [29] and Young [30]): positive involvement, negative involvement, strong participation, and multi-district consistency. Positive Involvement: A ranked voting method has positive involvement if the following property holds. Given a set of ranked ballots, suppose that the voting method in question selects alternative a as the winner. Suppose that the selection process were to be re-held with additional voters, all of whom rank a as their unique first choice. Then a must still be selected if none of the original voters change their ballots, no matter how the additional voters rank the alternatives below a on their ballots. Negative Involvement: A ranked voting method satisfies negative involvement if the following property holds. Given a set of ranked ballots, suppose that the voting method in question does not select alternative b as a winner. Suppose that the selection process were to be re-held with additional voters, all of whom rank all the alternatives, and all of whom rank b as their unique last choice. Then b must still not be selected if none of the original voters change their ballots, no matter how the additional voters rank the alternatives above b on their ballots. Strong Participation: A ranked voting method has strong participation if the following property holds. Given a set of ranked ballots, suppose that the voting method in question selects alternative a as the winner. Suppose that the selection process were to be re-held with additional voters, all of whom rank a ahead of alternative b. Then b must not be selected if none of the original voters change their ballots, no matter how the additional voters rank the other alternatives on their ballots. A voting method that has strong participation necessarily has both positive involvement and negative involvement, but not vice versa. Multi-district consistency: A voting method satisfies the property of multidistrict consistency if the following holds. Suppose the voters can be divided into two groups, or panels, in such a manner that, considering the ballots of each panel separately, the method would declare some alternative, say a, to be winner for each panel. Then, if the ballots are not changed, the method must declare a to be the winner if the two groups are combined into a single panel.

2 Literature Review

Cook and Kress [6] suggested evaluating each candidate with the most favorable scoring vector for him/ her. With this purpose, they introduced data envelopment analysis (DEA) in this context. The data envelopment analysis/ assurance region (DEA/AR) model proposed by these authors is as follows:

$$
\begin{aligned}
& Z_0 = max \sum_{r=1}^{s} u_{or} y_{or} \\
& s.t. \\
& \sum_{r=1}^{s} u_{or} y_{jr} \leq 1, \qquad j = 1, \dots, n, \\
& u_{or} - u_{or+1} \leq d(r, \epsilon), \quad r = 1, \dots, s-1, \\
& u_{os} \geq d(s, \epsilon).
\end{aligned}
\tag{2.1}
$$

where $\epsilon \geq 0$ and the functions $d(r, \epsilon)$, called the discrimination intensity functions, are non-negative and non-decreasing in ϵ and also, $d(r, 0) = 0$ for all $r \in \{1, \dots, s\}$. They show that in the special case where $d(r, \epsilon) = \epsilon$, their model is equivalent to the consensus model of Borda. After the problems are solved for all candidates, several (not only one) candidates often achieve the maximum attainable score 1. We call these candidates efficient candidates. We can judge that the set of efficient candidates is the top group of candidates, but cannot single out only one winner among them or rank them. To avoid this weakness, Cook and Kress [6], suggest maximizing the gap between the weights so that only one candidate is left DEA efficient. This has been found equivalent to imposing a common set of weights on all candidates and therefore Green et al. [12], propose the use of the cross-efficiency evaluation technique in DEA to choose the winner. Noguchi et al. [19], also use the cross-efficiency evaluation to get the best candidate and give a strong ordering constraint condition on weights. Hashimoto [13] also proposes using the DEA exclusion model (Andersen and Petersen [2]) with $d(., \epsilon) = \epsilon$, where ϵ is a positive non- Archimedean infinitesimal. Wang and Chin [28], discriminate efficient candidates by considering their least relative total scores. They also propose a model in which the total scores are measured within an interval. Soltanifar [22] introduces an interval efficiency which consists of efficiencies obtained from the optimistic and pessimistic viewpoints. A minimax regret-based approach (MRA) is used to compare and rank the efficiency intervals of candidates in his method. Obata and Ishii [20], suggest excluding non-DEA efficient candidates and using normalized weights to discriminate the DEA efficient candidates. Their method is subsequently extended to rank non-DEA efficient candidates in Foroughi and Tamiz [7] (see also Foroughi et al. [8]).Soltanifar [23] proposes a new methodology to rank the common weight models for the performance indices of only DEA efficient DMUs based on model (12). Also, Soltanifar et al. [24] propose a new methodology to rank the ranking models for the performance indices of only DEA efficient candidates based on model (12). Model (12) was used by Soltanifar and Hosseinzadeh Lotfi [25] to propose new Voting Analytic Hierarchy Process (VAHP) method for ranking efficient DMUs. Llamazares and Pena [14]have analyzed the principal methods proposed in the literature to discriminate efficient candidates. The main conclusion of their study is that none of the proposed procedures is fully convincing. More application of classical voting model can be found in Post [21], Galanis et al. [10], Ma et al. [17] and so on. Ebrahimnejad [9] published a paper in which he described ranking in voting systems. But our work follows a case study design, with in depth analysis voting model for groups with members of unequal power and proficiency. In this paper, we present a voting model for groups with members of unequal power and proficiency, and further provide some models for ranking efficient candidates by extending the ideas of some authors. Then, we select the best ranking model from among those models by a classical voting model. The rest of this paper is organized as follows. Section 3 provides a voting model for groups with members of unequal power and proficiency, based on the DEA approach. Section 4 contains a review and extension of the ideas of some authors and establishing ranking models to distinguish between efficient candidates. In Section 5, we employ a classical voting model to select the best ranking model from among those mentioned in Section 4 and we combine the results obtained from the ranking models. In Section 6, the new method is utilized to rank order five Iranian automobile manufacturing groups in terms job independence. The paper is concluded in Section 7.

3 A new voting model

In this section, we present a new voting model for groups with members of unequal power and proficiency. Suppose we intend to select s candidates from among n candidates$\{A_1, A_2, , A_n\}$, and rank them in terms of their priority $(n \geq s)$. Assume further that the voters have unequal power and proficiency, that is, the members are classified in m separate categories, such that the votes of the members in the ith category are of higher importance than those in the $i+1$ st category. Moreover, the system contains t voters. Table 1 provides a schematic presentation of this voting process. Considering v_{ir}^{j} as the number of rth place votes given to the pth candidate by the ith category members, we define:

$x_{ij}^{''} = \sum_{r=1}^{s} v_{ir}, i = 1, ..., m; j = 1, 2, ..., n$

$y_{rj} = \sum_{i=1}^{m} v_{ir}, r = 1, ..., s; j = 1, 2, ..., n$

If $u = (u_1, u_2, , u_m)$ and $w = (w_1, w_2, , w_s)$ are the weight vectors of the categories and voting places, respectively, the score obtained by the pth candidate is calculated as follows:

$$Z_p = \frac{\sum_{r=1}^{s} w_r y_{rp}}{\sum_{i=1}^{m} u_i(t - x_{ip}^{''})}, p = 1, ..., n$$

Once the weights are given or determined, candidates can be ranked in terms of their total scores. To avoid subjectivity in determining the relative importance weights, we suggest the following fractional model which determines the most favorable weights for each candidate:

$$Z_p^* = max \frac{\sum_{r=1}^{s} w_r y_{rp}}{\sum_{i=1}^{m} u_i(t - x_{ip}^{''})}$$

s.t.

$$\frac{\sum_{r=1}^{s} w_r y_{rj}}{\sum_{i=1}^{m} u_i(t - x_{ij}^{''})} \leq 1, j = 1, \ldots, n,$$

$$w_r - w_{r+1} \geq d(r, \epsilon), r = 1, \ldots, s-1,$$

$$w_s \geq d(s, \epsilon),$$

$$u_i - u_{i+1} \geq f(i, \epsilon), i = 1, \ldots, m-1,$$

$$u_m \geq f(m, \epsilon), \tag{3.2}$$

where $d(., \epsilon)$ and $f(., \epsilon)$ are referred to as discrimination intensity functions that are non-negative and monotonically increasing in a non-negative ϵ that satisfies $d(0, \epsilon) = 0$ and $f(0, \epsilon) = 0$. It is found that the choice of the functional form of $d(., \epsilon)$ and $f(., \epsilon)$ and the value of ϵ has significant impacts on the winner. First 'n' constraints denote that the final efficiency value must be in [0,1]. The denominator of the objective function show that the votes of the members in the ith category are of higher importance than those in the i+1st category. Moreover, the system contains t voters. The above model can be rewritten in a linear form as follows.

$$Z_p^* = max \sum_{r=1}^{s} w_r y_{rp}$$

$s.t.$

$$\sum_{i=1}^{m} u_i(t - x_{ip}^{''}) = 1$$

$$\sum_{r=1}^{s} w_r y_{rj} - \sum_{i=1}^{m} u_i(t - x_{ij}^{''}) \leq 0, j = 1, \ldots, n,$$

$$w_r - w_{r+1} \geq d(r, \epsilon), r = 1, \ldots, s-1,$$

$$w_s \geq d(s, \epsilon),$$

$$u_i - u_{i+1} \geq f(i, \epsilon), i = 1, \ldots, m-1,$$

$$u_m \geq f(m, \epsilon), \tag{3.3}$$

where $x_{ij} = (t - x_{ij}^{''})i = 1, 2, m; j = 1, 2, , n$. In this paper, for simplicity, we assume $d(r, \epsilon) = \epsilon d_r(r = 1, 2, s)$ and $f(i, \epsilon) = \epsilon f_i(i = 1, 2, m)$, in which d_r and f_i are determined by the manager and are the preferred values for the rth and ith gaps, respectively. Model (3.3) can be reached from model (3.2) by process similar to that reach LP model from fractional model in DEA. Now, we set:

$$w_r - w_{r+1} = w_r^{'}, r = 1, \ldots, s-1,$$

$$w_s = w_s^{'}$$

$$u_i - u_{i+1} = u_i^{'}, i = 1, \ldots, m-1,$$

$$u_m = u_m^{'}.$$

In other words, $w' = wA$ and $u' = uB$, where A and B are lower triangular matrices of order $(s*s)$ and $(m*m)$, respectively. In fact, we have the following representation.

$$[w_1, w_2, ..., w_s] \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{bmatrix} =$$

$$\begin{bmatrix} w_1 - w_2 \\ w_2 - w_3 \\ \vdots \\ w_s \end{bmatrix} = \begin{bmatrix} w_1' \\ w_2' \\ \vdots \\ w_s' \end{bmatrix}$$

There is a similar representation for $uB = u'$.It is clear that A and B are invertible matrices whose inverse matrices are lower triangular with entries of 1.
Considering the above, and setting $Y'^t = A^{-1}Y^t$ and $X'^t = B^{-1}X^t$, we have:

where X and Y are defined as follows, and t denotes the transpose.

$$Y = \begin{bmatrix} y_{11} & \dots & y_{1n} \\ y_{21} & \dots & y_{2n} \\ \vdots & \ddots & \vdots \\ y_{s1} & \dots & y_{sn} \end{bmatrix}$$

$$X = \begin{bmatrix} x_{11} & \dots & x_{1n} \\ x_{21} & \dots & x_{2n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \dots & x_{mn} \end{bmatrix}$$

Thus, model (3.3) can be rewritten as:

$$\begin{aligned} & Z_p^* = max \sum_{r=1}^{s} w_r' y_{rp} \\ & s.t. \\ & \sum_{i=1}^{m} u_i'(t - x_{ip}'') = 1 \\ & \sum_{r=1}^{s} w_r' y_{rj} - \sum_{i=1}^{m} u_i'(t - x_{ij}'') \leq 0, j = 1, \dots, n, \\ & w_r' \geq \epsilon d(r), r = 1, \dots, s, \\ & u_i' \geq \epsilon f(i), i = 1, \dots, m. \end{aligned} \tag{3.4}$$

The principal disadvantage of this procedure is that several candidates are often efficient, i.e., they achieve the maximum attainable score ($Z_p^* = 1$). To avoid this weakness, several authors have proposed methods for ranking the best performers. In the next section, we review various ranking models for discriminating between efficient candidates.

4 Preliminary concepts and some extensions

In this section, we review and extend the ideas of some authors to construct various ranking models for discriminating between efficient candidates.

4.1 Common weights analysis

Common weights analysis (CWA), proposed by Liu and Peng [16], aims to assist the manager in determining one set of weights attached to the performance indices, in order to have the best efficiency score for the group of efficient candidates. Then, the set of weights is regarded as one common set of weights across each efficient candidate, in order to compute its absolute efficiency score for the ranking of candidates. In order to solve the ranking problem thoroughly, the assessed target is further expanded to all candidates, including the inefficient candidates. The CWA model to determine the common set of weights for n candidates is formulated as follows:

$$\begin{aligned} & \Delta^* = max \sum_{j \in E} \Delta_j \\ & s.t. \\ & \sum_{r=1}^{s} W_r' y_{rj}' - \sum_{i=1}^{m} U_i' x_{ij}' + \Delta_j = 0, \quad j \in E \\ & \sum_{r=1}^{s} W_r' + \sum_{i=1}^{m} U_i' = 1, \\ & \Delta_j \geq 0, j \in E, \\ & W_r' \geq \epsilon d(r), r = 1, \dots, s, \\ & U_i' \geq \epsilon f(i), i = 1, \dots, m. \end{aligned} \tag{4.5}$$

where E is the set of efficient candidates. We note that the above-mentioned model gives a special non-dominated solution of the following multi-objective linear programming (MOLP)

Table 1: Voting process in a group with members of unequal power and proficiency.

Candidate	First place	...	rth place	...	sth place	
A_1	v^j_{11}	...	v^j_{1r}	...	v^j_{1s}	First category
$\vdots$						$\vdots$
A_j	v^j_{i1}	...	v^j_{ir}	...	v^j_{is}	ith category
$\vdots$						$\vdots$
A_n	v^j_{m1}	...	v^j_{mr}	...	v^j_{ms}	mth category

Table 2: Votes received by five automobile manufacturing groups in terms of job independence.

Candidate	First place	Second place	Third place	Forth place	Fifth place	
I	1	3	3	0	1	Top manager
	2	1	1	0	3	Expert
Z	2	1	1	1	3	Top manager
	2	1	2	1	1	Expert
B	1	2	1	2	2	Top manager
	2	0	2	2	1	Expert
S	2	1	2	1	0	Top manager
	1	2	2	2	0	Expert
P	2	0	0	5	1	Top manager
	0	3	0	2	2	Expert

problem.

$$
\begin{aligned}
&\Delta^* = min\{\Delta_j\}_{j\in E}\\
&s.t.\\
&\sum_{r=1}^{s} W'_r y'_{rj} - \sum_{i=1}^{m} U'_i x'_{ij} + \Delta_j = 0, \quad j \in E\\
&\sum_{r=1}^{s} W'_r + \sum_{i=1}^{m} U'_i = 1,\\
&\Delta_j \geq 0, j \in E,\\
&W'_r \geq \epsilon d(r), r = 1, \dots, s,\\
&U'_i \geq \epsilon f(i), i = 1, \dots, m.
\end{aligned} \tag{4.6}
$$

Indeed, we can use the above MOLP problem to search one common set of weights to create the best efficiency score of one group composed of efficient candidates. In the next subsection, we give some other methods for solving the above-mentioned MOLP based on the idea of Liang et al. [15].

4.1.1 Liang, Wu, Cook, and Zhu's idea

Here, we apply the idea proposed by Liang et al. [15] to introduce three common weights approaches. In an MOLP problem, it is generally impossible to find a solution that optimizes all objectives simultaneously. Therefore, the task of an MOLP solution process is not to find an optimal solution but, instead, to find non-dominated solutions and to help select a most preferred one. Loosely speaking, a solution, represented by a point in the decision variable space, is non-dominated if it is not possible to move the point within the feasible region to improve an objective function value without deteriorating at least one of the other objectives. In multiple criteria terminology, a non-dominated solution is also called an efficient solution. One fact we would like to point out here is that a non-dominated solution set for an MOLP problem will always contain, but is not limited to, the optimal solutions obtained by individually optimizing each of the objectives in the MOLP problem under the setting of single objective linear programming (LP). For more de-

Table 3: Efficiency scores obtained by the five manufacturing groups in terms of job independence.

Candidate	I	Z	B	S	P
Efficiency Score	1.000000	1.000000	0.999994	1.000000	0.999990

Table 4: Ranks obtained by efficient candidates in terms of job independence, evaluated by different ranking models.

Ranking model→ Efficient Candidate↓	(4.7)	(4.9)	(4.11)	(4.12)	(4.13)	(4.14)
I	1	1	1	0.99999	7.6999	1.0999946
Z	1	1	0.9999997	0.99998	9.3332	1.3333256
S	1	1	0.9999994	1	6.999944	1.2068917

Table 5: Votes received by five automobile manufacturing groups in terms of job independence.

Candidate	First place	Second place	Third place
model (4.7)	3	0	0
model (4.9)	3	0	0
model (4.11)	1	1	1
model (4.12)	1	1	1
model (4.13)	1	1	1
model (4.14)	1	1	1

Table 6: The score of various ranking models.

model (4.7)	model (4.9)	model (4.11)	model (4.12)	model (4.13)	model (4.14)
1.00	1.00	0.85	0.85	0.85	0.85

Table 7: Efficiency score obtained from our approach

Efficient candidate	I	Z	S
φ_p^*	2.07039	2.364215	1.977038

Table 8: Ranking obtained from our approach

Efficient candidate	I	Z	S
φ_p^*	2	1	3

tails about multi-objective optimization, readers are referred to Steuer [26] and Masud and Hwang [18]. In order to reach a special non-dominated extreme point, the MOLP formulation (3.4) can be written in min-sum approach, max-ordering approach, or minimizing the mean absolute deviation. These approaches do not need any inter-objective or other subjective preference information from the DM once the problem constraints and objectives have been defined. Thus, these approaches require the DM to be able to accept the solution obtained from the method. The advantage of adopting these approaches is that in the process of obtaining the solution, the DM will not be disturbed by the analyst, which is preferable from the point of view of the DM. But a major disadvantage, then, is that the analyst has to make many assumptions about the DM's preference. This is difficult to do with even the best and most knowledgeable analyst. Therefore, we can use an interactive MOLP method, such as the step method (or STEM) (Benayoun et al. [3]), the G-D-F algorithm (Geoffrion et al. [11]) and Zionts and Wallenius's method (Zionts and Wallenius ([31]), to reflect the DM's preferences in the process of searching the common set of weights.

Min-sum approach: The ideal point is defined as that multiplier bundle U? for which every candidate is efficient, that is, $\sum_{i=1}^{m}\sum_{r=1}^{s}\hat{U}_{ir}\hat{v}_{ir}^{j}$ or $\Delta_j = 0$. In the absence of such an ideal point, a reasonable objective is to treat Δ_j as goal achievement variables. To this end, we minimize $\sum_{j\in E}\Delta_j$. The model used in the current procedure was named CWA. Proposed by Liu and Peng [16], CWA aims to assist the manager in determining one set of weights attached to the performance indices, in order to have the best efficiency score for the group of efficient candidates. Then, the set of weights is regarded as one common set of weights across each efficient candidate, in order to compute its absolute efficiency score for the ranking of candidates. In order to solve the ranking problem thoroughly, the assessed target is further expanded to all candidates, including the inefficient candidates. The CWA model to determine the common set of weights for n candidates is formulated as follows:

$$
\begin{aligned}
&\Delta^* = min\sum_{j\in E}\Delta_j\\
&s.t.\\
&\sum_{r=1}^{s}W_r'y_{rj}' - \sum_{i=1}^{m}U_i'x_{ij}' + \Delta_j = 0, \quad j\in E\\
&\sum_{r=1}^{s}W_r' + \sum_{i=1}^{m}U_i' = 1,\\
&\Delta_j \geq 0, j\in E,\\
&W_r' \geq \epsilon d(r), r = 1,\dots,s,\\
&U_i' \geq \epsilon f(i), i = 1,\dots,m.
\end{aligned}
\tag{4.7}
$$

Note that models (4.5) and (4.7) are the same. Indeed, Liu and Peng [16] used the min-sum approach to solve MOLP (4.6) and reached model (4.5).

Max-ordering approach: Troutt [27] developed a maximum efficiency ratio DEA model in an effort to further prioritize the efficient candidates using a common set of weights. In a similar way, Liang et al. [15] proposed a model to derive a multiplier bundle that assigns the maximum possible score to the "worst"-performing candidate. They also used this model as a secondary goal in cross efficiency evaluation. Similar to their idea, we use the max-ordering approach to introduce a common weights model. The following model gives a common set of weights by solving MOLP (6) based on the max-ordering approach.

$$\Delta^* = min(max_{j\in E}(\Delta_j))$$

$s.t.$

$$
\begin{aligned}
&\sum_{r=1}^{s}W_r'y_{rj}' - \sum_{i=1}^{m}U_i'x_{ij}' + \Delta_j = 0, \quad j\in E\\
&\sum_{r=1}^{s}W_r' + \sum_{i=1}^{m}U_i' = 1,\\
&\Delta_j \geq 0, j\in E,\\
&W_r' \geq \epsilon d(r), r = 1,\dots,s,\\
&U_i' \geq \epsilon f(i), i = 1,\dots,m.
\end{aligned}
\tag{4.8}
$$

The above formulation can then be written as follows by introducing an auxiliary variable θ as follows:

$$
\begin{aligned}
&\Delta^* = min\theta\\
&s.t.\\
&\sum_{r=1}^{s} W_r^{'} y_{rj}^{'} - \sum_{i=1}^{m} U_i^{'} x_{ij}^{'} + \theta \geq 0, \quad j \in E\\
&\sum_{r=1}^{s} W_r^{'} y_{rj}^{'} - \sum_{i=1}^{m} U_i^{'} x_{ij}^{'} \leq 0, \qquad j \in E\\
&\sum_{r=1}^{s} W_r^{'} + \sum_{i=1}^{m} U_i^{'} = 1,\\
&W_r^{'} \geq \epsilon d(r), r = 1, \ldots, s,\\
&U_i^{'} \geq \epsilon f(i), i = 1, \ldots, m.
\end{aligned}
\tag{4.9}
$$

Minimizing the mean absolute deviation: With the aim of seeking to minimize the variation among the efficiencies of the candidates, we propose formalizing this concept through:

$$
\begin{aligned}
&\Delta^* = min\tfrac{1}{p} \sum_{j \in E} |\Delta_j - \bar{\Delta}_j|\\
&s.t.\\
&\sum_{r=1}^{s} W_r^{'} y_{rj}^{'} - \sum_{i=1}^{m} U_i^{'} x_{ij}^{'} + \Delta_j = 0, \quad j \in E\\
&\sum_{r=1}^{s} W_r^{'} + \sum_{i=1}^{m} U_i^{'} = 1,\\
&\Delta_j \geq 0, j \in E,\\
&W_r^{'} \geq \epsilon d(r), r = 1, \ldots, s,\\
&U_i^{'} \geq \epsilon f(i), i = 1, \ldots, m.
\end{aligned}
\tag{4.10}
$$

where $\bar{\Delta} = \frac{1}{p} \sum_{j \in E} \Delta_j$ and $|E|= p$.

The objective function in model (3.3) computes the mean absolute deviation of a set of data, namely, the average of the absolute deviations of data points from their mean. To show that this nonlinear model can be linearized, let $a_j = 1/2(|\Delta_j - \bar{\Delta}|-(\Delta_j - \bar{\Delta}))$ and $b_j = 1/2(|\Delta_j - \bar{\Delta}|+(\Delta_j - \bar{\Delta}))$. Then, model (4.10) becomes the following linear programming problem:

$$
\begin{aligned}
&\Delta^* = min\tfrac{1}{2} \sum_{j \in E} (a_j - b_j)\\
&s.t.\\
&\sum_{r=1}^{s} W_r^{'} y_{rj}^{'} - \sum_{i=1}^{m} U_i^{'} x_{ij}^{'} + \Delta_j = 0, \quad j \in E\\
&a_j - b_j = \Delta_j - \tfrac{1}{p} \sum_{j \in E} \Delta_j, j \in E\\
&\sum_{r=1}^{s} W_r^{'} + \sum_{i=1}^{m} U_i^{'} = 1,\\
&\Delta_j, a_j, b_j \geq 0, j \in E,\\
&W_r^{'} \geq \epsilon d(r), r = 1, \ldots, s,\\
&U_i^{'} \geq \epsilon f(i), i = 1, \ldots, m.
\end{aligned}
\tag{4.11}
$$

4.1.2 Common weights based on aggregated candidate

In this subsection, we introduce a model in which the manager can choose the most favorable weights for the group that comprises all candidates under the manager's governance. In other words, one set of weights that maximizes the group's comprehensive score is used as the common set of weights for all the units to obtain each individual's comprehensive score. Now, if we define aggregated candidate as a candidate to whom the number of votes given by the ith category in the rth place is $\sum_{j=1}^{n} v_{ir}^{j}$, then the following model gives this common set of weights.

$$
\Delta^* = min \sum_{r=1}^{s} \acute{W}_r (\sum_{j=1}^{n} \acute{y}_{rj})
$$

$s.t.$

$$
\begin{aligned}
&\sum_{i=1}^{m} U_i^{'} (\sum_{j=1}^{n} x_{ij}^{'}) = 1,\\
&\sum_{r=1}^{s} W_r^{'} y_{rj}^{'} - \sum_{i=1}^{m} U_i^{'} x_{ij}^{'} \leq 0, j \in E\\
&\sum_{r=1}^{s} W_r^{'} + \sum_{i=1}^{m} U_i^{'} = 1,\\
&W_r^{'} \geq \epsilon d(r), r = 1, \ldots, s,\\
&U_i^{'} \geq \epsilon f(i), i = 1, \ldots, m.
\end{aligned}
\tag{4.12}
$$

Now, by the optimal weight obtained from each of models (4.7), (4.9), (4.11) and (4.12), and substituting it in

$\hat{Z}_j = \frac{\sum_{r=1}^{s} \acute{W}_r^* \acute{y}_{rj}}{\sum_{i=1}^{m}}$

We can rank order the efficient candidates. In other words, $\hat{Z}_j$ will be a criterion for ranking efficient candidates.

4.2 Ranking based on Obata and Ishii's idea

Obata and Ishii [20] proposed that a larger range of place votes index values is desirable to the decision maker. They explained that the benefit of the weighted sum is from the indices value itself, rather than from the individual weights. Indeed, they think that they should use the most favorable vector, i.e., a vector which attains the maximum preference score, according to the policy of DEA. We extend above discussion to introduce a new ranking model as follows:

$$\hat{Z}_p^* = max(\sum_{r=1}^{s} \hat{w}_r \acute{y}_{rp} - \sum_{i=1}^{m} \hat{u}_i \acute{x}_{ip})$$
$$\text{s.t.} \sum_{i=1}^{m} \acute{u}_i \acute{x}_{ip} = 1$$
$$\sum_{r=1}^{s} \acute{w}_r \acute{y}_{rp} = 1$$
$$\sum_{r=1}^{s} \acute{w}_r \acute{y}_{rp} - \sum_{i=1}^{m} \acute{u}_i \acute{x}_{ip} \leq 0, j \in E$$
$$\acute{w}_r \geq 0, r = 1, \dots, s$$
$$\acute{u}_i \geq 0, i = 1, \dots, m$$
$$\acute{w}_r \geq \epsilon d_r, r = 1, \dots, s$$
$$\acute{u}_i \geq \epsilon f_i, i = 1, \dots, m$$
$$\hat{w} = \alpha \acute{w}, \alpha > 0$$
$$\parallel \hat{w} \parallel = 1$$
$$\hat{u} = \beta \acute{u}, \beta > 0$$
$$\parallel \hat{u} \parallel = 1 \ (4.13)''$$

where $\hat{w} = (\hat{w}_1, \hat{w}_2, \dots, \hat{w}_s)$, $\acute{w} = (\acute{w}_1, \acute{w}_2, \dots, \acute{w}_s)$, $\hat{u} = (\hat{u}_1, \hat{u}_2, \dots, \hat{u}_m)$, $\acute{u} = (\acute{u}_1, \acute{u}_2, \dots, \acute{u}_m)$, and $\parallel . \parallel$is a certain norm. Obata and Ishii [20], demonstrate that the previous model is equivalent to the following one:

$$\frac{1}{\hat{Z}_o^*} = min(\|\acute{w}\| - \|\acute{u}\|)$$
$$\text{s.t.} \sum_{i=1}^{m} \acute{u}_i \acute{x}_{ip} = 1$$
$$\sum_{r=1}^{s} \acute{w}_r \acute{y}_{rp} = 1$$
$$\sum_{r=1}^{s} \acute{w}_r \acute{y}_{rj} - \sum_{i=1}^{m} \acute{u}_i \acute{x}_{ij} \leq 0, j \in E$$
$$\acute{w}_r \geq \epsilon d_r, r = 1, \dots, s,$$
$$\acute{u}_i \geq \epsilon f_i, i = 1, \dots, m. \ (4.13)'$$

The following model is suggested as a ranking and $\hat{Z}_p^*$ is suggested as a ranking score for the efficient candidate, using the L_1-norm.

$$\begin{aligned}
&\Delta^* = min \frac{1}{\hat{Z}_p^*} = min \sum_{r=1}^{s} \acute{W}_r - \sum_{i=1}^{m} \acute{u}_i \\
&s.t. \\
&\sum_{i=1}^{m} \acute{u}_i \acute{x}_{ip} = 1, \\
&\sum_{i=r}^{s} \acute{w}_r \acute{y}_{rp} = 1, \\
&\sum_{i=r}^{s} \acute{w}_r \acute{y}_{rj} - \sum_{i=1}^{m} \acute{u}_i \acute{x}_{ij} \leq 0, j \in E, \\
&w_r' \geq \epsilon d(r), r = 1, \dots, s, \\
&u_i' \geq \epsilon f(i), i = 1, \dots, m.
\end{aligned} \tag{4.13}$$

4.3 Ranking based on Andersen and Petersen's idea

Andersen and Petersen [2] proposed the DEA exclusion model as one that discriminates DEA efficient DMUs. In this model, the DMU being evaluated is excluded from the comparison set. Applying this to ranked voting systems as a DEA/AR exclusion model, the formulation is as follows:

$$\hat{Z}_p^* = max \sum_{r=1}^{s} \acute{w}_r \acute{y}_{rp}$$

$s.t.$

$$\sum_{i=1}^{m} \acute{u}_i \acute{x}_{ip} = 1,$$

$$\sum_{i=r}^{s} \acute{w}_r \acute{y}_{rj} - \sum_{i=1}^{m} \acute{u}_i \acute{x}_{ij} \leq 0, j = 1, \dots, n; j \neq p,$$

$$\begin{aligned} &w_r' \geq \epsilon d(r), r = 1, \dots, s, \\ &u_i' \geq \epsilon f(i), i = 1, \dots, m. \end{aligned} \tag{4.14}$$

Now, $\hat{Z}_p^*$ is suggested as a ranking score for the efficient candidate.

5 A new ranking methods

In this section, we describe a new approach to carry out a ranking of the previously mentioned ranking models by the use of a voting model. In real world problems, it is very likely that applying model (3.4) to a voting system leads to several candidates being introduced as efficient. This accounts for the necessity of introducing ranking models, since managers are always interested in completely ranking the candidates present in the voting process. To this end, many authors have proposed different models to rank efficient candidates by employing various ideas and viewpoints, the results of which often differ from each other. But, the question arising here is which ranking model is better, and which model should be employed in a particular voting system. It does not seem very logical to select a ranking model without first considering the voting system under discussion. We believe that the ranking model should be chosen based on the voting system in question. In voting model (3.4), each candidate is allowed to be evaluated in its best condition, and this is the very policy of DEA. We follow the same policy in selecting a ranking model. That is, we allow the efficient candidates present in the voting process to select their ranking model of interest from among the existing ranking models. It is obvious that any efficient candidate will select the ranking model by which it will obtain the highest possible rank among the efficient candidates. Thus, each candidate will be able to rank the different ranking models. This process is a voting system, itself, for which model (2.1) presented by Cook and Kress [6] can be employed. The process will be as follows.

Suppose that in another voting process, each efficient candidate (as a voter) selects one of the mentioned ranking models (as a candidate). Indeed efficient candidates as voters express their opinion about the ranking models (4.7), (4.9), (4.11), (4.12), (4.13) and (4.14). In other words, the efficient candidates rank these models as first, second, , sixth rank based on the ranks they have been assigned by the models. Note that in this voting process, the efficient candidates play the role of voters, and the ranking models (4.7), (4.9), (4.11), (4.12), (4.13) and (4.14) serve as candidates in the classical voting process. Now, suppose that $R_j^{(7)}$, $R_j^{(9)}$, $R_j^{(11)}$, $R_j^{(12)}$, $R_j^{(13)}$, and $R_j^{(14)}$ are the ranks obtained by the jth efficient candidate as evaluated by the ranking models (4.7), (4.9), (4.11), (4.12), (4.13) and (4.14), respectively. Thus, the jth candidate can rank order each of the ranking models (4.7), (4.9), (4.11), (4.12), (4.13) and (4.14) according to the ranks $R_j^{(7)}$, $R_j^{(9)}$, $R_j^{(11)}$, $R_j^{(12)}$, $R?_j^{(13)}$, and $R_j^{(14)}$ obtained, from first to sixth.

Let y_{pj} be the number of the jth place votes of candidate (model) (p) ((p) = ((4.7), (4.9), (4.11), (4.12), (4.13) and (4.14); j=1,2,,6). We can use the following model to select the best ranking model.

$$
\begin{aligned}
& Z_p^* = max \sum_{r=1}^{6} u_{pr} y_{pr} \\
& s.t. \\
& \sum_{r=1}^{6} u_{pr} y_{jr} \leq 1, j = 1 \ldots, 6, \\
& u_{pr} - u_{p,r+1} \geq d''(r, \epsilon), r = 1, \ldots, 5, \\
& u_{p6} \geq d''(6, \epsilon).
\end{aligned} \tag{5.15}
$$

After the above process is followed, there are two courses of action available to the managers. One is to accept the results obtained by running model (k), where $Z_k^* = max\{Z_7^*, Z_9^*, Z_{11}^*, Z_{12}^*, Z_{13}^*, Z_{14}^*\}$, and make decisions based on them, since (k) is a model that is selected by the candidates and has been considered by them as the best ranking model for the voting system based on their aggregate votes. The second one is to present a new score for the final ranking of the candidates which is the result of aggregating the scores of different ranking models. This latter approach can be summarized as follows:

First stage (normalization): first we normalize the efficiency scores obtained from model (4.5) by dividing each score by the sum of the scores obtained. Assume $\theta_j^* = \frac{Z_j^*}{\sum_{i \in \{(4.7),(4.9),(4.11),(4.12),(4.13),(4.14)\}}}, j = (4.7), (4.9), (4.11), (4.12), (4.13) and (4.14)$

Second stage (presenting a new score for ranking the efficient candidates): suppose Z^*_{oj} is the score assigned to the efficient candidate o by the jth ranking model, $o \in E$ and j=(4.7), (4.9), (4.11), (4.12), (4.13) and (4.14). The score presented below is the result of aggregating the scores assigned by different ranking models and can be employed as a new criterion for ranking the efficient candidates.

$\varphi^*_o = \sum_{i\in\{(4.7),(4.9),(4.11),(4.12),(4.13),(4.14)\}} \theta^*_j \hat{Z}^*_{oj}$

Thus, the efficient candidate with the highest φ^*_o value will be selected.

It should be noted that the above process is an approach for ranking efficient candidates, and is never restricted to the models presented in this paper. It can be further employed by scholars with other ranking models.

6 A case study

In this section, we employ the new method to rank order five Iranian automobile manufacturing groups in terms of "job independence", drawing upon the opinions of 15 experts, including eight top managers and seven specialists with more than 15 years of experience. It is evident that the opinions of the top managers must be deemed higher than those of other experts. The votes received by the automobile manufacturing groups in terms of job independence are presented in Tables 2 below.

Concerning the results in Tables 4, the votes received by ranking models (4.7), (4.9), (4.11), (4.12), (4.13) and (4.14) as evaluated by the efficient candidates in terms of job independence are provided in Table 5.

Using model 3.4, the following efficiency scores are obtained for the five manufacturing groups in terms of job independence (Table 3).

As can be observed from the results in Table 3, candidates I, Z and S obtained the highest efficiency score possible, in terms of job independence. They are, therefore, efficient candidates in terms of the respective index. Table 4 demonstrate the results of ranking efficient candidates in terms of job independence, by each of the ranking models (4.7), (4.9), (4.11), (4.12), (4.13) and (4.14).

Taking into account the results in Tables 5, we can use model (2.1) to select the best ranking model among models (4.7), (4.9), (4.11), (4.12), (4.13), and (4.14), in terms of job independence,. Their scores are as follows (Table 6).

Therefore, models (4.7) and (4.9) are the best models as ranking models for this case study. The decision maker can, therefore, accept the results obtained by these models and make decisions based on these results, since each of these models is the one selected by the candidates themselves and has been chosen as the best ranking model for the voting system under discussion after aggregating the votes of candidates. As another choice, the decision maker can use the score obtained by aggregating the scores of different ranking models as a basis for decision making. To this end, we first normalize the efficiency scores of models (4.7), (4.9), (4.11), (4.12), (4.13), and (4.14).

$\theta^*_{(4.7)} = \frac{1.00}{1.00+1.00+0.85+0.85+0.85+0.85} = 0.185185$

$\theta^*_{(4.9)} = \frac{1.00}{1.00+1.00+0.85+0.85+0.85+0.85} = 0.185185$

$\theta^*_{(4.11)} = \frac{0.85}{1.00+1.00+0.85+0.85+0.85+0.85} = 0.157407$

$\theta^*_{(4.12)} = \frac{0.85}{1.00+1.00+0.85+0.85+0.85+0.85} = 0.157407$

$\theta^*_{(4.13)} = \frac{0.85}{1.00+1.00+0.85+0.85+0.85+0.85} = 0.157407$

$\theta^*_{(4.14)} = \frac{0.85}{1.00+1.00+0.85+0.85+0.85+0.85} = 0.157407$

Now, we present the following score (Table 7) for the efficient candidates by the process discussed in Section 4.

Considering the above results, we present the following ranking (Table 8) for the efficient candidates.

Note that other ranking models can be utilized in the above process.

7 Conclusions

In this paper, we presented a voting model for groups with members of unequal power and proficiency, as well as some models for ranking efficient candidates, by extending the ideas of some authors. Next, we proposed a new methodology to rank the ranking models mentioned for the performance indices of only DEA efficient candidates based on a voting model. Moreover, an approach for combining the results of ranking models was presented. Finally, we made use of the new method to rank order five Iranian automobile manufacturing groups in terms of job independence. It must be mentioned that further ranking models can be obtained by extending the ideas of other authors to discriminate between efficient candidates based on voting model (3.4). Also, other ranking models can be employed in the process introduced in Section 5 for discriminating efficient candidates.

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