Optimized Pricing Decisions In a Multi-Level Supply Chain With Various Power and Channel Structures: A Game-Theoretic Approach

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Abstract

This article tries to explain the optimization of pricing decisions in a multi-level supply chain with different channel under different power structure. The emphasis is placed on three-stage supply chain models with a single product. These models are composed of three parts; these are: one supplier, one manufacturer, and one retailer. In addition three different channel structures will be considered for this supply chain; these include: the decentralized, the semi-integrated, and the integrated channel. There are two types of power balance structures for both the decentralized and the semi-integrated channels. The first type is a leader_ follower power structure, modeled as a Stackelberg game. The second type is an independent power structure and is treated as a Nash game. Appropriate mathematical models are developed for optimal pricing decisions. The analytical methods were extended to specify the Nash game. Several research findings have been obtained. Optimized decision making in supply chain management, however, is yet in the early stages. This is due to the hybrid environment in which individual firms should simultaneously decide with various supply chain configuration decisions. In order to facilitate decision_ making in such a hybrid environment, the overall supply chain patterns in the construction industry are studied in this paper.

Keywords : Pricing; Supply chain management; Game theory; Decision making; Optimization.

1 Introduction

A supply chain is a set of entities like suppliers, manufacturers, and retailers, who finally supply a specific product. Supply chain management is a vital process of trade activities that lead supply chains to higher profit, a rapid production cycle, and lower prices; in other words, supply chain management optimizes the results of any supply chain. In addition, a supply chain consists of firms and their resources, personnel, activities, and technologies involved in providing a final product.

For the last two decades, supply chain management has become an area that has received significant attention from the business community. Supply chain management is a collection of procedures to coordinate and integrate all the members in the supply chain so that all of the activities in the supply chain will be effective.

In total, the three parts of supply chain management are as follows: the supply chain network, the supply chain configuration, and the supply chain coordination.

In this article, we focus on supply chain configu-

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ration. One of the main decisions in supply chain management involves supply chain configuration [5]. Decision-making in supply chain configuration can be placed into two categories: the first level is macro; in this level, the supply chain system and the decision of its members are considered. The second level is micro and this level tries to follow-up the decisions made in the previous level. Typical supply chain configuration decisions include multiple issues like pricing, advertising, inventory, service time, ordering policy, supplier selection, etc. This article will focus on the aspect of pricing decisions. Pricing is an important marketing strategy related to product positioning and it affects other marketing elements such as channel decisions, promotion, and so on [20].

The main issue of this article is pricing decisions in multi-level supply chains. This type of supply chain reflects a chain structure in the construction industry. This supply chain structure has three levels comprising the supplier, the manufacturer, and the retailer [12, 26].

To optimize the model and analyze decision-making in such multi-level situations (often with conflicting goals), game theory is the best choice. Game theory is a collection of analytical instruments that help people to find out the facts behind the behavioral patterns of phenomena [21].

This article tries to explain the optimization of pricing decisions in a multi-level supply chain with different channels. The emphasis is placed on three-stage supply chain models with a single product. These models are composed of three parts; they are: one supplier, one manufacturer, and one retailer. In addition, various channel structures will be considered for this supply chain. The first mode is a decentralized structure and the members independently decide. The second mode indicates a semi-integrated supply chain, with forward integration or backward integration performed by the manufacturer. The third mode is an integrated channel and the members full participate in the process. There are two types of power balance structures for both the decentralized and the semi-integrated channels. The first type is a leader–follower power structure, modeled as Stackelberg game. The second type is an independent power structure and is treated as a Nash game.

Non-cooperative games are used for both the decentralized and the semi-integrated channels. Then, we use the price model in order to explain the integrated channel. After that, we include the impact of the power structure. Some optimization software, such as Lingo, is always adopted to assist in solving the multi-level models. The results in this paper are calculated by Lingo.

2 Related Literature

2.1 The pricing decisions in a supply chain

The issue of pricing is one of the crucial elements that affect the profitability of the members in any supply chain. In economic considerations, pricing is introduced as a regulator for supply and demand [9]. Often, there is a conflict of objectives between the members of a supply chain. In such circumstances, the Pareto optimal pricing decisions are not always effective [7]. Supply chain pricing follows the main ideas of upright pricing. Both suppliers and manufacturers, by optimizing their positions in the supply chain, try to optimize the conditions for all the members [24]. Coordination could, therefore, be useful for the members of the supply chain. Pricing decisions, for supply chain members, are affected by various power and channel structures in different ways [6, 7]. So, the coordination among the different members in the channel must be considered [8]. The suitable power and channel structures are important for all the members in every supply chain. Several industries, by creating a forward integration between the manufacturers and their retailers, tend to work together in determining pricing and the other aspects of their business. In such a scenario, the supply chain is capable of reducing costs and offers lower prices to the end users [11]. The retailer could create greater profits when it makes the price decisions in partnership with the manufacturer [25]. In other research, the same results were found for the coordination between a manufacturer and a retailer [16]. Other studies too have drawn the same conclusions in various circumstances [1, 2, 3, 22].

2.2 The application of game theory in supply chain management

Today, game theory can be considered as a necessary tool in supply chain management, where

there are conflicting objectives [4]. Game theory thus faces interactive optimization issues [4]. There are conditions in marketing and economics that have used mixed strategies; for example: promotion models and search models [23]. The companies set their relations strategically with both the competitors and the supply chain partners [10]. The supply chain members apply their interest in their individual decisions; the supply chain, therefore, cannot always reach optimal decisions [10]. When there is a decentralized channel in the supply chain rather than a centralized channel, the overall profit will be lower. If the supply chain has an overall decision-making unit, it could optimize the decisions and maximize the total profits of the supply chain [10]. This article, by using game theory, tries to study optimal decision-making among multiple partners [10].

3   Research Setting

This article, investigates pricing issue only for one product in multi-level price models when the demand is sensitive to price. As previously mentioned, there are three types of channel structures in the supply chain. The first type is the decentralized channel, where each partner makes their decisions non-cooperatively and individually. The second type is a semi-integrated channel, where the manufacturer is semi-integrated with the other members in the supply chain. Finally, there is the integrated channel structure. In the integrated channel, vertical integration systems exist. We apply two power structures in both the decentralized and the semi-integrated channels. If the manufacturer is the leader of the supply chain, the model is designed as a Stackelberg game. If the members of the supply chain have equal power and make their own decisions non-cooperatively and simultaneously, then the model is designed as an independent power structure or a Nash game [7].

According to the above argument, the equilibrium prices both for the decentralized channel and the semi-integrated channel in the two types of power structures were calculated by game theory. In addition, this paper studies the profits of the members individually and as a whole for the supply chain.

3.1   Method

In this section, the appropriate mathematical models are developed for the optimal pricing decisions. We use non-cooperative game models like the Nash game and the Stackelberg game for the calculation and analysis.

3.1.1   The research questions:

This paper tries to answer the following questions:

1. How will the supply chain members decide about optimal pricing under the various power and channel structures?

2. Which structure is preferred by the members?

3. What kind of integration (forward or backward) is preferred by the manufacturer?

3.2   The calculation

In this supply chain, demand is a function of the retail price (P_r). We use the following non-linear demand function that is a common demand function in marketing literature:

$$D(P_r) = ap_r^{-b}, a>0, b>0. \qquad (3.1)$$

In this function "a" is the market scale parameter and "b" is price elasticity; both of them are always positive [18, 19, 15, 13].

Furthermore, we assume that all the members of the supply chain have the capacity to satisfy each other. We study a static model thereafter.

In order to specify the profit of the members in the supply chain, and according to the demand function, we assume that p_s represents the raw material prices and p_m represents the final product price. Then, m_m will be the profit margin of the manufacturer and m_r will be the profit margin for the retailer. The cost paid by the supplier is shown as c_s and the manufacturer's cost is shown as c_m. δ_s indicates the amount of raw materials consumed per unit of the product. If the retailer controls the retail price, the manufacturer controls the final product price, and the supplier controls the raw material price, we can extract the profit functions as follows:

1. The retailer's profit function:

$$\Pi_r(P_r) = m_r D(P_r), \qquad (3.2)$$

<div align="center">**Table 1:** Notation.</div>

Symbol	Description
M, m	Index of the manufacturer
S, s	Index of the Supplier
R, r	Index of the Retailer
P_m	Wholesale price charged by the manufacturer to the retailer
P_s	Raw material price charged by the supplier to the manufacturer
P_r	Retail price charged to the customers by the retailer
m	Manufacturer's Profit function
s	Supplier's Profit function
r	Retailer's Profit function
a	A constant in the demand function of the retailer, which represents the market scale
b	Coefficient of the product's price elasticity for the retailer
D	Retailer's annual demand
m_m	Manufacturer's profit margin
m_r	Retailer's profit margin
C_m	Production cost per unit product
C_s	Raw material cost paid by supplier
s	Usage amount of unit raw material per unit product
MS	Manufacture Stackelberg
VN	Vertical Nash
MR-S	Integration of Manufacturer and retailer- Stackelberg
MR-N	Integration of Manufacturer and retailer- Vertical Nash
SM-S	Integration of supplier and Manufacturer- Stackelberg
SM-N	Integration of supplier and Manufacturer- Vertical Nash
I	Integrated

$$M_r = P_r - P_m$$

2. The manufacturer's profit function:

$$\Pi_m(P_m) = m_m D(P_r), \qquad (3.3)$$

$$M_m = P_m - P_s \delta_s - C_m$$

3. The supplier's profit function:

$$\Pi_s(P_s) = (P_s - C_s)\delta_s D(P_r) \qquad (3.4)$$

Now, we can specify the optimal pricing decisions for all the members in the supply chain under the various power and channel structures.

3.2.1   The decentralized channel

In this channel, two power structures have been considered. The first is the leader–follower structure and the second is the independent power structure. In the first structure, the manufacturer is the leader and the Stackelberg game structure is used. In the second structure, all the members have equal status; for this, we use the Nash game.
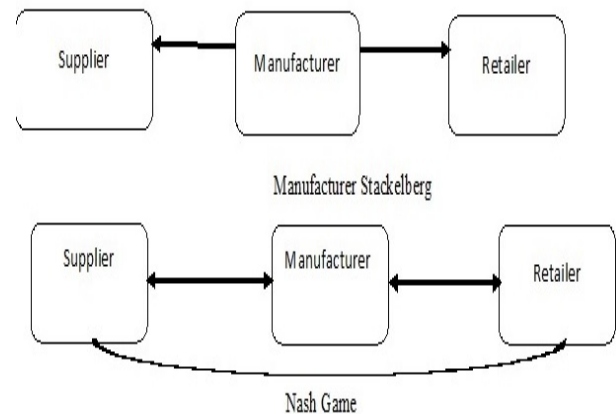
Figure 1: Stackelberg and vertical Nash for the decentralized channel

Manufacturer Stackelberg:

The manufacturer Stackelberg is actually composed of two sequential games. The first game is between the manufacturer and the supplier. The manufacturer, with regard to the supplier's reaction function, chooses its profit margins. The second game is between the manufacturer and the retailer. In this situation, the manufacturer, with

regard to the retailer's reaction function, chooses its profit margins. Initially, the Stackelberg game between the manufacturer and the retailer will be considered; consequently, the following outcome is observed:

We can derive the reaction function of the retailer from the profit function of the retailer as follows:

$$\frac{\partial \pi r}{\partial Pr} = D(P_r) + (P_r - P_m) \times \frac{\partial D\ (\text{Pr})}{\partial Pr} = 0 \quad (3.5)$$

Then, the retailer's reaction function can be derived as follows:

$$P_r = P_r(P_m) \quad (3.6)$$

The supplier will determine the price of the raw material according to the manufacturer's profit margin; thus the following outcome is seen:

$$P_m = m_m + P_s\delta_s + C_m, \quad (3.7)$$

$$P_r = P_r(P_m) \quad so \quad P_r = m_m + P_s\delta_s + C_{m.}$$

Therefore, the profit maximization of the supplier is as follows:

$$\frac{\partial \pi r}{\partial Pr}\ (P_rP_s) =$$

$$\delta_s D(P_r) + (P_s - C_s)\delta_s \times \frac{\partial D\ (\text{Pr})}{\partial Pr} \times \frac{\partial Pr}{\partial Ps} = 0 \quad (3.8)$$

Therefore, the supplier's reaction function for the pricing decisions is as follows:

$$P_s = P_s(P_r) = P_s(P_m) \quad (3.9)$$

By interleaving the supplier's reaction function and the retailer's reaction function into the manufacturer's profit maximization condition, we get the following outcome:

$$\frac{\partial \pi m}{\partial Pm} = [1 -_s \times \frac{\partial Ps}{\partial pm}]D(P_r) + (P_m + P_{ss} - C_m)$$
$$(3.10)$$
$$\times \frac{\partial D\ (\text{Pr})}{\partial Pr} \times \frac{\partial Pr}{\partial Ps} = 0$$

The Nash Game

In the vertical Nash game, all of the members in the supply chain are independent and make decisions non-cooperatively. This means that the supplier makes the pricing decisions according to other members' profit maximization conditions. The manufacturer and the retailer behave similarly. In addition, each level makes its own decisions based on the prices of the higher level.

As you can see, the first order condition in order to obtain equilibrium in the Nash game is the profit maximization condition. The profit maximization functions for each of the members are as follows:

$$\frac{\partial \pi r}{\partial Pr} = D(P_r) + (P_r - P_m) \times \frac{\partial D\ (\text{Pr})}{\partial Pr} = 0 \quad (3.11)$$

$$\frac{\partial \pi m}{\partial Pm} = D(P_r) + (P_m + P_s\delta_s - C_m) \times \frac{\partial D\ (\text{Pr})}{\partial Pr} = 0$$
$$(3.12)$$

$$\frac{\partial \pi s}{\partial Ps} = \delta_s D(P_r) + (P_s - C_s)\delta_s^2 \times \frac{\partial D\ (\text{Pr})}{\partial Pr} = 0$$
$$(3.13)$$

After replacing D(P_r) with Equation 3.1 and by solving Equations 3.6, 3.12, and 3.13, the optimal price and profit can be achieved. The results are shown in Table 2.

3.2.2   The Semi-Integrated Channel

In this situation, vertical forward or backward integration are performed. The members who do not participate in the integration, are independent. For the semi-integrated channel, both power structures have been considered. Then, we formulate the Stackelberg and the Nash games.

Manufacturer and Retailer Integration Structure:

In order to formulate the Stackelberg game, the manufacturer and the retailer act as leaders and the supplier acts as the follower. In the Nash game, the independent member and the integrated members have equal status.

Manufacturer-Retailer Stackelberg:

The integrated members are willing to make their own decisions according to supplier's reaction function. The profit function for the integrated members is as follows:

$$\Pi_{mr} = m_{mr}D(P_r) \quad (3.14)$$

While $m_{mr} = P_r - (P_s\delta_s) - C_m$ is profit margin of the integrated channel.

The supplier's reaction function can be derived from the supplier's profit function (Equation 3.5).

$$\frac{\partial \pi s}{\partial Ps} = \delta_s D(P_r) + (P_s - C_s)\delta_s^2 \times \frac{\partial D\ (\text{Pr})}{\partial Pr} = 0$$
$$(3.15)$$

Table 2: Results for Independent power structure.

Channels structures	Decentralized channel
	(VN)
Pm	$((b-1) (\delta s\ Cs + Cm))/(b-3)$
Ps	$((b-2) (\delta s\ Cs + Cm))/(s\ (b-3))$
Pr	$(b\ (\delta s\ Cs + Cm))/(b-3)$
Пm	$a/(b \wedge b\ )\ [(b-3)/(\delta s\ Cs + Cm\ ] \wedge (b-1)$
Π_s	$a/(b \wedge b\ )\ [(b-3)/(\delta s\ Cs + Cm\ ] \wedge (b-1)$
Пr	$a/(b \wedge b\ )\ [(b-3)/(\delta s\ Cs + Cm\ ] \wedge (b-1)$
П	$a/(b \wedge b\ )\ [(b-3)/(\delta s\ Cs + Cm\ ] \wedge (b-1)$
System efficiency	$3[(b-3)/(b-1)] \wedge (b-1)$

Continue Table 2.

Semi-integrated channel		Integrated Channel
(MR-N)	(SM-N)	
-	$((b-1) (\delta s\ Cs + Cm))/(b-2)$	$(b\ (\delta s\ Cs + Cm))/(b-1)$
$((b-1) (\delta s\ Cs + Cm))/(\delta s\ (b-2))$	-	-
$(b\ (\delta s\ Cs + Cm))/(b-2)$	$(b\ (\delta s\ Cs + Cm))/(b-2)$	-
-	$a/(b \wedge b\ )\ [(b-2)/(\delta s\ Cs + Cm\ ] \wedge (b-1)$	-
$a/(b \wedge b\ )\ [(b-2)/(\delta s\ Cs + Cm\ ] \wedge (b-1)$	-	-
$a/(b \wedge b\ )\ [(b-2)/(\delta s\ Cs + Cm\ ] \wedge (b-1)$	$a/(b \wedge b\ )\ [(b-2)/(\delta s\ Cs + Cm\ ] \wedge (b-1)$	$a/(b \wedge b\ )\ [(b-1)/(\delta s\ Cs + Cm\ ] \wedge (b-1)$
$2a/(b \wedge b\ )\ [(b-2)/(\delta s\ Cs + Cm\ ] \wedge (b-1)$	$2a/(b \wedge b\ )\ [(b-2)/(\delta s\ Cs + Cm\ ] \wedge (b-1)$	$a/(b \wedge b\ )\ [(b-1)/(\delta s\ Cs + Cm\ ] \wedge (b-1)$
$2[(b-2)/(b-1)] \wedge (b-1)$	$2[(b-2)/(b-1)] \wedge (b-1)$	1

So, the supplier's reaction function is as follows:

$$P_s\ = P_s\ (P_r) \qquad (3.16)$$

By using Equation 3.14, the manufacturer can obtain its optimal pricing decision:

$$\frac{\partial \pi mr}{\partial Pr} = [1 - \frac{\partial Ps}{\partial Pr} \times \delta_s] D(P_r) + \qquad (3.17)$$

$$(P_r - P_s \delta_s\ - C_m) \times \frac{\partial D\ (Pr)}{\partial Pr} = 0$$

After replacing $D(P_r)$ with Equation 3.1, the Stackelberg equilibrium results will be obtained. The results are shown in Table 3.

Manufacturer–Retailer Vertical Nash:

Here, independent power means that the integrated members and the supplier have equal power. The Nash game thus takes shape.
The integrated members try to maximize their total profit based on the supplier's raw material price. The supplier makes the pricing decision and also maximizes its profit according to the profit margin of the integrated members. By using Equations 3.5 and 3.14, we can extract the equilibrium conditions for the Nash game.

$$\frac{\partial \pi s}{\partial Ps} = \delta_s D(P_r) + (P_s - C_s)\delta_s^2\ \times \frac{\partial D\ (Pr)}{\partial Pr} = 0 \qquad (3.18)$$

$$\frac{\partial \pi mr}{\partial Pr} = D(P_r) + (P_r - P_s \delta_s\ - C_m) \times \frac{\partial D\ (Pr)}{\partial Pr} = 0 \qquad (3.19)$$

After solving Equations 3.18 and 3.19 simultaneously, the Nash equilibrium results for price and profit can be obtained (see Table 2).

Supplier and Manufacturer Integration Structure:

Supplier and Manufacturer Stackelberg:

In this condition, the manufacturer and the supplier integrate. They will be the leaders of the

Table 3: Results for Leader-follower power structure.

Channels structures	Decentralized channel
	(MS)
Pm	(b∧2 (δ_s C_s+ C_m))/(b-1)∧2
Ps	((b∧2-b+1) Csδs+bC_m)/(δs (b-1)∧2)
Pr	(b∧3 (_s C_s+ C_m))/(b-1)∧3
Πm	(a(b-1)∧2b)/b∧3b [(b-1)/(δ_s C_s+ C_m)]∧(b-1)
Π_s	(a(b-1)∧b)/b∧(3b-1) [(b-1)∧2/(δ_s C_s+ C_m)]∧(b-1)
Πr	a/(b∧(3b-2))[(b-1)∧3/(δ_s C_s+ C_m)∧(b-1)]
Π	[(b-1)∧3/(δ_s C_s+ C_m)]∧(b-1)
System efficiency	((3b∧2- 3b+1)(b-1)∧(2(b-1)))/b∧2b

Continue Table 3.

Semi-integrated channel	
(MR-N)	(SM-N)
-	b(δ_s C_s+ C_m)/(b-1)
((b∧2-b+1) Csδs+bC_m)/(δs (b-1)∧2)	-
(b∧2 (δ_s C_s+ C_m))/(b-1)∧2	(b∧2 (δ_s C_s+ C_m))/(b-1)∧2
-	(a (b-1))/b∧2b
a/(b∧(2b-1)) [(b-1)∧2/(δ_s C_s+ C_m)]∧(b-1)	[(b-1)∧2/(δ_s C_s+ C_m)]∧(b-1)
(a (b-1))/b∧2b [(b-1)∧2/(_s C_s+ C_m)]∧(b-1)	a/(b∧(2b-1)) [(b-1)∧2/(δ_s C_s+ C_m)]∧(b-1)
[(b-1)∧2/(δ_s C_s+ C_m)]∧(b-1)	[(b-1)∧2/(δ_s C_s+ C_m)]∧(b-1)
((2b-1)(b-1)∧((b-1)))/b∧b	((2b-1)(b-1)∧((b-1)))/b∧b

Continue Table 3.

	-	Integrated Channel
Pm	-	b(δ_s C_s+ C_m)/(b-1)
Ps	-	-
Pr	-	-
Πm	-	a/(b∧b) [(b-1)/(δ_s C_s+ C_m)]∧(b-1)
Π_s	-	-
Πr	-	-
Π	-	a/(b∧b) [(b-1)/(δ_s C_s+ C_m)]∧(b-1)
System efficiency	-	1

supply chain. The retailer also acts as the follower; therefore, we can formulate the Stackelberg game between them. The members who are integrated are willing to set their wholesale price based on the reaction function of the retailer. In return, the retail price is also specified according to the price given by the integrated members.

The profit function for the integrated members is as follows:

$$\Pi_{SM} = (P_m - C_m - C_s\delta_s)D(P_r) \quad (3.20)$$

By replacing Equation 3.20 with Equation 3.7, we have

$$\frac{\partial \pi sm}{\partial Pm} = D(P_r) + (P_m - C_m - C_s\delta_s) \quad (3.21)$$

$$\times \frac{\partial D\ (Pr)}{\partial Pr} \times \frac{\partial Pr}{\partial Ps} = 0$$

The results are shown in Table 3.

Supplier and Manufacturer Vertical Nash:

We adjust the Nash game for the integrated members in backward integration and the independent member in the independent power structure. The retailer regulates its retail price to

maximize its own profit based on the given price by the integrated members. On the other hand, the integrated members, in order to maximize their profits, determine their price based on the retail price. By using Equations 3.3 and 3.20, we can derive the equilibrium conditions for the Nash game:

$$\frac{\partial \pi \text{sm}}{\partial \text{Pm}} =$$

$$D(P_r) + (P_m - C_m - C_s\delta_s) \times \frac{\partial D\ (Pr)}{\partial Pr} = 0 \quad (3.22)$$

Simultaneously solving the above equation and Equation 3.6, we get the results of the Nash equilibrium. (The results are shown in Table 1).

3.2.3 Integrated Channel

In the integrated channel, all the members in the supply chain integrate together. There is a full vertical integration for maximizing the entire system's profits. This structure protects the members against conflict. Here, we assume that there is only one retail price (p_r). Therefore, the total profit function for the supply chain is as follows:

$$\Pi = (P_r - C_m - C_s\delta_s) \times D(P_r) \qquad (3.23)$$

By solving the above equation, we obtain the optimal retail price. All the results for the integrated channel can be seen in Table 3.

4 Conclusion and Discussion

4.1 Results

All the results and the related quantities are summarized in Table 2 and Table 3. The results of the leader–follower power structure are shown in Table 2 and the results for the independent power structure are shown in Table 3. To achieve meaningful results, we will consider the price elasticity or b to be larger than 3. The results of the integrated channel are included in both tables.

4.2 Conclusion

In this paper, we investigate the effects of the power and the channel structures on the prices and profits for the supply chain members, individually and for the whole system.
The results demonstrate that the manufacturer in the decentralized channel or the integrated members in the semi-integrated channel are preferred

as the leaders of the chain. In this situation, the equilibrium profits are greater and the equilibrium prices are lower. Forward integrations between the manufacturer and the retailer do not improve their profits all the time. If the price elasticity is higher than 3 (b > 3), this integration can improve their profits. If the product cost becomes greater than a certain level (0.2749), the chain members' profits will increase. In addition, the market sensitivity to the retail price will be increased.

4.3 Discussion

There are many implications of these results; in this section, we will discuss them. Here, the impact of the different power structures and the channel structures are discussed.

The Effects of the Channel Structure:

The supply chain tends to choose the structure that makes the most profit.

1. When there is semi-integration structure in a three-level channel with the retailer, it always creates a larger profit for all the members toward the decentralized channel. When price elasticity: $b \geq 3.5396$.

$$\frac{2\text{b}^2 - 2\text{b} + 1}{\text{b}^2}\left(\frac{(\text{b}-1)^3}{\text{b}^2\,(\text{b}-2)}\right)^{\text{b}-1} \leq 1,\ b \geq 3.5396.$$

2. Whenever the degree of integration is higher, the efficiency is also higher.

$$\frac{(3\text{b}^2 - 3\text{b} + 1)(\text{b}-1)^{3\text{b}-3}}{3\text{b}^{2\text{b}-2}(b-3)^{b-1}} = \frac{\pi^{MS}}{\pi^{VN}} \geq 1,$$

$$\frac{2b-1}{2b}\left(\frac{(b-1)^2}{b\,(b-2)}\right)^{b-1} = \frac{\pi^{MR-S}}{\pi^{MR-N}} \geq 1.$$

So $\pi^{MS} \geq \pi^{VN} \& \pi^{MR-S} \geq \pi^{MR-N}$.

The Effects of the Power Structure:

1. The leader–follower structure in the decentralized channel is preferred. This structure leads the supply chain to lower equilibrium prices and larger profits.

$$\frac{P_r^{ms}}{P_r^{VN}} = \frac{P_m^{ms}}{P_m^{VN}} = \frac{b^2(b-3)}{(b-1)} =$$

$$\frac{b^3 - 3b}{b^3 - 3b^2 + 3b - 1} \le 1$$

$$\frac{P_s^{ms}}{P_s^{VN}} - 1 =$$

$$-(b+1) \times \frac{\delta s Cs + Cm}{(b-1)^2((b-2)\,\delta s Cs + Cm)} \le 0$$

According to the above statement,

$$P_r^{ms} \le P_r^{VN}$$
$$P_m^{ms} \le P_m{}^{VN}$$
$$P_s{}^{ms} \le P_s^{VN}$$

2. In a semi-integrated channel, the integrated manufacturer-retailer acts as a leader and the supplier acts as a follower as preferred by the members of the supply chain. The leader–follower structure, rather than independent power, is preferred by the members.

$$\frac{P_r^{MR-S}}{P_r^{MR-N}} = \frac{b(b-2)}{(b-1)^2} = \frac{b^2 - 2b}{b^2 - 2b + 1} \le 1$$

$$\frac{P_s^{MR-S}}{P_s^{MR-N}} - 1 =$$

$$-\frac{\delta_s C_s + C_m}{(b-1)^2((b-1)\delta_s C_s + C_m)} \le 0$$

Thus, $P_r{}^{MR-S} \le P_r{}^{MR-N}$, $P_s{}^{MR-S} \le P_s{}^{MR-N}$ The total profits in the manufacturer and retailer Stackelberg and the profits in the manufacturer and retailer Nash are shown as follows:

$$\frac{P_{mr}^{MR-S}}{P_{mr}^{MR-N}} = \frac{(b-1)^{2b-1}}{b^b (b-2)^{b-1}} \ge 1.$$

The above equation is a decrement function of b.

5 Contribution and the Areas for Future Research

5.1 Contribution

The formulation of game-theoretic models in a multi-level supply chain for the pricing problem is presented. This model and the solution procedures are the decision support tools for managers to determine optimal decisions and understand the interaction between all the members in the supply chain. This paper has provided a comparative study for the pricing problem under the various power and channel structures. We have proved that forward integration does not always lead to increased profits. On the other hand, it was proved that the leader–follower power structure brings about better results.

5.2 Future research

In this article, the competition at each level is not covered. A more general model with multiple members in each level could be developed.
This model focuses on pricing as a market-strategic variable. Considering the other market variables like distribution cost is recommended.
This research focuses on non-cooperative game approaches. Other types of game models could be interesting for future research.

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