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Research Article

A Divergence Measure for Combining Super-Efficiency Scores in Performance Measurement of Two-Stage Production Systems

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Abstract

In the conventional data envelopment analysis (DEA) internal sub-processes of the production units are ignored. The current paper develops a network-DEA super-efficiency model to compare the performance of efficient network systems. A new ranking method is developed by aggregating the computed super-efficiency scores with a J-divergence measure. The proposed approach is then applied to evaluate wheat productions in Iran provinces.

Keywords : Data envelopment analysis; Network DEA; Super-efficiency; J-divergence; Ranking.

1 Introduction

Data envelopment analysis (DEA) is a non-parametric approach for evaluating efficiency score of peer decision making units (DMUs) which consume multiple inputs to produce multiple outputs. Non-parametric approaches assume no specific form for production function and as the result, measuring efficiency scores is based on an estimated production frontier using observed inputs and outputs data.

Following the pioneering work of Farrell [11], DEA as a non-parametric technique is introduced by Charnes et al. [7] for evaluating efficiency scores of units in constant returns to scale (CRS) technology. Banker et al. [3] introduced a new model for measuring the efficiency score of systems under variable returns to scale (VRS) technology. Recent citation based studies indicate that the DEA has a significant growth in many application areas which is accompanied by new theoretical developments, see for example Emrouznejad et al [12] and Cook and Seiford [10] among the others.

In standard DEA models, a DMU is treated as a black-box which converts some inputs to some outputs and interrelationship between sub-processes are totally ignored. So, basic DEA models cannot use directly for performance evaluation of multi-stage production systems. To overcome this deficiency, network DEA models are introduced to deal with production systems with some sub-processes.

In recent years, many researchers studied the issue of modeling production units with network structures in DEA. Zhu [32] studied the two-stage structure where the first stage consumes some inputs to produce some intermediate outputs which used to produce final outputs in the second stage. The proposed model is then used to measure the

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efficiency of the best 500 companies as ranked by Fortune. Lewis and Sexton [22] also used a similar method to study the performance of Major Baseball League. Kao and Hwang [17] developed a relational model to measure the efficiency of a two-stage production unit in which the product of two-stage efficiencies is equal to the total system efficiency. Li et al. [23] introduced a model for two stage systems in which a second stage has exogenous inputs in addition to intermediate product. Kao [16] presented a model to evaluate network systems with parallel structure. Kao [15] generalized the model of two stage systems for systems with more than two stages (series system). Also, Kao [15] presented a relational model for measuring the efficiency score of a production unit with general network structure. Although, this model is applicable for all network systems, it is not a unified model and the constraints of the model need to rearrange by any variation in the number of inputs, outputs and sub-processes. Lozano [24] presented a simple model for general network systems to derive cost efficiency and scale efficiency of the units. Although Lozano's model is applicable for most of network structures, including two stages or parallel systems, it doesnt cover systems with more than two stages. To overcome this issue, Kazemi Matin and Azizi [18] introduced a general network DEA model which is capable to model and evaluate all network production systems in a unified development. Chen et al. [9] showed that the multiplier and envelopment network DEA models have different results in presenting divisional efficiency.

Also, they mentioned that proper benchmarks cannot be derived from most of network DEA models. Kao [14] evaluated the efficiency score of general multi-stage systems, where each stage consumes exogenous inputs in addition to intermediate products. Most of the real world production process are multi-stages and dealing with network production processes has increased growth in recent DEA literature. One related issue in this context is ranking multi-stage production units with network DEA models.

Similar to Multi-Criteria Decision Analysis (MCDA), ranking units in the DEA has become an acceptable technique. Many ranking methods with different criteria are available in the DEA literature. If we consider these models and the availability of a model in DEA commercial software as an indication of popularity then we find Anderson and Peterson (AP) [1] super-efficiency method as the most popular model for ranking units; see for example Cook and Seiford [10]. In AP ranking method, in contrast to DEA efficiency models, each observation is excluded from its own reference set and it is possible to compute efficiency scores greater than one.

In classic DEA, the concept of super-efficiency is related to differentiate the performance of efficient units. Although in the DEA evaluation of network production process, by considering the internal processes, there are a few efficient units, but the super-efficiency scores still contain useful information about the production process. Comparing the efficiency and the super-efficiency distributions is an interesting idea for ranking efficient units and also identifying outliers, i.e. efficient units with high super-efficiency score. See for example Banker and Chang [2] for more details.

In contrast with its popularity, the super-efficiency ranking method has its limitations which need to be considered in applications. Seiford and Zhu [25] indicated under what circumstances the VRS super-efficiency model can be infeasible. Lee et al. [21] presented a model which calculates the same efficiency score as the classic super-efficiency model for feasible units, but the units which are infeasible under classical super-efficiency model are feasible under their model. Chen [8] presented one model to calculate the efficiency score of inefficient DMUs and the super-efficiency score of efficient ones as well.

One important issue in using traditional super-efficiency scores for ranking boundary units is its self-appraisal behavior. In computing this, score each unit is compared with the production frontier composed by the other observations. This is a one-sided evaluation in which efficiency of the other observations relative to the new production frontier is totally ignored. We suggest using these scores in an aggregated measure to achieve a new ranking criterion. For aggregation step, we will benefit of some useful measures from information theory.

The concept of entropy introduced by Shannon [26] plays the central role in information theory and can be shown to be a good measure of randomness or uncertainty. This concept has been widely used in different scientific areas, like statistics, soft computing and decision making.

In a glance look to the literature, we can see some key works in this field. Kullback and Leibler [20] introduced a measure associated with two probability distributions of a discrete random variable. They also developed the idea of Jeffrey's [13] invariant. Taneja [29] presented geometric mean divergence measure. Kerridge [19] introduced an expression similar to Shannon's entropy for two probability distributions.

During past decade, researchers interested scalar parametric generalizations of the above classic measures in information theory. These measures have deep relations with statistics in applications.

In the recent DEA literature, there are considerable works which directly use entropy techniques in performance measurement. Soleimani-damaneh and Zarepisheh [28] proposed a procedure based on Shannon's entropy for combining the efficiency scores of different DEA models, to select the best model. Bian and Yang [6] also used Shannon's entropy for resource and environmental efficiency analysis of 30 provinces in China. Recently, Shuai and Wu [27] used DEA and grey entropy to analyze the impact of Internet marketing on hotel performance. Yang et al. [30] also used Shannon's entropy to determine the weights for ultimate cross efficiency scores. Most recently, Zhou et al. [31] proposed a non-radial DEA approach by integrating the entropy weight and optimal solution of SBM model for environmental efficiency analysis of the power industry in China. These show the potential of entropy based techniques for combining the computed scores and weights in DEA models to introduce aggregated results. We will benefit of this feature in aggregating two-stage super efficiency scores and introducing a new ranking procedure with the ability of detecting outliers. An application in the wheat farming industry in Iran provinces will also used to illustrate the approach.

In this paper a two-stage network DEA model and its super efficiency version under VRS technology is presented. By eliminating each efficient observation, relative efficiency scores associated with the new production set are computed. Two efficiency and super-efficiency distributions are then compared by using the J-divergence measure for ranking efficient units.

An application in wheat production of Iranian provinces is then used to illustrate the method. In the application, each province is considered as a parallel system with two processes in which each process is composed of two series processes. The rest of this paper unfolds as follows. In section 2, a brief review of some basic network DEA models under VRS technology are presented. A two-stage super-efficiency model is also introduced. Section 3 is devoted to introduce a new ranking in aggregating efficiency and super-efficiency scores with J-divergence measure. An application in wheat farming in 30 provinces in Iran is used to illustrate the method in section 4. Section 5 concludes.

2 DEA and Network DEA and super-efficiency models

At first we suppose there are n $DMUs$, and $DMU_j; j = 1, ..., n$ represents unit j whose input and final output vectors are $x_j = (x_{1j}, ..., x_{mj})$ and $y_j = (y_{1j}, ..., y_{sj})$, respectively. The efficiency score of DMU_k in DEA is calculated as the ratio of its weighted output to weighted input subject to non-negative and universal weights. The CCR model by Charnes et al. [7] is used to estimate a CRS production function while the BCC model introduced by Banker et al. [3] allows for the VRS assumption of the estimated production function. With the above notations, the CCR model is as follows, when DMU_k is under evaluation unit:

$$
\begin{aligned}
e_k = \max & \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \qquad (2.1) \\
s.t \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ik}} \leqslant 1, j = 1, ..., n, \\
& v_i, u_r \geqslant 0, r = 1, ..., s, i = 1, ..., m
\end{aligned}
$$

In optimality, e_k shows technical efficiency for DMU_k . If $e_k = 1$, DMU_k is defined CCR-efficient and for $e_k < 1, DMU_k$ is said CCR-inefficient. The BCC model computes efficiency score in a VRS technology and can be presented as follows.

$$
\begin{aligned}
\max \quad & \frac{\sum_{r=1}^{s} u_r y_{rk} - u_o}{\sum_{i=1}^{m} v_i x_{ik}} \qquad (2.2) \\
s.t \quad & \frac{\sum_{r=1}^{s} u_r y_{rj} - u_0}{\sum_{i=1}^{m} v_i x_{ik}} \leqslant 1, j = 1, ..., n \\
& v_i, u_r \geqslant 0, r = 1, ..., s, i = 1, ..., m \\
& u_o free
\end{aligned}
$$

The super-efficiency score for DMU_k in VRS technology can be obtained by solving the following linear programming model.

$$
\begin{aligned}
&\max \sum_{r=1}^{s} u_r y_{rk} - u_o, \qquad (2.3)\\
&s.t \quad \sum_{i=1}^{m} v_i x_{ik} = 1,\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - u_o \leqslant 1,\\
&\quad j = 1, ..., n, j \neq k,\\
&\quad v_i, u_r \geqslant 0, r = 1, ..., s, i = 1, ..., m\\
&\quad u_o free
\end{aligned}
$$

Note that the main difference between model (2.3) and the conventional BCC model is that in super-efficiency evaluation for DMU_k, the unit k is removed from its own reference set and as the result for efficient units we may obtain a super-efficiency score greater than one, while inefficient units has equal efficiency and super-efficiency scores. See Anderson and Peterson [1] and Zhu [33] for more details.

The super efficiency score is often used for ranking efficient units based on the computed super-efficiency scores. Although it is originally proposed for outlier identification and using this methodology to identify outliers performs satisfactorily in DEA applications. Banker and Gifford [5] suggested the use of the super efficiency scores to filter data and remove those identified outliers to obtain more reliable efficiency estimation. Banker and Datar [4] applied this method for outlier identification in cost efficiency evaluation for 117 hospitals. Andersen and Petersen [1] employed the same Banker and Gifford model and prescribed the use of the super-efficiency score for ranking efficient units in DEA. Banker and Chang [2] using simulation experiments show that AP super-efficiency scores for ranking efficient observations does not perform satisfactorily. In contrast, the evidence supports super-efficiency based procedure for outlier identification.

In this paper, we will extend this procedure for outlier identification in multi-stage production processes by considering super-efficiency evaluation in network DEA models.

In conventional DEA evaluations, production units are considered as black-boxes, i.e. units consume some inputs to produce some outputs, and possible internal processes and intermediate products are totally ignored. If we decide to consider these intermediate products to achieve a more realistic evaluation of the units, then the conventional super-efficiency DEA model is needed to be modified and new super-efficiency network DEA model is need to be developed. Before proceeding further in this development, we consider some basic network DEA models.

One of the most common structures of network systems in the DEA is a two-stage network, which is depicted by figure 1. Here, z_d are denoted as

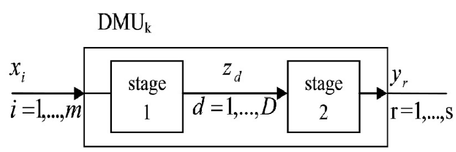

Figure 1: Two stage system

the dth intermediate products of process 1 which are the outputs of process 1 and are consumed as the inputs of process 2 to produce the final outputs.

The proposed model for efficiency evaluation of two-stage systems by Kao and Hwang [17] in VRS technology can be presented as follows:

$$
\begin{aligned}
&E_k = \max \sum_{r=1}^{s} u_r y_{rk} - w_o - u_o, \qquad (2.4)\\
&s.t \quad \sum_{i=1}^{m} v_i x_{ik} = 1,\\
&\sum_{d=1}^{D} w_d z_{dj} - w_0 - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, j = 1, ..., n,\\
&\sum_{r=1}^{s} u_r y_{rj} - u_o - \sum_{d=1}^{D} w_d z_{dj} \leqslant 0, j = 1, ..., n,\\
&\quad w_d, v_i, u_r \geqslant 0,\\
&\quad r = 1, ..., s, i = 1, ..., m, d = 1, ..., D,\\
&\quad w_o, u_o free
\end{aligned}
$$

In the literature, parallel network, which is shown in figure 2 is also considered as a basic structure of network production systems. Note that in parallel structure we have the following relations between inputs/outputs of sub-processes and total

Table 1: Descriptive statistics on the wheat-farming data set

	Irrigation farming			
	Cultivated area	Harvested area	Wheat production	Consumed seed
Min	105	100	283	13
Max	502325	384678	1307213	118362
Average	87040.4	161878	295846	20728.73
	Rainfed farming			
	Cultivated area	Harvested area	Wheat production	Consumed seed
Min	0	0	0	0
Max	516635	512798	512203	59601
Average	80178.87	140144.37	150416.9	21088.53

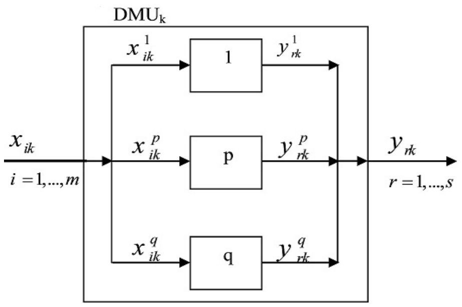

Figure 2: Parallel system

input/output of the stages.

$$\sum_{p=1}^{q} x_{ik}^{p} = x_{ik}, i = 1, ..., m,$$
$$\sum_{p=1}^{q} y_{rk}^{p} = y_{rk}, r = 1, ..., s.$$

Kao [15] also proposed transferring a general network structure into series stages, which contains some parallel processes. The parallel production systems are also studied in Kao [16] in VRS technology with following model:

$$\begin{aligned} \max\ & \sum_{r=1}^{s} u_r y_{rk} - u_o^p, \qquad (2.5) \\ s.t\ & \sum_{r=1}^{s} u_r y_{rj}^p - \sum_{i=1}^{m} v_i x_{ij}^p - u_o^p \leqslant 0, \\ & j = 1, ..., n \& p = 1, ..., q, \\ & \sum_{i=1}^{m} v_i x_{ik} = 1, \\ & v_i, u_r \geqslant 0, r = 1, ..., s \& i = 1, ..., m \\ & u_o^p free \end{aligned}$$

In the rest of the paper and to develop a network supper-efficiency model, we use the above mentioned network DEA model proposed by Kao and Hwang [17] for efficiency evaluation of two-stage production systems with VRS technology. The proposed models are then used in an empirical application in wheat farming and the computed scores are combined by using a divergence measure for ranking production units. The wheat farming application will discuss in section 4 in more details.
Similar to AP model, if we remove the under evaluation unit, DMU_k from the reference set we have the following super-efficiency version of the above introduced two-stage network model.

$$\begin{aligned} E_{kk} = \max & \sum_{r} u_r y_{rk} - w_o - u_o, \qquad (2.6) \\ s.t \quad & \sum_{i} v_i x_{io} = 1, \\ & \sum_{d} w_d z_{dj} - \sum_{i} v_i x_{ij} - w_o \leqslant 0, \\ & j = 1, ..., n, j \neq k, \\ & \sum_{r} u_r y_{rj} - \sum_{d} w_d z_{dj} - u_o \leqslant 0, \\ & j = 1, ..., n, j \neq k, \\ & w_d, v_i, u_r \geqslant 0, \qquad (2.7) \\ & r = 1, ..., s, i = 1, ..., m, d = 1, ..., D, \\ & w_o, u_o free. \end{aligned}$$

Here, E_{kk} shows the super-efficiency score of the observed unit k, when it is removed from its own

Table 2: The results of efficiency calculated by the general model and BCC model

Provinces (DMUs)	Two-stage Efficiency (VRS)	BCC Efficiency	Provinces (DMUs)	Two-stage Efficiency (VRS)	BCC efficiency
Azerbaijan, East	0.2496	0.368	16. Fars	0.8513	1
Azerbaijan, West	0.4813	0.648	17. Qazvin	0.4933	0.560
Ardabil	0.5798	0.858	18. Qom	1	1
Isfahan	0.5394	0.540	19. Kurdistan	0.3382	0.496
Ilam	0.2619	0.288	20. Kerman	0.6279	0.628
Bushehr	0.0811	0.133	21. Kermanshah	0.5718	0.716
Tehran	1	1	22. Kohgiluyeh and Boyer-Ahmad	0.2679	0.413
Chahar Mahaal and Bakhtiari	0.4652	0.466	23. Golestan	0.9188	1
Khorasan, South	0.3982	0.647	24. Guilan	1	1
Khorasan, Razavi	0.5721	1	25. Lorestan	0.3356	0.439
Khorasan, North	0.4004	0.459	26. Mazandaran	0.6380	0.684
Khuzestan	0.4615	0.575	27. Markazi	0.4549	0.717
Zanjan	0.3243	0.625	28. Hormozgan	0.9305	0.931
Semnan	0.6846	0.685	29. Hamadan	0.4825	0.665
Sistan and Baluchestan	0.4980	0.574	30. Yazd	0.7535	0.834

reference set. Note that we always have $E_k \leqslant E_{kk}$ and it is possible to obtain super-efficiency score that exceed one. Note that because of the multi-stage structure of production units, the network DEA efficient frontier is different from the one under the standard DEA, while production units are considered as black-boxes.
Model (2.6) not only can be used for estimating the super-efficiency score of the efficient observed unit, but it is also applicable to evaluate the effect of eliminating each unit on the efficiency scores of the other observations. We will benefit of this feature to introduce a new ranking method by combining these scores using a divergence measure from information theory. The method will develop in the next section.

3 Combining super-efficiency scores using a J - Divergence measure for ranking efficient units

The main drawback of the AP method which also appears in the above introduced two-stage super efficiency model (2.6), is the validity of their criteria for ranking all units, despite the fact that in computing E_{kk}, each unit is evaluated according to different weights in a self appraisal scenario. The objective function value in this model could be interpreted as a one-sided relation of DMU_k with the production frontier made by the other units. We suggest to consider the other side of this relation by computing the efficiency score of the other units ($DMU_j; j \neq k$) relative to the new efficient frontier, constructed without DMU_k in the two-stage production system. The following version of the model (2.6) provides these val-

Table 3: The results of super efficiency calculated by two-stage and BCC models

Provinces (DMUs)	Two-stage super efficiency	BCC super efficiency	Provinces (DMUs)	Two-stage super efficiency	BCC super efficienc
Azerbaijan, East	0.2496	0.368	16. Fars	No feasible solution	No feasible solution
Azerbaijan, West	0.4813	0.648	17. Qazvin	0.4933	0.560
Ardabil	0.5798	0.858	18. Qom	1.157	1.173
Isfahan	0.5394	0.540	19. Kurdistan	0.3382	0.496
Ilam	0.2619	0.288	20. Kerman	0.6279	0.628
Bushehr	0.0811	0.133	21. Kermanshah	0.5718	0.716
Tehran	1.638	1.748	22. Kohgiluyeh	0.2679	0.413
			and Boyer-Ahmad		
Chahar Mahaal	0.4652	0.466	23. Golestan	0.9210	1.217
and Bakhtiari					
Khorasan, South	0.3982	0.647	24. Guilan	2.411	2.411
Khorasan, razavi	0.5721	1.007	25. Lorestan	0.3356	0.439
Khorasan, North	0.4004	0.459	26. Mazandaran	0.6387	0.684
Khuzestan	0.4615	0.575	27. Markazi	0.4549	0.717
Zanjan	0.3827	0.625	28. Hormozgan	0.9310	0.931
Semnan	0.6846	0.685	29. Hamadan	0.4825	0.665
Sistan and	0.4980	0.574	30. Yazd	0.7535	0.834
Baluchestan					

ues:

$$
\begin{aligned}
E_{lk} = \max \ & \sum_r u_r y_{rl} - w_o - u_o, \qquad (3.8)\\
s.t \ & \sum_i v_i x_{il} = 1,\\
& \sum_d w_d z_{dj} - \sum_i v_i x_{ij} - w_o \leqslant 0,\\
& j = 1, ..., n, j \neq k,\\
& \sum_r u_r y_{rj} - \sum_d w_d z_{dj} - u_o \leqslant 0,\\
& j = 1, ..., n, j \neq k,\\
& w_d, v_i, u_r \geqslant 0,\\
& r = 1, ..., s, i = 1, ..., m, d = 1, ..., D,\\
& w_o, u_o free.
\end{aligned}
$$

Here, E_{lk} denotes the computed score for unit l where unit k is excluded from the observations. The linear program (3.8) is solved for every observed unit k and gives vector $\mathbf{E}^k = (E_{1k}, ..., E_{nk})$, containing information which reflects how eliminating unit k influences on the efficiency scores of the observations. In the other words, $\mathbf{E}^k$ includes information about the situation of all observed units relative to the efficient frontier of the *new production set* when unit k excluded from the observations.

Comparing vectors $\mathbf{E} = (E_1, ..., E_n)$ and $\mathbf{E}^j = (E_{1j}, ..., E_{nj}); j = 1, ..., n$ provides use the efficiency distribution it j and its influence on efficiency distribution of all observations. We'll benefit from this information for ranking efficient observed two-stage units and also for identifying potential outliers.

As mentioned in the introduction, using entropy techniques become more popular in aggregating computed results in different DEA models.

Here, we suggest a new aggregated ranking procedure technique, in an alternative use of a two-stage super-efficiency DEA model by using the J - divergence measure from information theory.

The proposed approach is also useful for identifying outlier observations.

3.1 J-divergence measure for ranking two-stage production units

In the rest of this section, we use a revised version of Kullback and Leibler's measure of information to rank the efficient units. Based on its features, this measure has many names such as, relative information, directed divergence, cross entropy, function of discrimination etc, and it has found many applications in information theory and statistics.

We use a symmetric version of this numerous information index, which is known as the J - divergence measure. Recall that for our ranking purpose we aim to compare two efficiency distributions $\mathbf{E}$ and $\mathbf{E}^j$ for $j = 1, ..., n$. In information theoretic approaches, information indices like J-divergence measure are entropy-based measures that quantify discrepancies between distributions; so it is useful for our comparison purpose. Having access to efficiency distributions $\mathbf{E}$ and $\mathbf{E}^j$, we would like to precisely measure differences between these factors. Any increase in the computed J-divergence measure means that the associated two-stage unit j has a bigger influence on the efficiency distribution of the observations.

Now, to obtain the divergence measure based upon two-stage efficiency and super-efficiency scores we suggest the following steps:

Step1. (Normalization) Set $\bar{E}_j = \dfrac{E_j}{\sum_{i=1}^{n} E_i}$ and $\bar{E}_{lj} = \dfrac{E_{lj}}{\sum_{i=1}^{n} E_{ij}}$ for $l, j = 1, ..., n$.

Step2. Compute J-divergence measure of efficiency distributions $\mathbf{E}$ and $\mathbf{E}^j$ as $J\left(E \,||\, E^j\right) = \sum_{i=1}^{n} (\bar{E}_i - \bar{E}_{ij}) \log \dfrac{\bar{E}}{\bar{E}_{ij}}$.

Note that it is assumed that $0 \log 0 = 0 \log \dfrac{0}{0} = 0$

$J\left(E \,||\, E^j\right)$ satisfies the basic properties of distance functions; i.e. non-negativity, symmetry and additivity; see Jeffreys [13] and Kullback and Leibler [20] for more details.

It is also easy to verify that $J\left(E \,||\, E^j\right)$ is a monotonic measure by this means that if $E^j \neq E^k$ and $E^j \leqslant E^k$ then $J\left(E \,||\, E^j\right) \leqslant J\left(E \,||\, E^k\right)$.

These useful features in addition to the property of the logarithm function which is quite sensitive to even small changes make the J - divergence measure sensitive to even small changes in the computed E^j vectors. So, J-divergence measure could be considered as a powerful measure in aggregating super-efficiency scores and ranking units in single or multi-stage production processes. The next section is devoted to describing the proposed ranking approach in wheat farming performance evaluation of 30 provinces in Iran.

4 An application in wheat farming

This section analyzes wheat farming efficiency in provinces of Iran in 2008-2009 crop years, which is started on 22 September 2008 and ended on 22 September 2009. In the mentioned time, Iran consisted of 30 provinces which were managed by the government. To estimate super efficiency of provinces and evaluate the possible impact of removing one province from the production set on the estimated efficiencies distributions, we apply the provided two-stage efficiency and super-efficiency models for the network structure depicted in figure 3.Then we compare the achieved results with the proposed J-divergence ranking criteria. Provided discussions on the results illustrate the approach and includes the possibility of being an outlier for some of the provinces.

The computed efficiency and super efficiency scores of models (2.4) and (2.6) for the two-stage network will be compared to suggest the rank positions for every province.

Figure 3 shows inside of sample wheat farming production unit as a network production system with four processes include in two parallel processes, which is composed of two sub-processes in series structure. In this application, the two parallel processes are considered as irrigation farming and rainfed farming and the two series processes are sowing-growing and harvesting, respectively. In figure 3, x_{1k}, x_{2k}, y_{1k} are used to show inputs and final output of the system. Table 1 summarizes descriptive statistics of the data set on Iran wheat farming in 2008-2009 crop years. Data are gathered by the Iranian Ministry of Agricultural Jihad [34]. The inputs of the system, which are also considered as the inputs of the first and third processes, are cultivated area (based on hectare) and consumed seed (based on ton). There is one intermediate product in the

Table 4: The results of applying the model (3.8) for the three two-stage efficient units

Provinces (DMUs)	**E**	$\mathbf{E}^7$	$\mathbf{E}^{18}$	$\mathbf{E}^{24}$
1	0.2496	0.3290	0.2496	0.2496
2	0.4813	0.5436	0.4813	0.4813
3	0.5798	0.6556	0.5798	0.5798
4	0.5394	0.8658	0.5409	0.5394
5	0.2619	0.4011	0.2652	0.2621
6	0.0811	0.0811	0.0921	0.1557
7	1	1.638	1	1
8	0.4652	0.6956	0.4733	0.4652
9	0.3982	0.5588	0.4101	0.5104
10	0.5721	0.5897	0.5721	0.5721
11	0.4004	0.6142	0.4004	0.4004
12	0.4615	0.4724	0.4615	0.4615
13	0.3243	0.4026	0.3243	0.3394
14	0.6846	0.9762	0.7030	0.6846
15	0.4980	0.6943	0.5135	0.4991
16	0.8513	0.8513	0.8513	0.8513
17	0.4933	0.7893	0.4950	0.4935
18	1	1	1.157	1
19	0.3382	0.3919	0.3382	0.3382
20	0.6279	0.9409	0.6386	0.6279
21	0.5718	0.6265	0.5718	0.5718
22	0.2679	0.3911	0.2739	0.3084
23	0.9188	0.9505	0.9188	0.9188
24	1	1	1	2.411
25	0.3356	0.4287	0.3356	0.3356
26	0.6380	0.9519	0.6494	0.6384
27	0.4549	0.5813	0.4549	0.4549
28	0.9305	1	1	0.9305
29	0.4825	0.5381	0.4825	0.4825
30	0.7535	0.9825	0.7863	0.7550

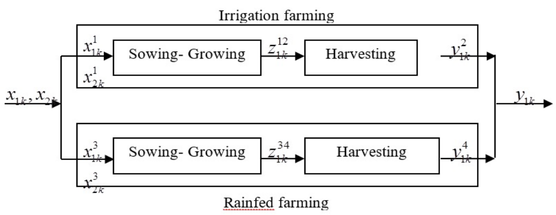

Figure 3: network structure of wheat farming in provinces

system which is the output of the sowing-growing process as well as the input of the harvesting process. The intermediate product is harvested area (based on hectare). The output of the system, which is also the output of the second and fourth process, is wheat production (based on ton). Yazd, Sistan-Baluchestan and Hormozgan are the provinces which do not have rainfed farming. Both of irrigation and rainfed farming are used in the other provinces.

4.1 Results

Efficiency scores are calculated using Lingo software, under the assumption of variable returns to scale. The results of the two-stage network model (model 2.4) and BCC model (model 2.2) for determining the efficiency of the wheat production in Iran provinces are reported in Table 2.

The results of applying the two-stage model (model 2.4) and the two-stage super-efficiency model (model 2.6) for determining super efficiency of the wheat production in the provinces are summarized in Table 3. Some interesting and considerable points are unfolded in these results:

(i) It is possible for some provinces to get different rank position when are evaluating with two-stage models in comparison with evaluating by traditional DEA model. For example, Azerbaijan, East (DMU1) has a better BCC rank position than Ilam (DMU4) but in two-stage models this relation is reversed. This is because of considering internal processes in the network evaluations.

(ii) As it is expected, both efficiency and super-efficiency scores in two-stage models are less than or equal to the case of traditional DEA models in all observed units.

(iii) Similar to traditional DEA, in two-stage production systems, super-efficiency scores are also always greater than or equal to efficiency scores.

(iv) In contrast to classical DEA models, the efficiency score of some inefficient DMUs is different with their super-efficiency scores in two-stage models. These are the units that are efficient in one of their stages: Zanjan (DMU13) and Golestan (DMU23), Mazandaran (DMU26) and Hormozgan (DMU28).

(v) It is possible to have infeasibility in both super-efficiency models. See for example Fars (DMU16).

As it is shown in Table 2, there are only three efficient provinces (10 %) with the highest efficiency score equal to 1, when internal sub-processes are considered in evaluating provinces with the two-stage model. For the other provinces, their computed super-efficiency scores could be used in ranking.

To achieve a complete ranking of the provinces, we need to distinguish between the three efficient units. As we suggest, the J-divergence measure is a useful tool for this purpose. The results of applying the extended super-efficiency model (3.8) for two-stage efficient DMUs are reported in Table 4.

Now we apply the provided J-divergence measure for this data set which yields to: $J\left(E\,||\,E^{7}\right) = 0.0136$, $J\left(E\,||\,E^{18}\right) = 0.006$, $J\left(E\,||\,E^{24}\right) = 0.0289$

and leads to the fallowing ranking positions which is reported in Table 5. In this application, the proposed ranking positions for efficient units are consistent with those provided by two-stage super-efficiency scores in the model (3.8).

Table 5: Ranking efficient provinces by J-divergence measure

Efficient Provinces	Rank position
Tehran	2
Qom	3
Guilan	1

Note that the computed J-divergence measures for efficient provinces are relatively close. In the case of big difference between some of the computed J-divergence measures with the other, units with big J-divergence values could be considered as potential outliers.

Finally, we can see that the introduced two-stage efficiency and super-efficiency models based on the network structure of the wheat farming in provinces provide a better discrimination power than classical DEA models. These help us to make a complete ranking of the provinces and detect inefficient provinces in wheat production.

5 Conclusion

Traditional DEA models cannot apply directly for production systems with network structure. In this paper a basic two-stage network model and its super-efficiency model is presented. To achieve a complete ranking of efficient two-stage units, in an alternative application of AP model, super-efficiency distributions associated with eliminating of any efficient unit is compared with the efficiency distributions before elimination. To do this, for the ranking purpose, a new aggregated measure is introduced using the J - divergence measure, which prevents the self appraisal behavior of the standard AP model in ranking.

For illustration purpose, wheat farming in Iranian provinces is evaluated from both efficiency and super-efficiency ranking viewpoints. The results show better discrimination in using network DEA in the application. Some new and interesting points are also unfolded in the results which show dealing with super-efficiency models in multi-stage production process could be considered as an interesting challenge for the future studies.

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