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Research Article

Benchmarking Automotive After-sales Service Companies with Dependent Criteria-Application of Data Envelopment Analysis

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Abstract

Benchmarking is a tool for evaluating organizational performance with a learning approach from others. The importance of benchmarking in every industry is clear for anyone. In the automotive industry, the performance of after-sales service agencies in Iran is evaluated every year by Iran Standard and Quality Inspection company. One of the ways to continuously improve in after-sales service agencies is benchmarking of successful and efficient examples in the network. In this paper, a benchmarking model is developed considering that the repair index and customer satisfaction are interdependent. To improve the accuracy and operationality of benchmarking, some constraints have been added to the model with the opinion of experts. Considering the dependent parameters, a data envelopment analysis model has been proposed and this model has been implemented to benchmark 20 after-sales service agencies of a car company. By solving the model and comparing it with the results of the original model, it was observed that the considered conditions changed the benchmarking and increased the accuracy. This paper discusses the concept of the impact and importance of dependent parameters in benchmarking, and with this concept, a benchmarking model for automotive after-sales service agencies is presented.

Keywords : Data Envelopment Analysis; Benchmarking; Dependent parameters; Automotive after-sales service.

1 Introduction

PErformance evaluation is one of the most basic tasks of managers of different organizations, for this purpose, various methods and models have been proposed by researchers [1]. One of the most widely used methods of performance evaluation is the Data Envelopment Analysis (DEA). Based on the basic concepts [2], Charens et al. measured the efficiency of decision-making units in 1978 using a linear programming model to esti-

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mate the efficiency frontier [3]. After that, many books and papers on the concept of DEA and DEA applications in various industries and topics were presented. The DEA method is a linear and non-parametric programming method that compares the outputs and inputs of Decision-Making Units or DMUs with each other. This method is a good tool for measuring and evaluating the relative efficiency of DMUs. Traditional statistical methods usually work with strict approaches and in these methods, DMUs are evaluated by comparing their parameters according to the average of parameters of DMUs, while DEA is a maximalist approach, it compares and evaluates the parameters of each DMU with only the specifications of the best DMU. Lai et al showed that data envelopment analysis has two advantages over the traditional performance evaluations approach [4]:

- Woight calcdlated baseu on the inputs and outpuus of each unit and there is no need te evaluate the weight of the inputs or outptts or prioritize them.
- We can calcelate relative efficiuncy, because the DEA method involves multiple inputs and outputs,

Therefore, data envelopment analysis is recommended to assist traditional performance assessment and benchmarking methods and for providing guidance for managing decision-making units [5]. Various experiences show that this method is a powerful tool for performance evaluating and benchmarking to improve and enhance the efficiency of organizations. This method has been used in various studies, for example [6, 7, 8, 9, 10]. The data envelopment analysis method has been widely used in performance evaluating and benchmarking studies since it was proposed by Charnz et al., But one of areas that has paid very little attention in researchers in data envelopment analysis models is the dependence parameters especially the output parameters of DMUs, because in the real world, parameters of DMUs have interdependence and correlation, for example, suppose the purpose is evaluating the efficiency of bank branches, one of the output parameters is the amount of bank income through loan installments that this parameter is directly related to the of given loans, here two basic questions are raised, first, why not consider both parameters as one gathered parameter? Second, the values of the parameters in each DMU are known, so what helps to consider this dependency? To answer the first question, the accuracy of measuring the efficiency of decision-making units is considered, because the parameters are interdependent and not exactly equal (in other words, their correlation coefficient is not equal to 1 and other factors affect them). To answer the second question, it should be noted that to measure efficiency, indeed, this dependence may not be significant (the efficiency of decision-making units is the result of the ratio of outputs to inputs of the decision-making unit, these parameters are obtained, even if the parameters are dependent). But it is very important for benchmarking, a virtual DMU cannot be benchmark for an inefficient DMU without the dependency of the parameters being logically observed. Therefore, the innovation of this paper is benchmarking by considering dependent parameters in data envelopment analysis. The envelopment model is used for benchmarking or obtaining a benchmark for inefficient DMUs, which is by calculation of the coefficients of each DMU. according to the purpose of this paper (benchmarking) the final model is the envelopment model. Like other businesses, after-sales service agencies, have input and output parameters, but these parameters have certain conditions, for example, some of them are interdependent, some are more important and some of them cannot be changed. In this paper, after-sales service agents have been selected as a case study and a benchmarking model has been developed. In the second section, the subject literature is presented and showed that the subject of benchmarking of DMUs withs dependent parameters was missed in the literature on data envelopment analysis. The necessity of this issue can be seen in the case study. The after-sales service agencies of a car company are considered as decision-making units, and further examination shows that its two outputs have a high correlation coefficient. Therefore, in this paper, a data envelopment analysis model has been developed for benchmarking for car after-sales service agencies. For this purpose, after reviewing the literature, mathematical modeling is presented in the third section, then in the fourth section the numerical results are presented, and finally, in the fifth section, discussion and conclusion are presented.

2 Literature Review

The history of benchmarking may back to the 1800s and the textile industry [11] and has undergone many changes, especially with the advent of quality management principles. The experience of using benchmarking as an effective and practical management tool began in the 1980s at Xerox due to the loss of market share and pressure from competitors, especially Japanese companies. Successful lessons from Xerox have led many other companies to use this new approach to increase efficiency, productivity, and consequentialism in order to gain a competitive advantage [12]. The benchmarking method spread rapidly and became the most widely used method of competitiveness [13]. This method is widely used as a method of efficiency improving [14], eliminating errors in the process, new product development [15] and improving customer satisfaction [16] was used. Accordingly, benchmarking has several definitions in the literature. In 1989, Camp presented a comprehensive and common definition of benchmarking as "a search to achieve the most acceptable industry exercises that would result in the exceptional results by implementing these best practices" [12]. According to the literature, benchmarking has more than 42 definitions [17] but it can be stated that there is not yet a suitable and comprehensive definition for benchmarking [18]. But Peng Wong and Yew Wong stated in 2008 that, in the opinion of most authors, benchmarking is one of the management tools that use a systematic process to find the best benchmark, innovative ideas, and efficiency on a path of continuous improvement [19]. The target of this improvement is to find a way to do similar tasks with greater efficiency, identify and implement methods to increase process efficiency, and determine the number of outputs [17]. As mentioned, based on the basic concepts [2], Charens et al. measured the efficiency of decision-making units in 1978 using a linear programming model to estimate the frontier of the production technology [3]. Efficiency means "working well", is influenced by internal indexes such as profit per unit, price per unit, and so on, which is expressed as the ratio of output to input. Data envelopment analysis is a linear programming method that uses the data of decision-making units to construct an efficiency frontier. The above frontier is based on data in the form of inputs and outputs, and in fact, the value of inefficiency of each decision-making unit is the distance of the unit to the efficiency frontier [20]. Data envelopment analysis calculates the deviation of each DMU from the efficiency frontier by plotting the performance frontier according to the Production Possibility Set (PPS). The production possibility set is defined as follows [21].

$$\mathrm{PPS} = \{(\mathrm{x}, \mathrm{y}) \mid \mathrm{x} \geq \sum_{\mathrm{j}=1}^{\mathrm{N}} \breve{}_{\mathrm{j}} \mathrm{x}_{\mathrm{j}}, 0 \leq \mathrm{y} \leq \sum_{\mathrm{j}=1}^{\mathrm{N}} \breve{}_{\mathrm{j}} \mathrm{y}_{\mathrm{j}}\} \tag{2.1}$$

Data envelopment analysis models are generally divided into two main parts: CCR and BCC, the basic CCR model proposed by Charnes and Cooper [20] and the BCC model in 1984 by Bunker et al. The basis of the CCR model and the addition of a new constraint were presented [22]. Data envelopment analysis models are divided into envelopment and multiplication categories in terms of modeling. The first model of data envelopment analysis is called multiplication. The basis of this model is the definition of efficiency as the ratio of one output to one input. For example, in the CCR model, instead of using the ratio of one output to one input, the ratio of the weighted sum of outputs (virtual output) to the weighted sum of inputs (virtual input) is used to calculate technical efficiency [20]. Equation (2) presents the CCR multiplication DEA model.

$$
\begin{array}{l}
M_o \ = Max \ \sum_{r=1}^{s} u_r.y_{r,o} \\
s.t. \quad \sum_{i=1}^{m} v_i.x_{i,o} = 1 \\
\sum_{r=1}^{s} u_r.y_{r,j} \ - \ \sum_{i=1}^{m} v_i.x_{i,j} \ \leq \ 0 \\
\qquad j = 1, ..., n \\
u_r \ , \ \ v_i \ \geq \varepsilon \qquad r = 1, ..., s, i = 1, ..., m
\end{array}
$$

(2.2)In Equation (

Equation (2,2) relates to the evaluation of n DMUs, where each DMU has m inputs and s outputs, which $y_{rj}$are outputs and x_{ij} are inputs of DMUs, in Figure 1 a schematic model of a DMU is presented.

Figure 1: Structure of DMU

By calculating the dual of multiplication model, the envelopment model is obtained. In Equation (2,3), the CCR envelopment model is provided.

$$
\begin{array}{l}
E_o \ = \ Min \ \theta \ - \sum_{r=1}^{s} \varepsilon.s_r^+ - \sum_{i=1}^{m} \varepsilon.s_i^- \\
s.t. \quad \sum_{j=1}^{n} \lambda_j.x_{ij} + s_i^- = \theta x_{i,o} \\
\qquad i = 1, ..., m \\
\sum_{j=1}^{n} \lambda_j.y_{rj} - s_r^+ = y_{r,o} \qquad r = 1, ..., s \\
\lambda_j \ , s_r^+, s_i^- \geq 0 \quad , \quad \theta \quad Free
\end{array}
$$

(2.3)

Equation (2,3) looks for a point in the production possibility set that consumes less input than the input and produces more output than the output of DMUo. This point is always a point on the efficiency frontier, comparing it with the unit under evaluation determines the efficiency. Obviously, DMUo is efficient if and only if Eo=1. After introducing DEA models, many books and papers were presented on DEA and the application of DEA in various organizations and subjects. Many papers on the applications of DEA in various industries have been presented by researchers. Most of researchers have included the development of computational methods for basic models or development internal structure of DMUs [23]. Regarding the development of base models [17, 24, 25, 26], and also in recent years researchers have focused on the development of models based on the internal structures of DMUs, for example [27, 28, 29, 30, 31]. As mentioned, researchers have focused on the structure of DMUs and developing methods for solving or analyzing results (such as performance prediction or benchmarking) and little attention has been paid to conditions of DMU parameters. For example, the dependency of parameters can have a high impact on the accuracy of benchmarking. By searching this topic in databases few papers can be found below some of them are mentioned. Siti Fatimah Mahmudah in 2017 were proposed a two-phase DEA model, which is in the first phase, the performance of the DMUs has been calculated and ultimately corrected by the effects of environmental parameters. They used this model to evaluate the efficiency of elementary schools in 2014-2015 [32]. NIU et al. in 2018, provided a data envelopment analysis model to evaluate the efficiency of wind turbine farms in China, they divided DMUs into two sub-processes, wind turbines, and power generation optimization. They first calculated the efficiency of each of two processes separately, and finally calculated the total performance process using the relationship between the dependence of these two sub-processes [33]. Najahi et al. In 2014, showed the impact of some parameters such as the organization's size on their performance, they used the data of the Tehran Stock Exchange from 2007 to 2011 to prove their theory and showed the effectiveness of the dependency with Company size [34]. Ji et al. In 2015, provided a model for evaluating the performance of DMUs that has interdependent parameters. They converted all dependent parameters using the Choquet integral into a parameter and used them in the final model [35]. Podinovski and Chameeva identify an additional theoretical problem that may arise if such bounds are used in a multiplier model with weight restrictions. Namely, they show that the use of small lower bounds may lead to the identification of an efficient target with negative inputs. they suggest a corrected model that overcomes this problem

[36]. Pidovski proves that, for any weight restrictions, the optimal weights of the multiplier model show DMUo in the best light in comparison to the entire technology expanded by the weight restrictions. This result is consistent with the fact that the dual envelopment DEA model benchmarks DMUo against all DMUs in the technology, and not only against the observed DMUs. his development overcomes previous concerns about the use of weight restrictions of certain types in DEA models and provides their rigorous and meaningful interpretation [37]. Gner examines the sustainable efficiencies of busiest European airports considering their success in reducing emissions during landing and take-off cycles. He has constructed a weight-restricted Data Envelopment Analysis model, and both physical and sustainable efficiencies of each airport were examined during nine years between 2010 and 2018. He used a combinative Analytic Hierarchy Process - Criteria Importance Through Intercriteria Correlation approach to defining criteria weights, and a new modified approach was proposed and used to include these weights in Data Envelopment Analysis. His findings proved the necessity of weight restriction when one would examine the impact of emissions produced on sustainable airport efficiency [38]. Medeiros and et. provided a paper intending to evaluate weights restrictions influence on efficiencies results and to perform a sensitivity analysis of efficiency scores using additional benchmarking techniques. they apply the Cross-Efficiency Analysis and the Ratio-based Efficiency Analysis benchmarking methods, to provide relevant quantitative information to compute relative efficiency scores and perform peer evaluations among utilities even if they are outside of the efficient frontier. The Brazilian electricity distribution system is selected as their study case. Results from their analysis show that the diversity of concession areas significantly influences the stability of efficiency scores [39]. As described, the subject of dependent parameters in the DEA literature are less investigated by researchers, and most articles have examined the external dependent parameters and their effect on performance, with our search, only one paper [35]

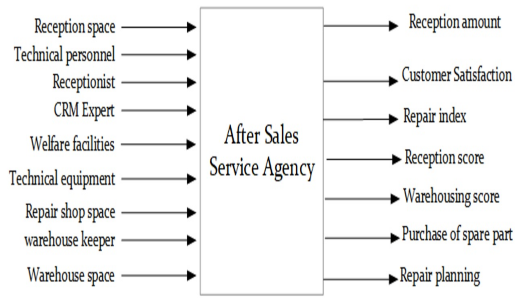

Figure 2: Structure of after-sales service agenciesU

was found that they also converted all parameters into a parameter and solved the model, according to the purpose of this paper, which finds a benchmark for DMUs, It cannot be used this method, so this paper has been attempted to cover this gap in the subject literature.

3 Modeling

3.1 Introducing case study

In this paper, we consider the car after-sales service agencies as DMUs. 20 agents were selected from a car provider after-sales service agency. Iran Standard and Quality Inspection Company (ISQI), based on the "Instruction of terms, conditions, and evaluation of after-sales service in the automotive industry", evaluates the after-sales service agencies of all automotive companies. In this instruction, the processes and performance functions are evaluated and rated. In this paper, the input and output parameters of the DMUs are obtained from the evaluation data of 2019.

3.2 The structure of DMUs

Figure 2 shows the structure, input and output parameters of the case study DMUs. It can be seen that after-sales service agencies have 9 input parameters and 7 output parameters. Definition of each parameter and how to calculate them are provided in follow.

(i) Reception space: Score on a scale of 100 for the status of the reception area in accordance with the standard.

(ii) Technical personnel: Score on a scale of 100 for the conditions of technical personnel (skills, experience and knowledge) in accordance with the standard.

(iii) Receptionist: Score on a scale of 100 for reception expert requirements (skills, experience and knowledge) in accordance with the standard.

(iv) CRM Expert: Score on a scale of 100 for CRM expert requirements (skills, experience and knowledge) in accordance with the standard.

(v) Welfare facilities: Score on a scale of 100 for the status of customer amenities in accordance with the standard.

(vi) Technical equipment: Score on a scale of 100 for the status of technical equipment in accordance with the standard.

(vii) Repair shop space: Score on a scale of 100 for the condition of the repair shop space in accordance with the standard.

(viii) warehouse keeper: Score on a scale of 100 for warehouse keeper requirements (skills, experience and knowledge) in accordance with the standard.

(ix) Warehouse space: Score on a scale of 100 for the condition of the warehouse space in accordance with the standard.

(x) Repair index: Score on a scale of 1000 for fixing all vehicle defects in a sutible time.

(xi) Reception score: Score on a scale of 100 regarding the status of customer reception conditions.

(xii) Warehousing score: Score 100 on the status of ordering and warehousing.

(xiii) Purchase of spare part: Amount of purchasing spare parts from the company in Rials.

(xii) Repair planning: Score on a scale of 100 for the status of repair planning.

(xv) Customer Satisfaction: Customer satisfaction score on a scale of 1000.

(xvi) Reception amount: Number of customers.

3.3 Identify the dependent parameters and the relationships of parameters

It should be noted that in the multiplication model (Equation (2,2)), some of the parameters have higher importance than other parameters and have higher weight representation for assessing performance, for example, customer satisfaction is more important than the warehousing score, this is not representing the triviality of warehousing in the evaluation of performance, but reflects the very high importance of customer satisfaction parameter. Therefore, these constraints are also added to the multiplication model. Considering that these weights are also effective for evaluating efficiency, therefore, it should be added to the primary and multiplication model, by adding these constraints Equation (3.4) is obtained.

$$
\begin{array}{l}
M'_o \ = Max \ \sum_{r=1}^{s} u_r.y_{r,o} \\
s.t. \quad \sum_{i=1}^{m} v_i.x_{i,o} = 1 \\
\sum_{r=1}^{s} u_r.y_{r,j} \ - \ \sum_{i=1}^{m} v_i.x_{i,j} \ \leq \ 0 \\
\qquad j = 1, ..., n \\
u_a \geq k u_b \qquad \forall (a,b) \in s \\
v_a \geq k' v_b \qquad \forall (a,b) \in m \\
u_r \ , \ v_i \ \geq \varepsilon
\end{array}
\tag{3.4}
$$

Because other conditions of parameters are important in determining the benchmark, at first, the dual form of Equation (3.4) should be calculated and then the constraints of other conditions of parameters should be added. In Equation (3.5) the dual form of Equation (3.4) is presented. In Equation (3.4) CX and CY are related to weight restrictions coefficients.

$$
\begin{aligned}
&E'_o = Min\ \theta - \textstyle\sum_i \varepsilon.S_i^- - \sum_j \varepsilon.S_j^+ \\
&s.t.\ \textstyle\sum_{j=1}^n \lambda_j.x_{i,j} + \sum_{ci} \mu i_{ci} CX_{ci} + S_i^- = \theta x_{i,o} \\
&\quad i = 1, ..., m \\
&\textstyle\sum_{j=1}^n \lambda_j.y_{r,j} - \sum_{cr} \mu j_{cr} CY_{cr} - S_j^+ = y_{r,o} \\
&\quad r = 1, ..., s \\
&\lambda_j\ , \mu i_{ci}, \mu j_{cr}, S_i^-, S_j^+ \geq 0
\end{aligned}
$$

(3.5)

Due to the nature of the automobile after-sale work, as well as the evaluation model, "Repairs Index" and "customer satisfaction" that both measured by a questionnaire notified in the "Instructions for measuring consumer satisfaction in the automotive industry". The initial assumption of experts is that these two parameters have a significant correlation coefficient, of course, this hypothesis is proved, by examining and testing the Spearman correlation coefficient. According to the non-normal distribution of parameters, the nonparametric Spearman test was used. SPSS Statistics 24 software was used to perform the correlation coefficient test. The correlation coefficient value was 0.761 and the P-Value value was 0.001, which shows that the correlation coefficient test shows the relationship between these two parameters with a high value of 0.761 at the level. 99.99% is significant, so this should be applied to the model. Assume that the two output parameters (for example, variables a and b) are interdependent, the two parameters have a relationship such as Equation (3.6). In this model, it is assumed that the relationship is linear regression. For other functions, a similar relationship can be written. It should be noted that the direction of the equation can change based on the type of parameters.

$$y_{a,j} \geq \beta y_{b,j} + \alpha \tag{3.6}$$

On the other hand, some parameters have special conditions that according to the benchmarking, these conditions should also be considered, for example, according to the conditions (personnel training, equipment, vehicle conditions, etc.), according to experts, in repair index, an increase of more than 10% in this index cannot be expected from the agency. Also, repair shop space must be constant. In evaluating the efficiency, the amount of space used by the dealership should be considered as input, but during benchmarking, the dealer cannot be given a benchmark that increases or decreases the repair shop space. Adding these conditions and constraints model changes to Equation (3.7).

$$
\begin{aligned}
&E'_o = Min\ \theta - \textstyle\sum_i S_i^- - \sum_j S_j^+ \\
&s.t. \textstyle\sum_{j=1}^n \lambda_j.x_{ij} + \sum_{ci} \mu i_{ci} CX_{ci} + S_i^- = \theta x_{i,o} \\
&\quad i = 1, ..., m \\
&\textstyle\sum_{j=1}^n \lambda_j.y_{rj} - \sum_{cr} \mu j_{cr} CY_{cr} - S_j^+ = y_{r,o} \\
&r = 1, ..., s \\
&\textstyle\sum_{j=1}^n \lambda_j.y_{aj} \geq \sum_{j=1}^n \lambda_j.(\beta y_{bj}) + \alpha \\
&(a, b)\ \ Dependent\ \text{Variables} \\
&\textstyle\sum_{j=1}^n \lambda_j.y_{1j}\ \ \leq y_{1o}\ \text{Repair index} \\
&\textstyle\sum_{j=1}^n \lambda_j.x_{6j}\ \ = x_{6o}\ \text{Repair shop space} \\
&\lambda_j\ , \mu i_{ci}, \mu j_{cr}, S_i^-, S_j^+ \geq 0 j = 1, ..., n \\
&ci \in Indices\ for\ weight\ restrictions\ of \\
&input\ parameters \\
&cr \in Indices\ for\ weight\ restrictions\ of \\
&ouput\ parameters
\end{aligned}
$$

(3.7)

Theorem 3.1. *If* $\forall r\ y_{a,o} \geq \beta.y_{b,o} + \alpha$ *then model number 6 is always possible.*

$$
\begin{aligned}
&\mu i_{ci} = 0\ ,\ \mu j_{cr} = 0\ ,\ \lambda_j = e_o\ ,\ \theta = 1\ , \\
&s_i^- = 0\ ,\ s_j^+ = 0 \\
&x_{io} = x_{io} \qquad \forall i \\
&y_{ro} = y_{ro} \qquad \forall r \\
&\forall r \qquad y_{a,o} \geq \beta.y_{b,o} + \alpha \\
&y_{1o} \leq y_{1o} \\
&x_{6o} = x_{6o}
\end{aligned}
$$

Based on our case study data, $\forall r\ y_{a,o} \geq \beta.y_{b,o} + \alpha$ is always established

4 Numerical results

In this paper, data of after-sales service agencies of a car company in 2019 have been used. In Table 1, descriptive statistics of parameters are presented. In the first stage, to determine the efficiency results of each decision-making unit, the multiplication model (Equation (3.4)) is implemented and solved. Gams 23.4 software has been

Table 1: Descriptive statistics of model parameters

parameter	Minimum	Maximum	Mean	Std. Deviation
Reception space	50	100	84	20
Technical personnel	50	100	87	11
Receptionist	58	100	86	15
CRM Expert	33	100	81	19
Welfare facilities	50	100	87	15
Technical equipment	65	100	83	9
Repair shop space	75	100	93	10
warehouse keeper	50	100	87	17
Warehouse space	50	100	84	16
Repair index	608	848	723	62
Reception score	47	84	69	9
Warehousing score	41	93	67	15
Purchase of spare part	1744780190	46593650570	9857054188	10214055180
Repair planning	19	81	47	16
Customer Satisfaction	534	735	671	47
Reception amount	103	2112	540	520

Table 2: Results of efficiency of DMUs

DMU	Performance in Equation (2,2)	Status in Equation (2,2)	Performance in Equation (3.4)	Status in Equation (3.4)
DMU1	0.76	Inefficient	0.64	Inefficient
DMU2	1	Efficient	0.8	Inefficient
DMU3	0.72	Inefficient	0.68	Inefficient
DMU4	1	Efficient	0.72	Inefficient
DMU5	1	Efficient	0.74	Inefficient
DMU6	1	Efficient	1	Efficient
DMU7	1	Efficient	0.89	Inefficient
DMU8	1	Efficient	0.9	Inefficient
DMU9	1	Efficient	0.69	Inefficient
DMU10	0.82	Inefficient	0.82	Inefficient
DMU11	1	Efficient	0.73	Inefficient
DMU12	1	Efficient	0.71	Inefficient
DMU13	0.93	Inefficient	0.73	Inefficient
DMU14	0.8	Inefficient	0.71	Inefficient
DMU15	1	Efficient	0.66	Inefficient
DMU16	1	Efficient	0.68	Inefficient
DMU17	0.89	Inefficient	0.76	Inefficient
DMU18	0.92	Inefficient	0.63	Inefficient
DMU19	0.79	Inefficient	0.69	Inefficient
DMU20	1	Efficient	0.88	Inefficient

Table 3: Curve estimation results

Model	R Square	Constant	b1	b2	b3
Linear	0.658	0.158	0.856		
Logarithmic	0.592	0.945	0.53		
Inverse	0.575	1.22	-0.325		
Quadratic	0.66	1.967	-4.928	4.574	
Cubic	0.661	0.937	0	-3.238	4.103
Compound	0.626	0.326	3.323		
Power	0.612	0.984	0.745		
S	0.596	0.372	-0.458		
Growth	0.626	-1.121	1.201		
Exponential	0.626	0.326	1.201		

Table 4: Results of final model

DMU	θ	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10	DMU11	DMU12	DMU13	DMU14	DMU15	DMU16	DMU17	DMU18	DMU19	DMU20
DMU1	0.865	0	0	0	0	0	0.327	0	0	0	0	0	0	0	0	0.118	0	0.272	0	0	0.366
DMU2	0.811	0	0	0	0	0	0.956	0	0	0	0	0	0	0	0	0	0	0.036	0	0	0
DMU3	0.972	0	0	0	0	0	0.162	0	0	0	0	0	0	0	0	0.229	0	0	0	0	0.617
DMU4	0.925	0	0	0	0	0.243	0.389	0	0	0	0	0	0	0	0	0	0.022	0	0	0	0.438
DMU5	0.967	0	0	0	0	0.47	0.167	0	0	0	0	0	0	0	0	0	0	0.096	0	0	0.296
DMU6	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DMU7	0.898	0	0	0	0	0	0.926	0	0	0	0.056	0	0	0	0	0	0	0	0	0	0
DMU8	0.91	0	0	0	0	0	0.976	0	0	0	0	0	0	0	0	0	0	0	0	0	0.018
DMU9	0.915	0	0	0	0	0	0.396	0	0	0	0	0.128	0	0	0	0	0	0	0	0	0.549
DMU10	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
DMU11	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
DMU12	0.922	0	0	0	0	0	0.332	0	0	0	0	0.463	0	0	0	0	0	0	0	0	0.202
DMU13	0.962	0	0	0	0	0	0.162	0	0	0	0.717	0	0	0	0	0	0	0.004	0	0	0.144
DMU14	0.948	0	0	0	0	0	0.345	0	0	0	0	0	0	0	0	0	0	0	0	0	0.741
DMU15	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
DMU16	0.982	0	0	0	0	0	0.201	0	0	0	0	0	0	0	0	0.64	0	0	0	0	0.119
DMU17	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
DMU18	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
DMU19	0.913	0	0	0	0	0	0.437	0	0	0	0.38	0	0	0	0	0	0	0.163	0	0	0.134
DMU20	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

used for coding and solving linear programming. Table 2 shows the performance of each DMU. To determine the effect of the weight constraint, Table Table 2 presents the results of both Equation (2,2) and Equation (3.4).

The results show that in Equation (2,2) (basic model of data envelopment analysis), 12 DMUs are efficient, but when weight constraints (importance of parameters in evaluation by experts) are added and Equation (3.4) is implemented, only DMU6 is efficient. Considering the results of Equation (3.4), we go to the next step and benchmarking should be done for DMUs, for this, Equation (3.7) should be implemented. First, the situation of the two dependent parameters and the relationship function between them must be determined. By more evaluation of data, the results show that the parameters of customer satisfaction and repair index have a correlation coefficient equal to 0.761, so to apply this constraint in the final model (to increase the accuracy of benchmarking) the relationship between these two parameters must be specified. It should be noted that this shows that an agent cannot have a low score in the repair index and we expect to be able to achieve a high score of customer satisfaction, one of the purposes of this article is that this condition in defining the benchmark for agencies to be considered.

It can be logically concluded that customer satisfaction is the result of the repair index, therefore, the repair index is considered as an independent parameter and customer satisfaction is considered as a dependent parameter. By performing the curve estimation analysis, the results are obtained in Table 3.
According to R^2, it can be seen that linear regression provides a good estimate of the data, so we consider linear regression as a function between two parameters.

- Customer Satisfaction = 0.856 × Repair index + 0.158

Since the ultimate goal is customer satisfaction (and its highness is desired), so to add to the model, we modify the above equation as follows, it should be noted that the direction of inequality is defined according to the conditions and type of business.

- Customer Satisfaction ≤ 0.856 × Repair index + 0.158

Finally, the last constraint of the model is modified as Equation (4.7).

$$\sum_{j=1}^{n} \breve{\ }_{j}.y_{6,j} \geq \sum_{j=1}^{n} \breve{\ }_{j}.(0.856.y_{1,j}) + 0.158$$

(4.8)

After completing the model (Equation (3.7)), by solving the model, the results of are obtained as Table 4.

In results of the multiplication model (Table 2), only the efficient unit is DMU6, and in the corresponding envelopment model of this DMU, all other units must benchmark from this DMU but some conditions of this agent did not apply to other agents, for example, the customer satisfaction score of this agent was 749, and an inefficient agent with a customer satisfaction score of 590 (DMU1) could not benchmark this score without considering its repair index. Considering its progress as can be seen in the final model (Table 4), the DMU1 has benchmarked on other DMUs. The constraints of the parameters have also been influential, for example, DMU18 had a score of 707 in repair index, whereas if it were to be benchmarked on DMU6, this score would have reached 848, but given the nature of the work, it is clear that in a period of planning time cannot expect this score to be greater than 778, it should be noted that it is possible to reach a higher number (with a very low probability) but due to the normal conditions of goal setting and benchmarking with this number is illogical.

5 Conclusion

TBenchmarking is used as a method to improve the quality of products, services, strategies, etc. in an organization or company. Benchmarking is a common tool that is widely considered as one of the ways to improve the competitiveness and efficiency of organizations in their working life, also this method is one of the most effective methods of continuous improvement in management that managers for the growth of organizations, they pay special attention to this method. In this paper, we tried to improve the accuracy of the benchmarking by considering some assumptions *whichexistintherealworld.* It should be noted that if the appropriate benchmark is not selected for decision-making units, the path of progress of the organization will be diverted and will not achieve the desired results. In this paper, a benchmarking model for after-sales service agencies based on data envelopment analysis was developed. In the structure of after-sales service agencies, there are different input and output parameters, but *basedontheproposedstructure* repair index has a direct relation with customer satisfaction, the purpose of this paper was to benchmarking that the model does not use a virtual DMU as a benchmark for other DMUs that the logical and statistical relationship between these two parameters is not observed, the benchmark cannot be defined in any way we did not expect customer satisfaction to increase without a reasonable increase in the repair index. As mentioned in the previous section, the dependence and relationship between the parameters should be considered in benchmarking. On the other hand, it should be noted that by applying these constraints, more DMUs became efficient, because the model showed that some constraints and parameters are being out of control of agencies and that should be considered in evaluating their efficiency. This paper attempts to develop a benchmarking model for after-sales service agents with acceptable accuracy, although it should be noted that some improvements in this paper can be used as future studies. Here are a few:

- It is very important to pay attention to the structure of decision-making units. In this paper, the internal structure of DMUs is considered as a black box, while the model can be developed for network or other structures

- In this paper, the dependence of one-sided parameters of DMUs *inputoroutput* is considered, the dependencies can be combined, that is, the output and input parameters are interdependent.

- Another subject that has been considered by researchers in recent years in the field of data envelopment analysis is the combination of machine learning methods with DEA models. Can be predicted benchmarking by developing the proposed model with machine learning methods.

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