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Research Article

Single Assignment Capacitated Hierarchical Hub Set Covering Problem for Service Delivery Systems Over Multilevel Networks

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Abstract

Hierarchical hub location problems locate hub facilities on the first and second of three levels under the best economic conditions. The present study introduced a novel hierarchical hub set covering problem with capacity constraints. This study showed the significance of fixed charge costs for locating facilities, assigning hub links and designing a productivity network. Set covering was imposed to use the minimum number of variables and coverage constraints. The proposed model employs mixed integer programming to locate facilities and establish links between nodes according to the travel time between an origin-destination pair within a given time bound. The demand nodes are fairly covered at each service level and no demand node remains unanswered. The formulation was linearized by replacing the non-linear constraints with less intuitive linear structures. The resulting linear model was solved using GAMS and tested on the Australian Post dataset and the Iranian Airport dataset.

Keywords : Small Hub location; Hub set covering; Hierarchical; Capacity constraint.

1 Introduction

ub location problems find the locations of hub facilities to assign demand nodes to them and to obtain a minimum total cost for the established network. There are two assignment styles for links over the hub networks. Single assignment allocates any demand node to exactly one hub; multi-assignment connects any non-hub node to more than one hub. The single assignment hierarchical hub location problem operates on three levels to locate hub facilities at any level of the hierarchy and allocate demand nodes to exactly one hub and one central hub. Travel time between hubs and central hubs and between two central hubs is discounted to profit from economies of scale. In classical hub networks, hub facilities in the top level are usually connected to the complete network and links in the second and third levels form a star network. These problems were first developed by Hakimi [1] to find the optimal location of a facility in a communication network to minimize total routing cost. He proved that the best locations of facilities in a network are at the p-medians of the corresponding weighted graph [2]. Farahani et al. [3] focused on reviewing the most recent advances in hub location problems (HLP). They reviewed all variants of a HLP (network,

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continuous, discrete) and discussed mathematical models, solution methods, main specifications, and applications of HLPs and introduced case studies that illustrate real-world applications of HLPs. The present study builds upon their work and divides the literature on HLPs into four main sections.

1.1 Hierarchical hub location problem

Elmastas [4] presented a 3-level hierarchical hub location problem and tested the proposed model in a cargo company in Turkey. Yaman [5] altered the style developed by Elmastas. He first added the top level network [4] to linking hub facilities with a star network. In the second level, he connected the hubs to each other. The network structure for the complete network in Yaman [5] has the same hub connection as in the first level. Next, where Yaman [5] minimized the total routing cost in a time constraint for the objective function, Elmastas [4] minimized the total fixed charge cost. Alumur et al. [6] proposed a mixed-integer programming formulation and carried out comprehensive sensitivity analysis on the Turkish network. They showed that the locations of airport hubs are less sensitive to cost parameters than the locations of ground hubs and it was possible to improve service quality at no additional cost in multimodal networks. Dukkanci and Kara [7] proposed a hierarchical multimodal hub network structure with characteristics of a hub covering problem and a service time bound, So that the hierarchical network consists of three layers as a ring-star-star network. They developed a heuristic solution algorithm based on the subgradient approach to solve the problem. Torkestani et al.[8] presented a novel mixed-integer mathematical programming formulation for a hierarchical multi-modes transportation hub location problem. The main issue was how to tackle the hub nodes and edges disruption in a dynamic system.

1.2 Hub covering problem

Hub covering location problems developed by Campbell [9] locate hub facilities and allocate demand nodes to the nearest hub facilities. He presented two models for hub location problems (p-hub center and hub covering) with different objective functions. There are two types of hub covering problem. In one, the hub set covering minimizes the fixed charge cost of covering hubs or uses the least number of hubs and all demand nodes. In the other, the hub covering maximizes the number of covered non-hubs for a predetermined number of hubs. Campbell [9] introduced the first mixed- integer formulation for both problems. Kara and Tansel [10] studied a single assignment hub set covering problem and showed that it is NP-hard. Wagner [11] proposed new formulations for hub covering problems with single and multiple assignments. Ernst et al. [12] presented new formulations for single and multiple assignment hub set covering problems by using as the covering radius. These formulations were an improvement over Kara and Tansel [10] because they require less CPU processing time. Tan and Kara [13] focused on cargo delivery systems. They used their model formulation for different cargo delivery companies in Turkey. Calik et al. [14] studied a single assignment hub set-covering problem over incomplete hub networks and presented an integer programming formulation with a heuristic solution based on tabu search. Peker and Kara [15] extended the definition of coverage, introducing partial coverage, which changes with distance. They presented a new and efficient mixed-integer programming models that are also valid for partial coverage for single and multiple allocations. Ebrahimi-Zade et al.[16] proposed a mixed integer model for a multi-period single-allocation hub set covering problem in which the covering radius is a decision variable and validated through a real world case study. They extended a genetic algorithm (GA) for solving that. Silva and Cunha [17] presented a tabu search heuristic algorithm for the uncapacitated single allocation p-hub maximal covering problem, and reported the optimal solutions for larger instances of benchmark problems. Karimi [18] introduced a new hub location-routing problem. So that, his model minimizes the total cost of hub location and vehicle routing, subject to predefined travel time, hub capacity, vehicle capacity, and simultaneous pickups and deliveries.

1.3 Uncapacitated and capacitated hub location problems

OKelly [19] developed the first uncapacitated single allocation p-hub median problem (USAPHMP). Skorin-Kapov et al. [20] suggested a mixed integer programming formulation for the OKelly [19] problem. Sohn and Park [21] and Ebery [22] improved and linearized the USAPHMP. A hubs capacity to provide services is important to decision-making and designing the network in hub location problems because a target for these problems is to respond to requests by demand nodes. Yaman [23] installed a two-level telecommunication network and studied the uncapacitated hub location problem with modular arc capacities. They minimized the cost of installing hubs and capacity units on arcs. Yaman and Carello [23] considered the amount of traffic passing through a hub to be that hubs capacity; this was an extension of Yaman [23]. Yaman [24] identified hub locations and hub links to decrease costs in a hub median problem with regard to capacity of arcs. [24] also developed two formulations and an innovative algorithm to solve the problem and compare the quality of the solutions. Correia et al. [25] revised and modified a well-known formulation for a capacitated single allocation hub location problem. They showed that the previous formulation for balance limitations (constraints) has disadvantages. Stanojevi et al. [26] proposed a hybrid optimization method, consisting of an evolutionary algorithm and a branch-and-bound method for solving the capacitated single allocation hub location problem. Merakl and Yaman [27] considered a capacitated multiple allocation hub location problem with hose demand uncertainty in which the routing cost is a function of demand and capacity constraints are imposed on hubs, demand uncertainty has an impact on both the total cost and the feasibility of the solutions. They developed two different Benders decomposition algorithms for their problem. Correia et al. [25] focused on the development of a modeling framework for multi-period stochastic capacitated multiple allocation hub location problems and considered a planning horizon divided into several time periods.

1.4 Applications

Hub location problems have used hierarchical networks for freight transportation [29, 30, 31, 5, 32], public transportation [33, 34, 35, 36, 37], air transportation [38, 13, 39], maritime transportation [40, 41, 42, 43] and telecommunication systems. This paper presents a single allocation ca-

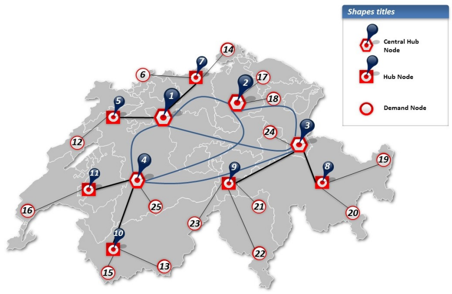

Figure 1: A three-level network on 25 nodes with 10 hubs and 4 central hubs.

pacitated hierarchical hub set location covering model with delivery time constraints (SA-DT-CHSC.) The model allows traffic demand from an origin to a destination to meet up to four hubs on its route. The non-hub links to exactly one hub and the hub nodes to a central hub. The central hubs are linked over a complete network, creating a direct link from any top-level facility to any other facility. Figure 1 shows the transportation network with two levels of hub facilities to illustrate the problem having the best resolution. The network has 25 nodes. The non-hubs are denoted by circles, hubs by squares and central hubs by hexagons. The covering radii of the hubs are denoted by incomplete circles. There are different routes and traffic paths shown. One visits 4 hubs and shows traffic demand from node 12 to node 21; its flow path is 12 - 5 - 1 - 3 - 9 - 21. Another path encounters 2 hubs crossing from node 17 to node 25 with a flow path of 17 - 2 - 4 - 25.
Hierarchical hub location problems are used in distributed systems, transportation, waste management, health services, emergency services and telecommunications. Creating a pattern and comprehensive plan will facilitate accurate and logical decision-making that is closer to reality. The complexity and challenging nature of management decisions in this field will emerge to decrease direct and indirect costs to the service

provider systems.

This paper focuses on the hierarchical structure and set covering problem at the same time. In addition, we are approaching the actual situation by imposing capacity constraints to the problem formulation. This formulation provides a logical approach that gives priority to costs and will prevent indiscriminate increases in priority costs. These costs are divided into routing and fixed charges. The shortest path between an origin and destination must be prioritized so that companies can decrease investment expenditure while providing services to meet demand. Fixed charge costs affect long-term decision-making by executive management. This study focuses particular attention on different situations and output network structures. The output of the proposed model will illustrate that a hierarchical structure is sometimes necessary and sometimes inappropriate.

Previous studies have focused on general aspects of the given time bounds, but have not separately considered travel time in the second and third levels. The travel time plays a significant role at these levels because some traffic demand requires few second-level services and can use these services in a short time. Traditionally, studies have not considered that travel time constraints may be higher on low level routes. Covering problems are related to travel time at all levels; thus, a combined model of hierarchical hub covering problem is proposed to resolve this problem by defining covering constraints for each level of service. The model does not allow the service time at each level to exceed the specified upper bound.

Hub nodes in all levels tend to gather together to minimize routing costs. In many studies, the models were designed with a routing cost objective that ends up increasing the number of facilities to decrease costs. Today, at least one communication path exists between demand nodes. Companies and transportation organizations must choose the best places to set up facilities. It is important to minimize the total fixed charge cost for organizations to allow them to devote minimum investment to these activities. OKelly [44] first introduced the fixed charge cost as a hub node setup cost for one level of hub facilities. Limited budgets to establish facilities impel researchers to find an optimal number of facilities to meet the predetermined fixed charge costs. The proposed study extends the work done by OKelly to a 3-level network.

As seen in literature, there is a very large variety of applications for hierarchical hub location problems, which is at the strategic decision level of organizations. In the meantime, for the development of the service system, covering problems are of particular importance. Despite, the many studies have been done on covering problems; the many research gaps remain to achieve these issues to the fullest extent. One of these cases occurs when a demand unit receives its service at a first level, then it go to the next second level, but the demand unit is faced with a lack of capacity. This leads to traffic congestion or outages of the service. This situation in telecommunications networks is very common. Therefore, the problem formulation must consider simultaneously the aspects of covering with the capacity of the facilities. Also, the service delivery in the covering problems is significant in the time frame, and it should be noted that failing to comply with this issue will have two dilemmas, first the demand centers will be discontented, second, the traffic increases and the queue is created. In transportation system, regardless of its type, when a vehicle moves from an origin toward a destination, after arrival in the destination and completing their service, the destination is redefined as origin. Therefore, the vehicle serves the destination demand now. So, the delay in the service system will be increased lead times for demand centers. Hence, that is very particular important, we have paid attention on the service delivery time, hierarchical structure and facilities capacity in the form of covering problems, simultaneously.

The proposed model designed new capacitated constraints for hierarchical networks so that all traffic demands are met on each level of service. All demands are met in the least time and best location in this capacitated approach. The hierarchical hub set covering problem of the present research locates the hubs and central hubs, the demand nodes with the nearest hub nodes, and connects a hub to a central hub with a unique covering situation so that no demand node remains unanswered. The new hub location problem is

defined based on the time bound at each level of hierarchy and considers fixed charge costs of establishing the hubs, central hubs, and links between them.
The structure of this paper is organized as follows. Section 2 suggests a new formulation for the hierarchical hub location problem. Section 3 explains the computational analysis using the Australia Post (AP) and Iranian Airport Data (IAD) data sets. It will be shown that some results from the proposed model do not coincide with similar results available in the literature. Section 4 concludes the paper and recommends future research.

2 Mathematical formulation

This section introduces the notations, parameters and decision variables for the proposed model and present a mathematical formulation for SA-DT-CHSC problems. It is assumed that I is a set of nodes, $H \subseteq I$ is a potential hub set and $C \subseteq H$ is a set of possible locations for central hubs. In hub location problems, discount factor α is the decrease in establishment link costs (or travel time) between hubs $\alpha \in [0, 1]$ [45]. Yaman [5] denoted the reduction coefficients of travel time between hubs and central hubs as α_H and between central hubs as α_C. It was assumed that $\alpha_H \geq \alpha_C$ [5]. The upper bound of the delivery time is denoted by β, which is the maximum time required for any flow demand to arrive at its destination.

2.1 Model parameters

The decision variables and other parameters of the model are as follows:

fh_j = fixed charge cost of opening hub in node $j \in H$.
fc_l = fixed charge cost of opening central hub in node $l \in C$.
f^*_{ij}= fixed charge cost of opening hub links between hubs $i \in H$ and $j \in H$.
f^{**}_{kl}= fixed charge costs of opening hub links between central hubs $k \in C$ and $l \in C$.
t_{ij}= travel time from node $i \in I$ to node ,$j \in I$ where t_{ij}=t_{ji} and $t_{ii} = 0$.
f_{ij}= flow demand from node $i \in I$ to node ,$j \in I$ where f_{ij}=f_{ji} and $f_{ii} = 0$.
C_{ij}= routing cost of each unit of flow from node $i \in I$ to node ,$j \in I$ where C_{ij}=C_{ji} and $C_{ii} = 0$.
Γh_i= potential hub capacity of node $i \in I$.
Γc_i= potential central hub capacity of node $i \in I$.
S_i= time required for all traffic originating at node $i \in I$ to prepare for travel; after this time in the telecommunications context, let $S_i = 0$ for all $i \in I$ [5, 46].

2.2 Decision variables

The decision variables for models are as follows:
$X_{ijl} = 1$ if node i is assigned to a hub at node j and a central hub at node l; 0 otherwise.
$g_{kl} = 1$ if node k and node l have central hub so that $k \neq l$; 0 otherwise.
r^*_j= covering radius of hub j.
r^{**}_l= covering radius of central hub l.
D_l = time in which all demand flows arrive at their destination central hub l.
v^i_{kl}= amount of trade demand passing central hub $k \in C$ to central hub $l \in C; l \neq k$, where is the origin.
If the $X_{jjl} = 1$ for some $j \in H$ where node j is assigned to a central hub at node l, it means that node j is a hub node; if $X_{lll} = 1$ for some $l \in C$, it means that node l is a central hub node.

2.3 SA-DT-CHSC model

This study suggests a nonlinear mathematical model for SA-DT-CHSC. Central hub links and central hub capacity constraints in this formulation cause the model to be nonlinear. The objective function and constraints of the SA-DT-CHSC nonlinear problem (SA-DT-CHSC-NLP) are:
Objective function:
The Objective function will be obtained as follows:

$$Min = \sum_{j\in H}\sum_{\substack{l\in C\\ l\neq j}} fh_j X_{jjl} + \sum_{l\in C} fc_l X_{lll} + \sum_{j\in H}\sum_{\substack{l\in C\\ l\neq j}} f^*_{jl} X_{jjl} + \sum_{k\in C}\sum_{\substack{l\in C\\ l>k}} f^{**}_{kl} g_{kl}. \tag{2.1}$$

SA-DT-CHSC-NLP has designed the objective function as the the fixed charge costs of facilities and their links. This objective function is to minimize total establishment costs, including fixed charge costs of hubs, central hubs, hub links and central hub links using formulation (2.1).

Single assignment hierarchical hub constraints:

$$\sum_{j\in H}\sum_{l\in C} X_{ijl} = 1, \qquad \forall i \in I. \tag{2.2}$$

$$X_{ijl} \le X_{jjl}, \quad \forall i \in I, j \in H : j \ne i, l \in C. \tag{2.3}$$

$$\sum_{m\in H} X_{jml} \le X_{lll}, \quad \forall j \in H, l \in C : j \ne l. \tag{2.4}$$

$$X_{ljl} = 0, \qquad \forall j \in H, l \in C : j \ne l. \tag{2.5}$$

$$X_{ijl} = \{0,1\}, \forall i \in I, j \in H, l \in C. \tag{2.6}$$

The presented model corresponds to a single assignment. Each non-hub node should be allocated to exactly one hub facility and and each hub to one central hub facility. Constraints (2.2), (2.4) and (2.5) guarantee the single assignment for this model. Constraint (2.4) ensures that no hub can be linked to another nodes unless that node is a central hub node. Constraint (2.3) states that if a demand node is assigned to another node, then that node should be a hub node. Constraint (2.5) increases LP relaxation.

Top level hub-links constraints:

$$g_{kl} \le X_{lll}, \qquad \forall k \in C, l \in C. \tag{2.7}$$

$$g_{kl} \le X_{kkk}, \qquad \forall k \in C, l \in C. \tag{2.8}$$

$$X_{kkk}X_{lll} \le g_{kl}, \qquad \forall k \in C, l \in C : k \ne l. \tag{2.9}$$

$$g_{kl} \in \{0,1\}, \qquad \forall k \in C, l \in C. \tag{2.10}$$

Constraints (2.7), (2.8) and (2.10) ensure that central hub links are only established between two different central hubs. If two central hubs are open to the nodes of $k \in C$ and $l \in C$ and both X_{kkk} and X_{lll} are equal to 1, then direct connections exist between these two central hubs according to constraint (2.9).

Flow balance constraints:

$$\sum_{\substack{k\in C\\ l\ne k}} v^i_{lk} - \sum_{\substack{k\in C\\ l\ne k}} v^i_{kl} = \sum_{r\in I} f_{ir} \sum_{j\in H}(X_{ijl} - X_{rjl}), \forall i \in I, l \in C. \tag{2.11}$$

$$v^i_{kl} \ge 0, \forall i \in I, k \in C, l \in C : k \ne l. \tag{2.12}$$

If X_{ijl} =1, there is a direct link between node i and central hub l; hence, traffic from demand node i to other nodes will pass from central hub l. This flow is balanced by constraints (2.11) and (2.12).

Capacity constraints:

Attribute capacity has been added to the model by constraints (2.14) and (2.15). Γh_j is the potential hub capacity of node j and Γc_l is the potential central hub capacity of node l . Constraints (2.14) and (2.15) ensure the summation of all flow passing through each hub j and each central hub l does not equal more than Γh_j or Γc_l . Constraint (2.13) (cut constraint) is the upper bound for the amount of traffic between central hubs, because the traffic demand that passes from each central hub should not exceed its capacity and that of all demands by the origin node. It is logical that the amount of traffic between two nodes should not exceed the maximum flow between them; if there is no direct connection between them, there should be no direct flow. Formulation (2.13) was derived from Correia [25]. Constraint (2.13) is a depiction of the capacity problem; this constraint does not allow v^i_{lk} to be positive when $X_{ijl} = 0$.

$$\sum_{\substack{k\in C\\ l\ne k}} v^i_{lk} \le \sum_{m\in I} f_{im} \sum_{j\in H} X_{ijl}, \quad \forall i \in I, l \in C. \tag{2.13}$$

$$\sum_{l\in C}\sum_{i\in I}\sum_{m\in I}(f_{im} + f_{mi})X_{ijl} \le \Gamma h_j, \quad \forall j \in H. \tag{2.14}$$

$$\sum_{j\in H}\sum_{i\in I}\sum_{\substack{m\in I\\ m\ne j}} f_{im}X_{ijl} - \sum_{j\in H}\sum_{i\in I}\sum_{\substack{m\in I\\ m\ne j}} f_{im}X_{ijl}X_{mjl} + \sum_{i\in I}\sum_{\substack{k\in C\\ l\ne k}} v^i_{kl}\sum_{j\in H} X_{ijk} \le \Gamma c_l, \quad \forall l \in C. \tag{2.15}$$

Figures 2 and 3 illustrate the capacity constraints. Figure 2 contains an input node set where $n = 9$. Figure 3 shows the resulting network with 4 hubs and 2 central hubs. In the solution when $X_{175} = X_{275} = X_{775} = X_{355} = X_{689} = X_{489} = X_{889} = 1$, nodes 5, 7, 8 and 9 are chosen as hub nodes. When $X_{555} = X_{999} = 1$, nodes 5 and 9 are selected as central hub nodes. The capacity constraints for hub node 7 and central hub node 9 are: $(f_{12}+f_{13}+f_{14}++f_{19}+f_{91}++f_{31}+f_{21})X_{175}+(f_{21}+f_{23}+f_{24}++f_{29}+f_{92}++f_{12})X_{275} \leq \Gamma h_7$ and $(f_{41}+f_{42}+f_{43}++f_{49}-f_{48})X_{489}+(f_{61}+f_{63}+f_{64}++f_{69}-f_{68})X_{689}+(f_{81}+f_{82}+f_{83}++f_{89})X_{889}-f_{46}X_{489}X_{689}+v^1_{59}X_{175}+v^2_{59}X_{275}+v^7_{59}X_{775}+v^3_{59}X_{355} \leq \Gamma c_9$.

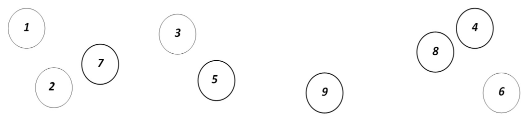

Figure 2: The node set with $n = 9$.

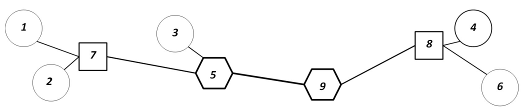

Figure 3: The output network of the model with $n = 9$, four hubs and two central hubs.

Covering and time bound constraints:

The delivery time bound constraints used were introduced by Kara and Tansel [10], Wagner [11], Ernst et al.[12], ahin and Sral [48], Yaman [5] and Korani and Sahraeian [46]. The total solution space was classified and the assignment of each non-hub node and each hub node to the nearest facility in the top level was modified. It was specified that each assignment between two nodes will be implemented when the nodes are a specific distance apart. The closeness criterion is equal to the shortest distance between nodes. The travel time and distance in the configuration model are similar to those from Yaman [5] and Korani and Sahraeian [46]. The closeness criteria for the first level is r_l^{**} and for the second level is r_j^* . The model creates links having the best travel times for each level. Constraints (2.16)-(2.21) have been designed such that constraint (2.16) ensures that each demand node is assigned to hub j when the related node meets closeness criterion r_j^* .

$$s_i + t_{ij}X_{ijl} \leq r_j^*, \forall i \in I, j \in H, l \in C. \quad (2.16)$$

$$\alpha_H t_{ij} X_{ijl} \leq r_l^{**}, \forall, j \in H, l \in C. \quad (2.17)$$

$$\begin{aligned} & r_j^* + r_k^{**} + \alpha_C t_{kl} X_{kkk} \leq D_l, \\ & \forall, j \in H, k \in C : k \neq j, l \in C. \end{aligned} \quad (2.18)$$

$$D_l + r_l^{**} + r_j^* \leq \beta, \forall, j \in H, l \in C : l \neq j. \quad (2.19)$$

$$r_l^{**}, r_j^* \geq 0, \qquad \forall, j \in H, l \in C. \quad (2.20)$$

$$D_l \geq 0, \qquad \forall, l \in C. \quad (2.21)$$

This hierarchical model has a nested hierarchy

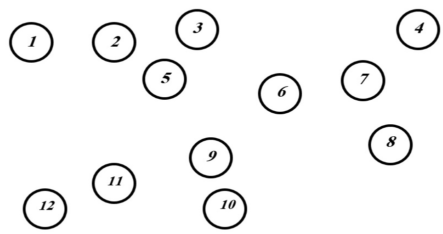

Figure 4: The node set I.

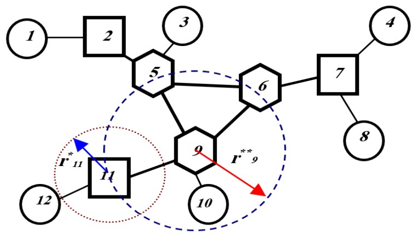

Figure 5: The resulting network of a solution.

where a higher-level facility supplies all services provided by a lower level facility plus at least one additional service [47]. It is possible to assign non-hubs to a central hub to acquire low service. Constraint (2.17) states that each hub node is allocated to central hub l to obtain top service according to closeness criterion r_l^{**} . Constraints (2.18), (2.19) and (2.21) impose given time bound β to the model, as suggested by Kara and Tansel [10], Wagner [11], and Yaman [5].

Constraint (2.18) shows the travel time for each traffic demand to reach the central hub of the destination and inserts travel times for each route in D_l. Constraints (2.19) and (2.21) ensure that the traffic from each origin arrives at the destination at or before time β.

Figures 4 and 5 show the covering and time bound constraints of the mathematical model. A node set in Figure 4 is used as the data set of the resulting network shown in Figure 5. The covering radii of level 1 are denoted by an incomplete circle at 9 and of level 2 by an incomplete circle at 11.

In this solution, central hubs open at nodes 5, 6 and 9 and nodes 2, 7 and 11 nodes are hub nodes. $X_{555} = X_{666} = X_{999} = 1$ for the central hub nodes and $X_{225} = X_{776} = X_{11119} = 1$ for hub nodes.

For the route between nodes 1 and 8, there are 4 covering constraints. In level 2, $t_{12}X_{125} \leq r_2^*$ and $t_{87}X_{876} \leq r_7^*$ are the hub covering constraints in accordance with mathematical relationship (2.16).

The central hub covering constraints in accordance with formulation (2.17) are $\alpha_H t_{25} X_{225} \leq r_5^{**}$ and $\alpha_H t_{76} X_{776} \leq r_6^{**}$.

The time bound constraint imposes an upper bound for delivery on this route (nodes 1 to 8), so $r_2^* + r_5^{**} + \alpha_c t_{56} X_{555} \leq D_6$ and $D_6 + r_6^{**} + r_7^* \leq \beta$ are considered to be constraints (2.18) and (2.19), respectively. An additional binary variable is required for linerazation as follows:

Y_{im}: 1 if nodes i and m have the same hub node in level 2; 0 otherwise.

As stated, this formulation is nonlinear with constraints (2.9) and (2.15). To linearize the model, for each nonlinear constraint a linearization procedure is proposed. Thus, $X_{lll}X_{kkk}$ is replaced by $X_{lll} + X_{kkk}1$ in constraint (2.9), $X_{ijl}X_{mjl}$ is replaced by Y_{im} in constraint (2.15),and X_{ijl} is deleted in the second summation of constraint (2.15). Constraints (2.23), (2.25), (2.26), (2.27) and (2.28) are added to the model. Constraints (2.9) and (2.15) can be replaced with constraints (2.22) to (2.28). The outcome of these methods is linear model *SA-DT-CHSC*. The constraints are as follows:

$$X_{kkk} + X_{lll} - 1 \leq g_{kl}, \forall, k \in C, l \in C : l \neq k. \tag{2.22}$$

$$X_{kkk} + X_{lll} \geq 2g_{kl}, \forall, k \in C, l \in C : l \neq k. \tag{2.23}$$

$$\begin{aligned} &\textstyle\sum_{j\in H}\sum_{i\in I}\sum_{\substack{m\in I\\ m\neq j}} f_{im}X_{ijl} - \\ &\textstyle\sum_{j\in H}\sum_{i\in I}\sum_{\substack{m\in I\\ m\neq j}} f_{im}Y_{im} + \\ &\textstyle\sum_{i\in I}\sum_{\substack{k\in C\\ l\neq k}} v_{kl}^i \leq \Gamma c_l, \forall, l \in C. \end{aligned} \tag{2.24}$$

$$\begin{aligned} &v_{kl}^i \leq M \textstyle\sum_{j\in H} X_{ijk}, \forall, i \in I, k \in C, \\ &l \in C : k \neq l. \end{aligned} \tag{2.25}$$

$$\begin{aligned} &X_{ijl} + X_{mjl} - 1 \leq Y_{im}, \forall, i \in I, m \in I \\ &: m \neq i, l \in C. \end{aligned} \tag{2.26}$$

$$\begin{aligned} &X_{ijl} + X_{mjl} \geq 2Y_{im}, \forall, i \in I, m \in I : m \\ &\neq i, l \in C. \end{aligned} \tag{2.27}$$

$$Y_{im} \in \{0,1\}, \forall, i \in I, m \in I. \tag{2.28}$$

Theorem 2.1 *Any feasible solution for SA-DT-CHSC-NLP is a feasible solution for SA-DT-CHSC.*

1. Suppose $\overline{X}$ is a feasible solution for SA-DT-CHSC-NLP. Because constraints (2.1) to (2.8), (2.10) to (2.14) and (2.16) to (2.21) are the same for two formulations, it is sufficient to prove that $\overline{X}$ is feasible for constraints (2.22) and (2.23) as for constraint (2.9). There are four cases to consider for linearization based on X_{lll} and X_{kkk}.

Case .1.1: If $X_{lll} = X_{kkk} = 1$, then constraint (2.22) causes $X_{lll} + X_{kkk} - 1 = 1$, otherwise $X_{lll}X_{kkk} = 1$. The left sides of constraints (2.9) and (2.22) are the same. X_{kkk} and X_{lll} equal 1 and direct connections exist between these two central hubs as ensured by constraint (2.23).

Case .1.2: If $X_{lll} = 0$, $X_{kkk} = 1$ then constraint (2.22) causes $X_{lll} + X_{kkk} - 1 = 0$, otherwise $X_{lll}X_{kkk} = 0$. The left sides of constraints (2.9) and (2.22) are the same. Constraint (2.23) ensures that a direct connection in the form of a central hub link does not exist between these two nodes.

Case .1.3: If $X_{lll} = 1$, $X_{kkk} = 0$ then constraints (2.22) causes $X_{lll} + X_{kkk} - 1 = 0$, otherwise $X_{lll}X_{kkk} = 0$. The left side of constraints (2.9) and (2.22) are the same. Constraint (2.23) ensures that a direct connection in the form of a central hub link does not exist between these two nodes.

Case .1.4: If $X_{lll} = 0$ and $X_{kkk} = 0$ then $X_{lll} + X_{kkk} - 1 = -1$, otherwise $X_{lll}X_{kkk} = 0$. The values for g_{kl} and g_{lk} are always greater than -1 because g_{kl} and g_{lk} are binary variables (-1 is always less than 0 or 1). The objective function is a minimization problem, so the minimum value of g_{kl} for equals zero. The right side of constraints (2.9) and (2.22) are the same and $g_{lk} = g_{kl} = 0$. Constraint (2.23) ensures that a direct connection in the form of a central hub link does not exist between the l and k nodes.

Corollary 2.1 *Constraints (2.26) and (2.27) operate similarly to constraints (2.22) and (2.23).*

2. Suppose $\overline{X}$ is a feasible solution for SA-DT-CHSC-NLP because constraints ((2.1) to (2.8), (2.10) to (2.14) and (2.16) to (2.21) are the same for the two formulations. It is sufficient to prove that $\overline{X}$ is feasible for constraints (2.24) and (2.25) as for constraint (2.15). There are 2 cases to consider for linearization based on v_{kl}^{i} and $\sum_{j\in H} X_{ijl}$. Since the model has a single assignment attribute, $\sum_{j\in H} X_{ijl}$ equals zero or one..

Case .2.1: If $\sum_{j\in H} X_{ijl} = 1$, then constraint (2.25) causes $0 \le v_{kl}^{i} \le M$, otherwise $0 \le v_{kl}^{i}\sum_{j\in H} X_{ijl} \le M$. The left side of constraints (2.15) and (2.24) are the same.

Case .2.2: If $\sum_{j\in H} X_{ijl} = 0$, then constraint (2.25) causes $v_{kl}^{i} = 0$, otherwise $v_{kl}^{i}\sum_{j\in H} X_{ijl}$ =0 . The left sides of constraints (2.15) and (2.24) are the same.

This theorem shows that each optimum solution for SA-DT-CHSC-NLP is an optimum solution for SA-DT-CHSC.

3 Computational Analysis

Next the computational analysis was run for IAD and AP data sets. Karimi and Bashiri [49] compiled the IAD data set, which contains the distances, travel times, flows, and capacities for 37 cities in Iran as well as fixed hub costs for these cities based on hub airport location. The method suggested by [5] was used to create subsets of the IAD to determine the effects of different parameters on the outcome of the proposed model (Yaman [5]; Korani and Sahraeian [46]). Demand set I comprises all cities in the IAD data. These include 17 cities in Iran with large populations. In addition, Set H is the set of 17 IAD city hubs that include the international airport and set C comprises 7 cities with the largest populations as potential central hubs. The model was solved for sets H and I.

The AP data set was first used by Ernst and Krishnamoorthy (1996) and is arranged according to postal delivery in Sydney, Australia. It consists of 200 nodes that represent the postal districts. This data set features 5 small sized groups for test hub problems with sizes of 10, 20, 25, 40, and 50 and 4 strategies. The strategies are designed for capacity and fixed charge costs of nodes. The groups are labeled light fixed cost and light capacity (LL), light fixed cost and tight capacity (LT), tight fixed cost and light capacity (TL) and tight fixed cost and tight capacity (TT). Because the proposed model is new, a sample with 10 nodes and 3 strategies (LL, LT, TT) was tested to verify the capacity and covering constraint effects.

All nodes were positional nodes for the hubs and central hubs ($I = H = C$) as done by Yaman [5], Calik et al. [14] and Korani and Sahraeian [46]. The travel time between two nodes is equal to their distance apart ($t_{ij} = d_{ij} and i,j \in I$). The formula employed by Calik et al. [14] and Alumur et al. [32] was used to calculate the fixed charge costs of the hub links as follows:

$$f_{ij}^{*} = \frac{\frac{d_{ij}}{f_{ij}}}{Max_{ij}\{\frac{d_{ij}}{f_{ij}}\}} \times 100, \qquad \forall i,j \in I; i \neq j. \tag{3.29}$$

where d_ij is the distance between nodes $i \in I$ and $j \in I$, and f_ij is the flow between nodes $i \in I$ and $j \in I$. The fixed charge costs of the opening central hubs and hub links between the central hubs is denoted as λ_1 where $f_{ij}^{**} = \lambda_1 f_{ij}^{*}$ and $fc_j = \lambda_1 fh_j$. The capacity of the central hubs is denoted as λ_2, where $\Gamma c_i = \lambda_2 \Gamma h_i$. Let $\lambda_1 \in \{2, 4\}$ and $\lambda_2 = 4$ for all nodes because the facilities and links in level 1 provide different

services than those in level 2 [5], so they need additional fixed costs and capacity.
The performance of the model was tested using $GAMS21.7$ and optimization solver $CPLEX11.0.0$ on an $Intel(R)core2$ processor ($2.10GHz$) with $3GBRAM$.
The effect of different values for β and the discount coefficients were tested on total cost. The proposed method solved the problem for the AP data at $n = 10$ and the IAD data at $n = 37$, where $\alpha_H = \{0.9, 0.8\}$ and $\alpha_C = \{0.9, 0.8\}$. The smallest β was obtained for each data set as done by Tan and Kara (2007), Calik et al. [14], Yaman [5] and Korani and Sahraeian [46]; this problem is feasible when $\alpha_H = 0.9$ and $\alpha_C = 0.8$.

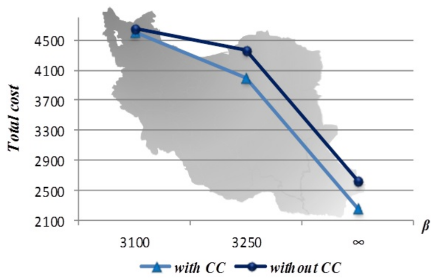

Figure 6: The total fixed charge costs for the IAD data, where $n = 37$, α_H=0.9, $\alpha_C = 0.8$, $\lambda_1 = 4$, $\lambda_2 = 4$.

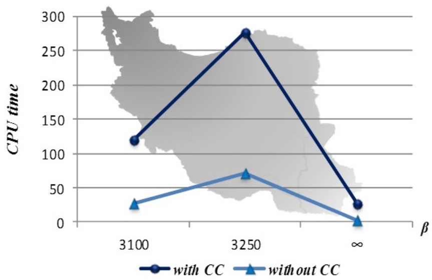

Figure 7: The CPU times for the IAD data, where $n = 37$, α_H=0.9, $\alpha_C = 0.8$, $\lambda_1 = 4$, $\lambda_2 = 4$.

The values for β for the AP data were calculated as an increase of 150, 4000 over units more than the smallest value for IAD for the light strategy and 10000 units more than the smallest value for IAD for the tight strategy. This means that $\beta \in \{3100, 3250 \text{ and } \infty\}$ for the IAD data and $\beta \in \{31000, 35000 \text{ and } \infty\}$ for AP data for the light strategy and $\beta \in \{45000, 55000 \text{ and } \infty\}$ for AP data for the tight strategy. Only feasible cases were reported.
Table 1 shows the total costs and computational times (CPU times or time) for the AP data and Table 2 shows these for the IAD data. Table 2 shows data for the capacitated problem and then for the uncapacitated problem. The total cost and CPU time of the problem strongly decreased without capacity constraints. Figures 6 and 7 were designed for total cost and CPU time, respectively, where $\alpha_H = 0.8, \alpha_C = 0.8, \lambda_1 = 4, \lambda_2 = 4$ and $\beta \in \{3100, 3250 \text{ and } \infty\}$. The total costs and CPU time dropped without capacity constraints (WOCC) over those with capacity constraints (WCC) as β increased. This indicates that the proposed model contains an accurate design for capacity constraints.

In Tables, the Inf equal infeasible, and the capacity and constraints abbreviations are CAP and CON respectively. Figures 8 and 9, respectively, show the total cost and number of open central hubs for AP data at $n = 10$, $\alpha_H = 0.9, \alpha_C = 0.8, \lambda_1 = 2, \lambda_2 = 4$ and different values for β. Figures 10 and 11, respectively, show the total cost and number of open hubs for IAD data at $n = 37, \alpha_H = \{0.9, 0.8\}, \alpha_C = \{0.9, 0.8\}, \lambda_1 = 2, \lambda_2 = 4$ and different values for β. The figures indicate that the total cost and facilities numbers of the problem decrease as β increases for all reduction factors. The increase in the number of facilities is necessary to decrease delivery time nor at any time.
A hierarchical structure is necessary to cover the demand nodes in the best possible time. Table 1 shows that the problem is infeasible for all cases where $(\alpha_H, \alpha_C) = (0.9, 0.9)$ and $\beta = \{31000 \text{ and } 45000\}$. Table 2 shows that the problem is infeasible for all cases where $(\alpha_H, \alpha_C) = (0.9, 0.9)$ and β =3100.

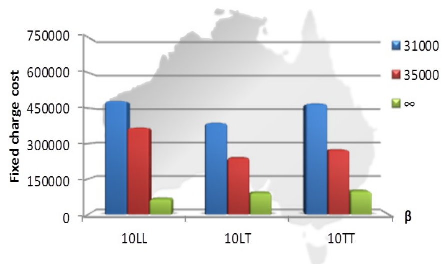

Figure 8: The total fixed charge costs for the AP data at $n = 10$.

Table 1: Total costs and CPU times for the AP data with $n = 10, \lambda_1 = 2$ and $\lambda_2 = 4$.

(α_H, α_C)	β	Strategy	Total Cost	CPU time
(0.9, 0.9)	31000	LL	Infeasible	Infeasible
	35000		388495.703	0.871
	∞		65837.124	0.734
(0.9, 0.8)	31000		481900.975	1.466
	35000		367589.957	1.302
	∞		65837.124	0.718
(0.8, 0.8)	31000		388495.703	1.566
	35000		314246.734	1.119
	∞		65837.124	0.748
(0.9, 0.9)	45000	LT	infeasible	Infeasible
	55000		244882.623	3.656
	∞		91718.385	5.348
(0.9, 0.8)	45000		388301.994	2.253
	55000		240930.496	5.236
	∞		91718.385	5.320
(0.8, 0.8)	45000		354326.524	2.833
	55000		212511.966	2.916
	∞		91718.385	5.339
(0.9, 0.9)	45000	TT	infeasible	Infeasible
	55000		278277.902	6.342
	∞		99554.106	3.226
(0.9, 0.8)	45000		470756.550	1.999
	55000		275006.616	4.082
	∞		99554.106	3.088
(0.8, 0.8)	45000		424526.703	3.576
	55000		242146.129	5.907
	∞		99554.106	3.186

Table 2: Total costs and CPU times for the IAD data with $n = 37$ and different values for λ_1 and $\lambda_2 = 4$.

(α_H, α_C)	β	*with CAP CONs*				*without CAP CONs*			
		$\lambda_1 = 4$		$\lambda_1 = 2$		$\lambda_1 = 4$		$\lambda_1 = 2$	
		Cost	*time*	*Cost*	*time*	*Cost*	*time*	*Cost*	*time*
	3100	*Inf*	*Inf*	*Inf*	*Inf*	*Inf*	*Inf*	*Inf*	*Inf*
(0.9, 0.9)	3250	5518.79	157.19	4128.79	121.61	5518.79	66.72	4128.79	45.44
	∞	2624	27.27	131	8.79	2256	1.45	1128	1.57
	3100	11128.9	148.14	7094.53	148.58	11128.9	38.90	7094.53	58.40
(0.9, 0.8)	3250	4482.26	82.59	3092.26	76.95	4482.26	45.26	3092.26	47.25
	∞	2624	27.582	1312	8.692	2256	1.563	1128	1.625
	3100	4662.24	121.05	3272.24	114.36	4610.21	27.85	3220.21	34.28
(0.8, 0.8)	3250	4362.11	276.23	2972.11	118.79	3996.09	70.32	2606.09	55.17
	∞	2624	27.72	1312	8.076	2256	1.968	1128	1.968

Table 3 shows the locations of the hubs and central hubs for the AP data. Table 4 shows the location of facilities for capacitated and uncapacitated problems for the IAD data. The tables make it easier to understand the problem of selecting hubs and central hubs in each situation. It is evident that the number of hubs and central hubs decreased as increased for both data sets. This is illustrated by Figures 9 for AP and Figure 11 for IAD where $\alpha_H = 0.9, \alpha_C = 0.8, \lambda_1 = 2, \lambda_2 = 4$. $\beta \in \{45000, 55000$ and $\infty\}$ in Table 3 is a good example of this because the number of facilities number decrease from 10 to 7 to 3 hubs and from 5 to 2 to 1 for central hubs. This proves the necessity of a hierarchical structure in the output network.

Table 3: Locations of hubs and central hubs for IAD data at $n = 37$ and different values for λ_1.

(α_H, α_C)	β	Strategy	Hub	Central Hub
(0.9, 0.9)	3100	LL	Infeasible	Infeasible
	3250		1,2,3,4,5,6,7,8,9,10	1,2,5,6
	∞		4,6	6
(0.9, 0.8)	3100		1,2,3,4,5,6,7,8,9,10	1,2,5,6,8,9,10
	3250		1,2,3,4,5,6,7,8,9,10	1,2,5
	∞		4,6	6
(0.8, 0.8)	3100		1,2,3,4,5,6,7,8,9,10	1,2,5,6
	3250		1,2,3,4,5,6,7,8,9,10	3
	∞		4,6	6
(0.9, 0.9)	45000	LT	infeasible	Infeasible
	55000		1,2,4,5,9,10	1,4,5
	∞		4,5,6	6
(0.9, 0.8)	45000		1,2,3,4,5,6,8,9,10	2,3,6,9,10
	55000		1,2,4,5,6,9,10	2,5
	∞		4,5,6	6
(0.8, 0.8)	45000		1,2,3,4,5,6,8,9,10	1,2,5,6
	55000		1,2,4,5,9,10	5
	∞		4,5,6	6
(0.9, 0.9)	45000	TT	infeasible	Infeasible
	55000		1,2,4,5,9,10	1,4,5
	∞		4,5,6	4
(0.9, 0.8)	45000		1,2,3,4,5,6,7,8,9,10	2,3,6,9,10
	55000		1,2,4,5,6,9,10	2,5
	∞		4,5,6	4
(0.8, 0.8)	45000		1,2,3,4,5,6,8,9,10	1,2,4,5
	55000		1,2,4,5,6,9,10	5
	∞		4,5,6	4

Table 4: Locations of hubs and central hubs for IAD data at $n = 37$ and different values for λ_1.

(α_H, α_C)	β	CC	Hub $\lambda_1 = 4$	CHub $\lambda_1 = 4$	Hub $\lambda_1 = 2$	CHub $\lambda_1 = 2$
(0.9, 0.9)	3100	WCC	Infeasible	Infeasible	Infeasible	Infeasible
	3250	$\lambda_2 = 4$	2,4,10,15,23,36	10	2,4,10,15,23,36	10
	∞		28	28	28	28
(0.9, 0.8)	3100		2,10,15,19,30,31,36	10,19,30	2,10,15,19,30,31,36	10,19,30
	3250		10,23,30,36	10	10,23,30,36	10
	∞		28	28	28	28
(0.8, 0.8)	3100		10,19,30,36	10	10,19,30,36	10
	3250		10,14,23,35	10	10,14,23,35	10
	∞		28	28	28	28
(0.9, 0.9)	3100	$WOCC$	Infeasible	infeasible	Infeasible	infeasible
	3250	$\lambda_2 = \infty$	2,4,10,15,23,36	10	2,4,10,15,23,36	10
	∞		2	2	2	2
(0.9, 0.8)	3100		2,10,15,19,30,31,36	10,19,30	2,10,15,19,30,31,36	10,19,30
	3250		10,23,30,36	10	10,23,30,36	10
	∞		2	2	2	2
(0.8, 0.8)	3100		10,14,19,30	10	10,14,19,30	10
	3250		10,14,19	10	10,14,19	10
	∞		2	2	2	2

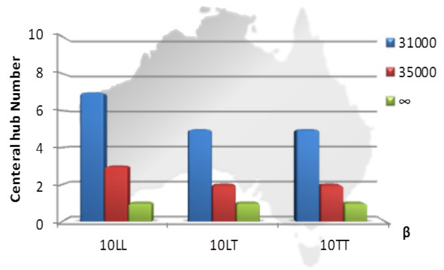

Figure 9: The number of opening central hubs for the AP data at $n = 10$.

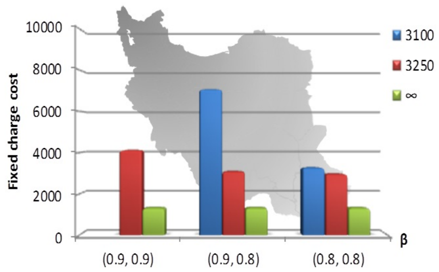

Figure 10: The total costs for the IAD data at $n = 37$.

Tables 3 and 4 show that the problems created more hubs than central hubs because the fixed charge cost of hubs is lower than that for central hubs and number of hubs increased as β decreased. This indicates the necessity of fixed charge costs in a hierarchical structure. Table 3 shows that 4 nodes have been selected as central hubs in 100% of cases and 10 nodes have been selected as central hubs in 67% of cases. Table 4 shows that the 10, 2 and 28 nodes were chosen as central hubs in the tight capacity. In the Table 4, the abbreviation of words is used, therefor, capacity constraints is CC and used CHub for central hub.
CPU time usually decreases as β increases; however, CPU times are suitable when discount factors equal 0.9. In 87.5% of instances (Table 4), the IAD data has a central hub. In two cases, when $(\alpha_H, \alpha_C) = (0.9, 0.8)$ and $\beta = 3100$, there are 3 central hubs. Table 4 shows that only 10 nodes were selected as central hubs in 69% of cases. In all instances, the number of hubs was greater than the number of central hubs. These results do not indicate a specific relationship; hence, the network output of the problem for all nodes of IAD data in Iran was examined. The

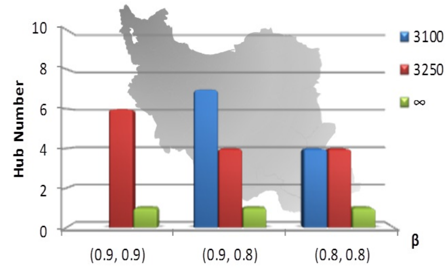

Figure 11: The number of opening hubs for the IAD data at $n = 37$.

results are discussed in detail in the next subsection.
Figure 12 shows a map of Iran with 37 demand nodes. The nodes for potential central hubs are denoted by hexagons and those for potential hubs by squares. The outcomes of the problem for IAD data where $n = 37$, $\alpha_H = 0.9$,$\alpha_C = 0.8$, $\lambda_1 = 4$ and $\lambda_2 = 4$ are shown in Figures 13, 14 and 15, respectively, for $\beta = 3100, \beta = 3250$ and $\beta = \infty$. It was found that the hierarchical structure must be applied to a hub location when the tight structure for covering and capacity is imposed on the problem. This is evident in Figures 13, 14 and 15 where the model chose the increasing numbers of hubs and central hubs as β decreased.
The number of facilities on each level of the hierarchical structure is determined by the specific problem conditions and not by predetermined constraints. Purposive and targeted investment is essential to modern business. The proposed approach will help companies to move in the right direction using strategic planning.

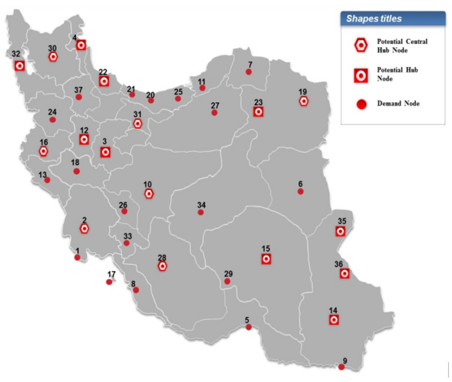

Figure 12: The map of Iran with 37 demand nodes, 17 cities as potential hub nodes and 7 cities as potential central hubs nodes.

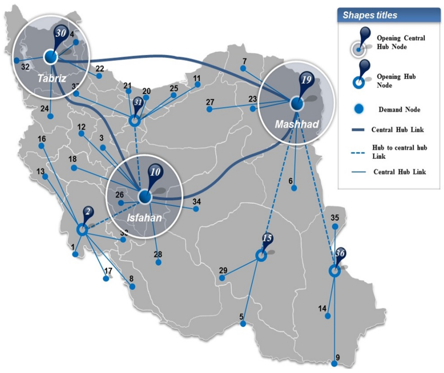

Figure 13: The hierarchical hub network for IAD data where $\alpha_H = 0.9$,$\alpha_C = 0.8$,$\lambda_1 = 4$, $\lambda_2 = 4$ and $\beta = 3100$.

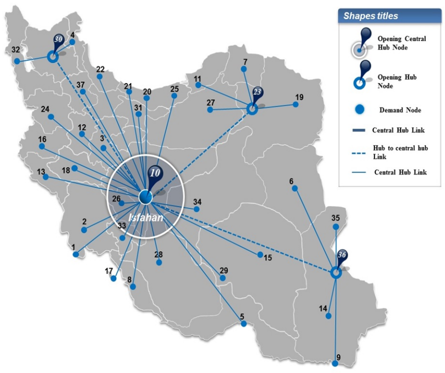

Figure 14: The hierarchical hub network for IAD data where $\alpha_H = 0.9$,$\alpha_C = 0.8$,$\lambda_1 = 4$, $\lambda_2 = 4$ and $\beta = 3250$.

The calculations were carried out on the AP data using 50 nodes so that the solution was produced in a short amount of time. When the problem was large in size and complexity, computation halted because the GAMS software ran out of memory.

4 Conclusion

The present study examined hierarchical hub problems and presented a new model for the capacitated hierarchical hub set covering problem that features a complete network in the first level and the star networks in the second and third levels. The model is nonlinear, so a mixed integer-programming model has been suggested to solve this problem. AP data and the IAD data were used to evaluate the model. The results showed

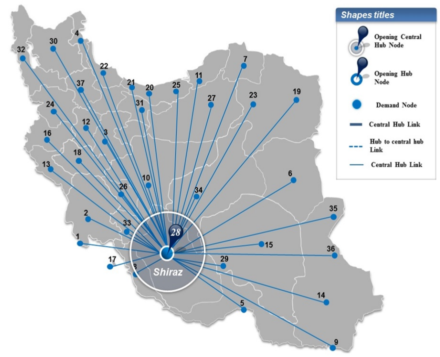

Figure 15: The hierarchical hub network for IAD data where $\alpha_H = 0.9$,$\alpha_C = 0.8$,$\lambda_1 = 4$, $\lambda_2 = 4$ and $\beta = \infty$.

the positive performance of the proposed model. This study showed that optimization of the fixed charge costs of facilities is more important than other objectives for hierarchical hub problems because the results of the problems target long-term strategic planning. The proposed model is very different from previously published models and uses simple and expressive constraints. It is called a capacitated hierarchical hub problem. In addition to being capacitated, the covering constraints are defined for each level separately, which makes the proposed model a hub set-covering problem. Such problems focus on strategy and large scale decision-making, which increases the importance of economies of scale. The choice of facilities is based on fixed charge cost without predetermining the number of facilities. This model allows a purposive and targeted investment approach by opening the facilities and introduces capacitated constraints for each level of the hierarchical structure. A linearized formulation was proposed. This problem is used to design telecommunication systems and that send signals to a large number of ground receivers that are widely scattered. Existing facilities should cover a comparatively wide service zone. For future research, we especially suggest the development of the hub network reliability aspects to the problem, where backup hub facility is defined. The set covering characteristic is good in the govern system, but this structure leads to increased pollution, because need more traffic and transportation in environment. Therefore, we propose the extension of the sustainable hub aspects to our model.

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