Science and Research Branch (IAU)

Generalized Fuzzy Inverse Data envelopment Analysis Models

A. Ashrafi *, M. Mansouri Kaleibar †‡

Abstract

Models in conventional Data Envelopment Analysis (DEA) applies accurate data to estimate efficiency scores, whereas cases are frequently arisen in empirical studies with imprecise data. Inverse DEA models can be used to estimate inputs for a decision making unit (DMU) when some or all outputs and efficiency level of this DMU are increased or preserved. This paper studies the inverse DEA for fuzzy data. Fuzzy data envelopment analysis (FDEA) models emerge as another class of DEA models to account for imprecise inputs and outputs for decision making units. Although several approaches for solving fuzzy DEA models have been developed, numerous deficiencies including the α-cut approaches and types of fuzzy numbers must still be improved. This paper proposes generalized inverse DEA in fuzzy data envelopment analysis. The practical application of these models is illustrated by a numerical example.

Keywords : Data envelopment analysis; Inverse DEA; Efficiency; Fuzzy DEA; Generalized fuzzy DEA; Multi-objective programming; Sample decision making unit.

---

1 Introduction

Data envelopment analysis is a mathematical tool which is used to analyze the efficiency of a set of decision making units. These units are described by multiple inputs and multiple outputs. The CCR model, is one of the best known models of this class [4]. Efficiency is defined, in this model, as the ratio of the weighted sum of outputs to the weighted sum of inputs; and the DMUs are evaluated under their best conditions, i.e., the CCR model Max imizes the efficiency of

the DMU under evaluation. After this model, many other models were introduced. One of the other popular models is the BCC model [3]. This model is under variable return to scale.

In recent years, inverse optimization of DEA models has been studied. For DEA models, constraint parameters are input and output values of DMUs. Wei et al. [24] proposed, for the first time, an inverse DEA model for short term-input and output estimation. An inverse DEA model was discussed to answer the following question: among a group of DMUs, if we increase certain inputs of a particular unit and assume that the DMU maintains its current efficiency value with respect to other units, how much more outputs could the unit produce? or, if the outputs need to be increased to a certain value and the efficiency of the unit remains unchanged, how much more

---

*Department of Mathematics, Statistics and Computer Science, Semnan University, Semnan, Iran.

†Corresponding author.Mozhganmansouri@semnan.ac.ir, Tel: +98(914)1288956.

‡Department of Mathematics, Statistics and Computer Science, Semnan University, Semnan, Iran.

inputs should be provided to the unit? In their developed inverse DEA model, the increases in input and output values were assumed to be non-negative values, and the inverse DEA model was transformed into and solved as a multi-objective linear programming (MOLP) problem.

The first type of inverse DEA models is a resource allocation problem. The resource allocation problem of DEA is an inverse DEA problem of determining the best possible inputs for given outputs such that the current efficiency value of a considered DMU with respect to other DMUs remains unchanged. Another type of inverse DEA models is an investment analysis problem. The investment analysis problem of DEA is an inverse DEA problem of determining the best possible outputs for given inputs such that the current efficiency value of a considered DMU0 with respect to other DMUs remains unchanged. [24].

After introducing inverse DEA by Wei et al. [24], this problem has been studied in many theoretical and applied publications, including Gattoufi, Amin, and Emrouznejad [6], Hadi-Vencheh and Foroughi [9], Hadi-Vencheh et al. [10], Jahanshahloo, Hadi-Vencheh, Foroughi, and Kazemi Matin [14], Jahanshahloo, Hosseinzadeh Lotfi, Shoja, Tohidi, and Razavyan [11], Lin [20], Lertworasirikul, Charnsethikul, and Fang [6], Li and Cui [[18], [19]], Ghobadi and Jahangiri [8], Yan, Wei, and Hao [26], Jahanshahloo, Soleimani-damaneh and Ghobadi [15].

Previous studies on inverse DEA problems mostly consider the input and output of DMUs are precise. In this paper, we deal with inverse DEA model for uncertain data. In most real world situations, the possible values of parameters of mathematical models are often only imprecisely or ambiguously known to the experts. It would be certainly more appropriate to interpret the experts understanding of the parameters as fuzzy numerical data which can be represented by means of fuzzy sets of the real line known as fuzzy numbers. Some papers propose a method using alpha cutting measure which changes fuzzy DEA model into primary firm model. Using such method, it is needed to solve several linear planning problems to estimate membership function and then assess performance of a decision making unit. In more general cases, the data for evaluation is imprecise. Thus, several studies proposed the fuzzy DEA model for input and output data [15]. However, while evaluating the model, there are still many places to be improved such as the selected special point, the types of fuzzy number, the α-cut or α-level approach and the type of the FDEA model. Furthermore, the target decision making units of traditional DEA models are limited to internal DMUs that cannot evaluate a sample DMU (SDMU). To date, only a few studies have discussed the evaluation methods for an SDMU. In this paper, we discuss the "generalized fuzzy inverse DEA model" for the case of constant return to scale and variable return to scale. This paper is organized as follows. In Section 2, introduces inverse DEA models. Section 3, presents the fuzzy inverse DEA models. In section 4 the Generalized Inverse DEA models is developed. We give the empirical example in section 5 and finally section 6 deals with conclusion.

2 Inverse DEA Models

This section shows the inverse Data Envelopment Analysis (Inverse DEA) for the case of constant return to scale and variable returns to scale.

2.1 The inverse DEA model, the case of constant return to scale

Assume that there are n DMUs and that the DMUs under consideration convert m inputs to s outputs. In particular, let the 0th DMU produces output $y_0 = (y_{10}, y_{20}, \cdots, y_{s0})$ using inputs $x_0 = (x_{10}, x_{20}, \cdots, x_{m0})$. To evaluate the relative efficiency, Yan et al. [26] provided the following general DEA model:

$$\max z_0$$
$$s.t \ \sum_{j=1}^{n} \lambda_j X_j - X_0 \in V^*$$
$$- \sum_{j=1}^{n} \lambda_j Y_j - z_0 Y_0 \in U^*,$$
$$\delta_1(e\lambda^T + \delta_2(-1)^{\delta_3}\lambda_{n+1}) = \delta_1$$
$$\lambda \in -K^*, \ \lambda_{n+1} \geq 0 \qquad (2.1)$$

Where $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n), e = (1, \cdots, 1) \in E^n$ and $\delta_1, \delta_2, \delta_3$ are parameters with 0–1values, and

it is easy to see that, if $\delta_1 = 0$; then model (2.1) is the CCR model. In addition V^*, U^* and K^* are the negative polar cone of V, U and K, respectively in which $V \subset E^m_+, U \subset E^s_+$ and $K \subset E^n_+$ are the preference cone of relative importance of inputs, outputs and DMUs, respectively. Suppose that for DMU_0, z_0^* is the optimal value of (2.1) the inputs of DMU0 has an increment from X_0 to α_0. To estimate the corresponding outputs level $\beta_0 = (\beta_{01}, \cdots, \beta_{0s})$ when the efficiency index to be maintained at z_0^* the following multiple objective programming problem [26] is considered.

$$\max \beta_0 = (\beta_{01}, \cdots, \beta_{0s})$$
$$s.t \sum_{j=1}^n \lambda_j X_j - \alpha_0 \in V^*$$
$$- \sum_{j=1}^n \lambda_j Y_j - z_0^* \beta_0 \in U^*,$$
$$- \beta_0 - Y_0 \in U^*,$$
$$\delta_1 (e\lambda^T + \delta_2(-1)^{\delta_3} \lambda_{n+1}) = \delta_1,$$
$$\lambda \in -K^*, \ \lambda_{n+1} \geq 0 \qquad (2.2)$$

To solve above Question, Wei et al. proposed the following MOLP model:

$$\max \beta_0 = (\beta_{01}, \cdots, \beta_{0s})$$
$$s.t \sum_{j=1}^n \lambda_j X_j \leq \alpha_0,$$
$$\sum_{j=1}^n \lambda_j Y_j \geq \beta_0 z_0^*,$$
$$\beta_0 \geq Y_0,$$
$$\lambda_j \geq 0 \quad j = 1, \cdots, n \qquad (2.3)$$

Similarly, we can determine the input oriented model [24].

2.2 The inverse DEA model, the case of variable return to scale

Inverse DEA models try to answer questions like: if DMU0, for instance, changes its current output into $\beta_0 = y_0 + \Delta y_0$, $\Delta y_0 \in \mathbb{R}^s$ then how much input is required to preserve the relative efficiency of DMU0. The following MOLP model is proposed in the literature for estimating the required input:

$$\min (\alpha_1, \alpha_2, \cdots, \alpha_m)$$
$$= (x_{10} + \Delta x_1, x_{20} + \Delta x_2, \cdots, x_{m0} + \Delta x_m)$$
$$s.t \sum_{j=1}^n \lambda_j x_{ij} \leq \theta_0^* \alpha_i i = 1, \cdots, m,$$
$$\sum_{j=1}^n \lambda_j y_{rj} \leq \beta_{r0} r = 1, \cdots, s,$$
$$\sum_{j=1}^n \lambda_j = 1,$$
$$\lambda_j \geq 0, \quad j = 1, \cdots, n \qquad (2.4)$$

Where θ_0^* is the optimal value of the model BCC and $\alpha_0 = x_0 + \Delta x_0$, $\Delta x_0 \in \mathbb{R}^m$ is the required input levels that guarantee unchanged relative efficiency for DMU0. Lertworasirikul et al. dealt with the inverse DEA Model as a non-linear program first and then moved into an MOLP model (2.4) due to the difficulty of solving a non-linear problem [17].

Assume that the relative efficiency of DMU0 is θ_0^* and the output value of DMU0 is perturbed into $\beta_0 = y_0 + \Delta y_0$. If (λ, α_0) is a weak efficient solution for MOLP model (2.1), where $\alpha_0 = x_0 + \Delta x_0$, then the relative efficiency of perturbed DMU -$DMU_{0'} = (\alpha_0, \beta_0) = (x_0 + \Delta x_0, y_0 + \Delta y_0)$- is also θ_0^*. Moreover, the aforementioned perturbation also does not affect the efficiency score of other DMUs (see [17], [7]).

3 Fuzzy Inverse DEA Models

In fuzzy DEA, it is assumed that some input values $\tilde{X}_{ij}$ and output values $\tilde{Y}_{ik}$ are approximately known and can be represented by fuzzy sets with membership functions $\mu_{\tilde{X}_{ij}}$ and $\mu_{\tilde{Y}_{ik}}$, respectively. Without loss of generality, we will assume that all observations are fuzzy, since crisp values can be represented by degenerated membership functions which only have one value in their domain. Hence, a fuzzy inverse DEA models can be for-

Table 1: The input–output set of Example 5.1

DMUs	Input1	Input2	Output1	Output2
1	(18,20,22)	(10,12,15)	(55,60,65)	(30,36,42)
2	(7,10,13)	(13,15,17)	(28,30,32)	(40,45,50)
3	(14,15,16)	(10,12,15)	(28,30,32)	(32,36,40)
4	(4,5,6)	(60,70,80)	(14,15,16)	(70,80,90)
5	(2,3,4)	(6,9,12)	(2,3,4)	(7.9,11)
6	(7.9,11)	(15,18,21)	(1,1,1)	(15,18,21)
7	(60,63,66)	(15,19 23)	(5,8,11)	(16,19 22)
8	(18,22,26)	(70,73,76)	(1,1,1)	(2,3,4)

Table 2: Efficiency values of DMU7 (Constant Return to Scale).

—WorstDMU	BestDMU	Center DMU	MaxDMU	MinDMU	1-cutDMU
— 0.1375	0.1434	0.1364	0.1398	0.1350	0.1364

Table 3: Results for $\alpha = (\alpha_{71}, \alpha_{72})$.

— Worst DMU	Best DMU	Center DMU	max DMU	min DMU	1-cut DMU
—(13.1845, —20.7631)	(13.5966, 21.0021)	(13.3866, 20.9821)	(13.5816, 20.9901)	(13.2743, 20.8753)	(13.3866, 20.9821)

Table 4: The input–output set of Example 5.2 .

DMUs	Input1	Input2	Output1	Output2
1	(4,56)	(6,8,10)	(6,7,8)	(6,9,12)
2	(5,7,9)	(5,6,7)	(3,5,7)	(5,6,7)
3	(4,6,8)	(3,4,5)	(7,8,9)	(4,6,8)

Table 5: Efficiency values of DMUB (Variable Return to Scale).

Worst DMU	Best DMU	Center DMU	max DMU	min DMU	1-cut DMU
0.8103	0.8451	0.8235	0.8356	0.8065	0.8235

mulated as

$$\max \beta_0 = (\beta_{01}, \cdots, \beta_{0s})$$

$$s.t \sum_{j=1}^{n} \lambda_j \tilde{X}_j \leq \alpha_0,$$

$$\sum_{j=1}^{n} \lambda_j \tilde{Y}_j \geq \beta_0 z_0^*,$$

$$\beta_0 \geq \tilde{Y}_0,$$

$$\lambda_j \geq 0 \quad j = 1, \cdots, n \qquad (3.5)$$

First, we introduce α-cuts of $\tilde{X}_j$ and $\tilde{Y}_j$. Let $S(\tilde{X}_j)$ and $S(\tilde{Y}_j)$ denote the support of $\tilde{X}_j$ and

$\tilde{Y}_j$. The α-cuts of $\tilde{X}_j$ and $\tilde{Y}_j$ are defined as

$$(\tilde{X}_j)_\alpha = \left\{ X_j \in S(\tilde{X}_j) \,\Big|\, {}^-_{\tilde{X}_j} \geq ff \right\} \quad \forall j$$

$$(\tilde{Y}_j)_\alpha = \left\{ Y_j \in S(\tilde{Y}_j) \,\Big|\, {}^-_{\tilde{Y}_j} \geq ff \right\} \quad \forall j \quad (3.6)$$

Note that $(X_j)_\alpha$ and $(Y_j)_\alpha$ are crisp sets. Using α-cuts, also called α-level sets, the inputs and outputs can be represented by different levels of confidence intervals. The fuzzy DEA model is thus transformed to a family of crisp DEA models with different α-level sets $\{(X_j)_\alpha \,|\, 0 < \alpha \leq 1\}$ and $\{(Y_j)_\alpha \,|\, 0 < \alpha \leq 1\}$ [16]. Similarly we can present the Eq. (2.1), (2.2) and (2.3) in fuzzy environment. But studies on Fuzzy DEA models still focus on the special DEA model and fuzzy number and often apply only a single fuzzy number and the α-cut approach to one FDEA model. The selected DMUs, after applying α-cut, still remain as special DMUs. More important to the above conclusions is that these evaluation methods still cannot analyze a fuzzy sample DMU (SDMU).To address the above limitations of the Fuzzy DEA model, the following section proposes a Generalized Fuzzy DEA model.

4 Generalized Fuzzy Inverse DEA Models

In this section, we extend Inverse DEA models in fuzzy environment to generalized models.

Definition 4.1 *Suppose DMU is one decision making unit in a decision making problem, all the DMU including the same input and output is called sample decision making unit (SDMU) based on this decision making problem [22].*

The distinctions between an SDMU and DMU are presented as follows:

- A DMU must be in the production possibility set, whereas SDMU may be outside of the production possibility set.

- The efficiency value of the DMU must be equal to or below 1, whereas that of the SDMU can be equal to, smaller than, or greater than 1.

- A DMU must appear in the constraints, whereas an SDMU can either appear within or not be among the constraints.

- The reference sets in FDEA model are the efficient FDMUs, while in the generalized FDEA model, they can be the efficient FD-MUs, normal FDMUs, inefficient FDMUs, special FDMUs, non-FDMUs. These five types of DMUs are called fuzzy sample DMUs (FSDMUs).

The FSDMU is replaced by one of the SDMUs of the FSDMU and FDMUi by one of the DMU is of the FDMUi. Once the model becomes a crisp DEA model, it can be solved using the appropriate software. The selected SDMU or DMUi can be any point of the domain. Among these points are the following seven special points: Best DMU, Worst DMU, Max DMU, Min DMU, Center DMU, 1-Cut DMU and Vertex DMU [22]. Fig.
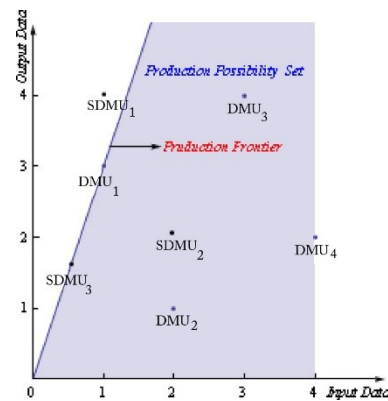

Figure 1: SDMUs of the CCR model

1 shows the SDMU of a CCR model. For the FDEA model, a number of studies found the efficiency value to be greater than 1. This condition results from the fact that all the constraints, including the target DMU, will select the worst DMUs, whereas the target DMU selects the best DMU to be evaluated. Thus, the evaluated DMU is an SDMU, not a DMU.

Fig. 2 shows an FCCR model with four FD-MUs. The target DMU is assumed to be an FDMU1. According to the ranking approach, all the FDMUs of the constraints will select the worst points, a_1, a_2, a_3, a_4, and the target FDMU1will use the best point A to be evaluated. Therefore, point A is an SDMU, not a DMU
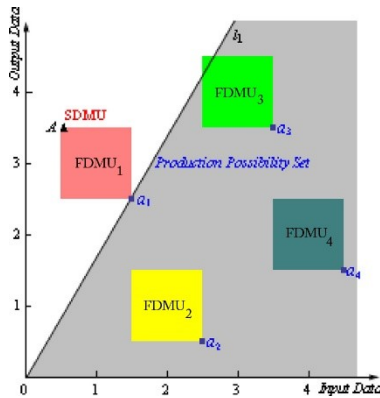
[22]. When evaluating the target FDMU of the

Figure 2: DMUs of the FCCR model

FDEA model, either the best or the worst DMU is selected. The remaining five special DMUs are never selected. In the proposed method, the seven special DMUs or any DMU of the domain that the decision maker prefers can be selected. After improving FDMUo to FSDMUo, the generalized fuzzy Inverse DEA model can easily be obtained. The models are shown in Eq. (4.7), (4.8) and (4.9).

Generalized fuzzy Inverse models, the case of constant return to scale

$$
\max z_0
$$
$$
s.t \sum_{j=1}^{n} \lambda_j X_{Sj} - X_{S0} \in V^*,
$$
$$
- \sum_{j=1}^{n} \lambda_j Y_{Sj} - z_0 Y_{S0} \in U^*,
$$
$$
\delta_1 (e\lambda^T + \delta_2(-1)^{\delta_3} \lambda_{n+1}) = \delta_1,
$$
$$
\lambda \in -K^*, \lambda_{n+1} \geq 0 \qquad (4.7)
$$

Output oriented model

$$
\max \beta_0 = (\beta_{01}, \cdots, \beta_{0s})
$$
$$
s.t \sum_{j=1}^{n} \lambda_j X_{Sj} \leq \alpha_0,
$$
$$
\sum_{j=1}^{n} \lambda_j Y_{Sj} \geq \beta_0 z_0^*,
$$
$$
\beta_0 \geq Y_{S0},
$$
$$
\lambda_j \geq 0 \quad j = 1, \cdots, n \qquad (4.8)
$$

Generalized fuzzy Inverse models, the case of variable return to scale

$$
\min (\alpha_1, \alpha_2, \cdots, \alpha_m) = (x_{10} + \Delta x_1
$$
$$
, x_{20} + \Delta x_2, \cdots, x_{m0} + \Delta x_m)
$$
$$
s.t \sum_{j=1}^{n} \lambda_j x_{Sij} \leq \theta_0^* \alpha_i \quad i = 1, \cdots, m,
$$
$$
\sum_{j=1}^{n} \lambda_j y_{Srj} \leq \beta_{r0} \quad r = 1, \cdots, s,
$$
$$
\sum_{j=1}^{n} \lambda_j = 1,
$$
$$
\lambda_j \geq 0, \quad j = 1, \cdots, n \qquad (4.9)
$$

Here, X_S and Y_S are the input and output data of the selected SDMU.

5   Numerical Example

In this section, a numerical example is presented to describe the proposed models. The purpose is to test the performance of our proposed model.

Example 5.1 *Consider eight DMUs with two fuzzy inputs and two fuzzy outputs. The data for this example is shown in Table 1. In this example, we suppose that the fuzzy number are triangular.*

Table 2 show the efficiency of DMU7 using Lingo, with different SDMUs. For instance, by evaluating DMU7 using input oriented of model (4.8), we have: (Table 3).

In Table 3, we now increase the outputs of DMU7 from $Y_7 = ((5, 8, 11), (16, 19, 22))$ to $((7, 10, 13), (19, 22, 25))$. As can be seen the first input has reduced and the second input of this DMU has increased.

Example 5.2 *Consider Table 4 which shows the data of three DMUs. Each DMU uses two fuzzy inputs and produces two fuzzy outputs. In this example, we suppose that the fuzzy number are triangular.*

Consider DMUB, for instance; the relative efficiency of DMUB has been shown in Table 5 using the BCC model. DMUB is the only inefficient DMU, other DMUs are efficient. Now, assume that this DMU needs to increase its second output

to $(5.5, 6.5, 7.5)$. The model (4.9) shows to what extent we should change the input of DMUB to preserve the relative efficiency of this DMU.

Using model (4.9), the relative efficiency of perturbed DMUB is the same as the relative efficiency of DMU B before perturbation. Moreover, efficient DMUs remain efficient after perturbation of DMUB.

6   Conclusion

The traditional inverse DEA models are used to determine the best possible values of inputs (outputs) for given values of outputs (inputs) of a considered DMU with crisp data. In the real world there are many problems which have fuzzy parameters. In this paper, generalized fuzzy inverse DEA models for the case of constant return to scale and variable returns to scale were proposed. The generalized fuzzy models is the generation of fuzzy models. It can not only evaluate the inner DMU, but also arbitrarily evaluate the given sample DMU. A numerical example is also provided to illustrate the proposed models.

References

[1] A. Alinezhad, A. Makui, R. Kiani Mavi, An inverse DEA model for inputs/outputs estimation with respect to decision maker's preferences: The case of Refah bank of IRAN, *Journal of Mathematical Science* 1 (2007) 61-70.

[2] A. Ashrafi, M. Mansouri Kaleibar, Cost, revenue and profit efficiency models in generalized fuzzy data envelopment analysis, *Fuzzy Information and Engineering* 9 (2017) 237-246.

[3] R. Banker, A. Charnes, WW. Cooper, Some models for estimating technical and scale inefficiencies in data envelopment analysis, *Management Science* 30 (1984) 1078-1092.

[4] A. Charnes, WW. Cooper, E. Rhodes, Measuring the efficiency of decision making units, *European Journal of Operational Research* 2 (1978) 429–444.

[5] W.W. Cooper, L.M. Seiford, K. Tone, Data envelopment analysis, a comprehensive text with models application references and DEA- solver software, Boston: Klawer Academic Publishers (2000).

[6] S. Gattoufi, GR. Amin, A. Emrouznejad, A new inverse DEA method for merging banks, *IMA Journal of Management Mathematics* 25 (2014) 73-87.

[7] M. Ghiyasi, On inverse DEA model: The case of variable returns to scale, *Computers and Industrial Engineering* 87 (2015) 407-409.

[8] S. Ghobadi, S. Jahangiri, Inverse DEA: Review, extension and application, *International Journal of Information Technology & Decision Making* 14 (2015) 805-824.

[9] A. Hadi-Vencheh, A. Foroughi, A generalized DEA model for inputs/outputs estimation, *Mathematical and Computer Modelling* 43 (2006) 447-457.

[10] A. Hadi-Vencheh, AA. Foroughi, M. Soleimani-damaneh, A DEA model for resource allocation, *Economic Modelling* 25 (2008) 983-993.

[11] GR. Jahanshahloo, FH. Lotfi, N. Shoja, G. Tohidi, S. Razavyan, The outputs estimation of a DMU according to improvement of its efficiency, *Applied Mathematics and Computation* 147 (2004) 409-413.

[12] GR. Jahanshahloo, FH. Lotfi, N. Shoja, G. Tohidi, S. Razavyan, Input estimation and identification of extra inputs in inverse DEA models, *Applied Mathematics and Computation* 156 (2004) 427-437.

[13] GR. Jahanshahloo, FH. Lotfi, N. Shoja, G. Tohidi, S. Razavyan, Sensitivity of efficiency classifications in the inverse DEA models, *Applied Mathematics and Computation* 169 (2005) 905-916.

[14] GR. Jahanshahloo, A. Hadi-Vencheh, AA. Foroughi, R. Kazemi Matin, Inputs/outputs estimation in DEA when some factors are

undesirable, *Applied Mathematics and Computation* 156 (2004) 19-32.

[15] GR. Jahanshahloo, M. Soleimani-damaneh, S. Ghobadi, Inverse DEA under intertemporal dependence using multiple-objective programming, *European Journal of Operational Research* 240 (2015) 447-456.

[16] C. Kao, ST. Liu, Fuzzy efficiency measures in data envelopment analysis, *Fuzzy Sets and Systems* 113 (2000) 427-437.

[17] S. Lertworasirikul, P. Charnsethikul, SC. Fang, Inverse data envelopment analysis model to preserve relative efficiency values: The case of variable returns to scale, *Computer & Industrial Engineering* 61 (2011) 1017-1023.

[18] X. Li, J. Cui, A comprehensive DEA approach for the resource allocation problem based on scale economies classification, *Journal of Systems Science and Complexity* 21 (2008) 540-557.

[19] X. Li, J. Cui, Inverse DEA model with considering returns to scale and elasticity, *In 11th International symposium on operations research and its applications in engineering, Technology and Management* (2013).

[20] HT. Lin, An efficiency-driven approach for setting revenue target, *Decision Support Systems* 49 (2010) 311-317.

[21] ST. Liu, A fuzzy DEA/AR approach to the selection of Flexible manufacturing systems, *Computers and Industrial Engineering* 54 (2008) 66-76.

[22] Z. Muren, M. Zhanxin, C. Wei, Generalized fuzzy data envelopment analysis methods, *Applied Soft Computing* 19 (2014) 215-225.

[23] RE. Steuer, Multiple criteria optimization: Theory, computation, and application, *Malabar, FL: Krieger Publishing* (1986).

[24] QL. Wei, J. Zhang, X. Zhang, An inverse DEA model for inputs/outputs estimate, *European Journal of Operational Research* 121 (2000) 151-163.

[25] QL. Wei, H. Yan, A data envelopment analysis (DEA) evaluation method based on sample decision making units, *International Journal of Information Technology & Decision Making* 9 (2010) 601-624.

[26] H. Yan, Q. Wei, G. Hao, DEA models for resource reallocation and production input/output estimation, *European Journal of Operational Research* 136 (2002) 19-31.

[27] LA. Zadeh, Fuzzy Sets as a basis for a theory of possibility, *Fuzzy Sets and Systems* 1 (1978) 3-28.

[28] J. Zhu, Imprecise data envelopment analysis (IDEA): A review and improvement with an application, *European Journal of Operational Research* 144 (2003) 513-529.

Ali Ashrafi has got PhD degree from University of Putra in 2011. He has been member of Faculty of Mathematics, Statistics and Computer Science, Semnan University, Semnan, since 2011. Main research interest include: Network and Fuzzy Data Envelopment Analysis, Nonlinear Programming and Semi Definite Programming.

Mozhgan Mansouri Kaleibar received B.Sc and M.Sc in Applied Mathematics (operational Research. She has got PhD degrees in Applied Mathematics (operational research) from Semnan University. She is lecturer in universities in Tabriz science 2010. Her research interest is: Data Envelopment Analysis, Game Theory, Goal programming, Performance Evaluation, Supply chain management, Fuzzy Theory, Fuzzy DEA, Dynamic DEA, Network DEA. Dr Mansouri Kaleibar is the author of many research articles published in scientific journals and conferences.