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An Approach to Identify and Evaluate Congestion in Data Envelopment Analysis

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Abstract

Congestion indicates an economic state where inputs are overly invested. Evidence of congestion occurs whenever reducing some inputs can increase outputs. In this paper, we present a new model to identify and evaluate congestion in Data Envelopment Analysis (DEA). We use output efficient DMUs to construct our proposed model to evaluate congestion. We also proposed a linear inequality and equality system to identify the occurrence of congestion. Finally, three numerical examples are presented to illustrate the use of our proposed method.

Keywords: Data Envelopment Analysis, Congestion, Efficiency.

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1. Introduction

Evaluation of decision making units (DMUs) is an important task especially from a managerial point of view. DEA is a nonparametric and mathematical programming based approach for evaluating the performance of a set of homogeneous DMUs using multiple inputs to produce multiple outputs. In performance analysis, in particular in DEA, the concept of congestion plays a seminal role in theory and application. Congestion is a special phenomenon in the production process which is defined in economics where outputs are reduced due to excessive amount of inputs or an increase in one or more outputs results in a reduction in one or more inputs. For an actual example of congestion, in a coal mine where a large crowd of the miners are working in a narrow underground, the amount of minerals excavated will be reduced [2].

Heretofore, various approaches have been presented in DEA for the treatment of congestion. The concept of congestion was first introduced in the literatures by Färe and Grosskopf [5] in the context of DEA. Subsequently an operationally implementable form was given by Färe et al. [6] and Cooper et al. [1-4]. Afterwards, Tone and Sahoo [16] developed a new slack-based approach to evaluate the scale elasticity in the presence of congestion with a unified framework. Wei and Yan [17] used DEA output additive models and proposed a necessary and sufficient condition for existence of congestion. Jahanshahloo and Khodabakhshi [7,8] provided an approach of input congestion based on the relaxed combinations of inputs. Later on, Khodabakhshi [10] provided a one-model approach of input congestion based on input relaxation model. Also Khodabakhshi [11-12] proposed a method to detect the input congestion in the stochastic DEA. Jahanshahloo et al. [9] and Khodabakhshi et al. [14] proposed some methods for

computing the congestion in DEA models with production trade-offs and weight restrictions. Sueyoshi and Sekitani [15] proposed a modified approach which is able to measure congestion under the occurrence of multiple solution. There exist some papers which reviewed congestion papers, as that of Khodabakhshi et al. [13].

In this paper, we present an approach to identify and evaluate congestion in DEA based on the definition of congestion. The rest of the paper unfolds as follows. In the next section, preliminary information is introduced to facilitate later discussions. In section 3, we present a procedure for identifying and evaluating congestion. The validity of the proposed model is demonstrated using three numerical examples in Section 4. Finally, Section 5 gives the conclusion of this paper.

2. Preliminaries

Suppose that there are n DMUs to be evaluated in terms of m inputs and s outputs. Let $x_{ij} (i = 1, 2, \dots, m)$ and $y_{rj} (r = 1, 2, \dots, s)$ be the input and output values of $DMU_j (j = 1, \dots, n)$.

It should be noted that evaluating congestion in customary models for T_{BCC} has been studied on T_{NEW} , which is a PPS without input disposability postulate, which can be defined as follows [16]:

$$T_{NEW} = \{ (\mathbf{x}, \mathbf{y}) | \mathbf{x} = \sum_{j=1}^n \lambda_j \mathbf{x}_j, \mathbf{y} \leq \sum_{j=1}^n \lambda_j \mathbf{y}_j, \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, j=1, \dots, n \}$$

$$\mathbf{y} \leq \sum_{j=1}^n \lambda_j \mathbf{y}_j, \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, j=1, \dots, n \}$$

The key to the debate concerning to treatment of congestion in DEA lies in the meaning and definition of congestion. The definition we use is as follows:

Definition 2.1- Congestion is said to occur when the output that is maximally possible can be increased by reducing one or more inputs without improving any other inputs

or outputs. Conversely congestion is said to occur when some of the outputs that are maximally possible are reduced by increasing one or more inputs without improving any other inputs or outputs [3]. We distinguish this from “technical inefficiency” with the latter defined as follows:

Definition 2.2- Evidence of “technical inefficiency” is present when reductions in one or more inputs, or increases in one or more outputs can be undertaken without worsening any other input or output. Evidently congestion can be regarded as a particularly severe form of technical inefficiency [2]. Technical aspects of efficiency are to be identified in this paper, as defined in the following:

Definition 2.3- Efficiency is achieved only when it is not possible to improve any input or output without worsening some other input or output [2].

3. Proposed approach

In this section we proposed an approach to evaluate and identify the occurrence of congestion.

The proposed algorithm to identify and evaluate congestion has the following procedures:

Step1: Let (x_p, y_p) be coordinate of DMU under evaluation. We solve the following model:

$$\begin{aligned}
 z_p^* &= \text{Max } e s^+ \\
 \text{s.t. } \sum_{j=1}^n \lambda_j x_j &\leq x_p \\
 \sum_{j=1}^n \lambda_j y_j - s^+ &= y_p \\
 \sum_{j=1}^n \lambda_j &= 1,
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 s^+ &\geq 0 \\
 \lambda_j &\geq 0, \quad j=1,2,\dots,n
 \end{aligned}$$

Where e is a row vector with all its elements being equal to one. Define the set E as follows:

$$E = \{j \mid z_j^* = 0, j = 1, 2, \dots, n\}$$

Step2: solve the following model for $DMU_p (p \in E)$:

$$\begin{aligned}
 z_p^{**} &= \text{Max } e s^- \\
 \text{s.t. } \sum_{j \in E} \lambda_j x_j &= x_p - s^-, \\
 \sum_{j \in E} \lambda_j y_j &= y_p, \\
 \sum_{j \in E} \lambda_j &= 1, \\
 s^- &\geq 0 \\
 \lambda_j &\geq 0, \quad j \in E
 \end{aligned} \tag{2}$$

Define the set E^+ as follows:

$$E^+ = \{j \mid z_p^{**} \neq 0 (p \in E) \& \lambda_j \neq 0, j \in E\}$$

Step3: To identify and evaluate congestion of $DMU_p (p=1,2,\dots,n)$, we apply the following model:

$$\begin{aligned}
 z_p^{***} &= \text{Max} \\
 \sum_{r=1}^s (y_r - y_{rp}) - \varepsilon \sum_{i=1}^m (x_{ip} - x_i) & \\
 \text{s.t. } \sum_{j \in E^{++}} \lambda_j x_j &= x, \\
 \sum_{j \in E^{++}} \lambda_j y_j &\leq y, \\
 x &\leq x_p, \\
 y &\geq y_p \\
 \sum_{j \in E^{++}} \lambda_j &= 1, \\
 s^- \geq 0, \lambda_j &\geq 0, \quad j \in E^{++}
 \end{aligned} \tag{3}$$

Where ε is a non-Archimedean small positive number and $E^{++} = E - E^+$.

Let (x^*, y^*, λ^*) be an optimal solution of (3).

(a) If (3) be infeasible then DMU_p not congested.

(b) If $\sum_{i=1}^m (x_{ip} - x_i^*) = 0$ then DMU_p not congested.

(c) If $\sum_{i=1}^m (x_{ip} - x_i^*) \neq 0$ then DMU_p is congested and value of congestion is as follows:

$$\text{congestion of the } i\text{th -input} = x_{ip} - x_i^*$$

Theorem 3.1- $DMU_p = (x_p, y_p)$ is congested if the following system has no congestion:

$$\begin{aligned} \sum_{j \in E} \lambda_j x_j &= x_p, \\ \sum_{j \in E} \lambda_j y_j &\leq y_p, \\ \sum_{j \in E} \lambda_j &= 1, \\ \lambda_j &\geq 0, \quad j \in E \end{aligned} \tag{4}$$

Proof: (4) is equivalent the following system:

$$\sum_{j \in E} \lambda_j x_j = x_p,$$

$$\begin{aligned} \sum_{j \in E} \lambda_j y_j + s &= y_p, \\ \sum_{j \in E} \lambda_j &= 1, \\ \lambda_j &\geq 0, \quad j \in E \\ s &\geq 0 \end{aligned} \tag{5}$$

If $s = 0$ then $p \in E$, so DMU_p is output efficient in T_{BCC} and not congested. If $s \neq 0$, according definition 2.2 DMU_p has technical inefficiency and not congested. If (5) has no solution then

$$\sum_{j \in E} \lambda_j x_j < x_p, \quad y_j > y_p \quad (j \in E) \tag{6}$$

One or more inputs of DMU_p must reduce until DMU_p reach to T_{NEW} . So according to (6) and definition 2.1, DMU_p is congested.

4. Numerical Example

In this section, we consider three numerical examples to illustrate our proposed method.

Example1. Consider a technology comprising of seven DMUs, where each uses one inputs to produce one outputs. The input–output data are displayed in Table 1. The statuses of all DMUs are shown in Figure1.

Table 1: Input – output data for Example 1.

DMU	I	O
A	1	0.5
B	2	2
C	3	2
D	5	1
E	4	1
F	4.1	2
G	4.5	1.2

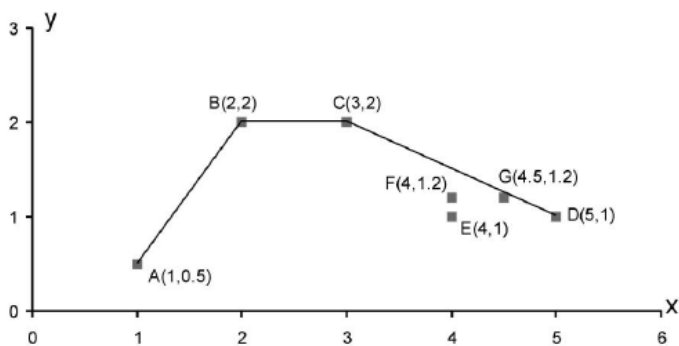


Figure1- Illustration of Data Set in Example1.

The results of the proposed approach have been presented in Table 2. The sets of E, E^+ and E^{++} are as follows:

$$E = \{A, B, C\}, E^+ = \{B\}, E^{++} = \{A, C\}$$

According to Definition 2.1, DMUs D, E, F, G are congested. The output that is maximally possible is the amount of output of DMU C. **Example2.** Consider a

technology comprising of six DMUs, where each uses two inputs to produce one outputs. The input–output data are displayed in Table 3.

The status of all DMUs depicted in Figure 2. Figure 2 illustrates a pyramid with R at its vertex, producing an input of $y = 10$. The output from all other DMUs is $y = 1$.

Table 2: The results of the proposed approach for Example 1.

DMU	Z^*	Z^{**}	$\lambda_j^* \neq 0$ (step2)	Congestion of I_1
A	0	0	-	0
B	0	0	-	0
C	0	1	λ_B	0
D	0.8	-	-	1
E	1	-	-	1
F	0.8	-	-	1.5
G	1	-	-	2

Table 3: Input–output data for Example 2.

DMU	I_1	I_2	O
A	5	1	1
B	1	5	1
C	5	10	1
D	10	5	1
G	7.5	7.5	1
R	5	5	10

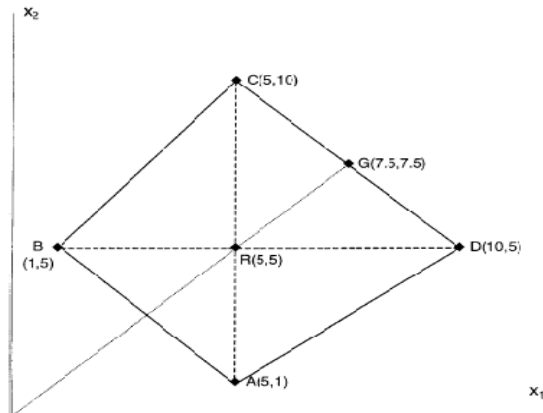


Figure2-Illustraion of Data Set in Example2.

The results of the proposed approach are presented in Table 4. The sets of E, E^+ and E^{++} are as follows:

$$E = \{A, B, R\}, E^+ = \{\}, E^{++} = \{A, B, R\}$$

As shown in **table 4**, DMUs C, G and D are congested. The output that is maximally possible is the amount of output of DMU R.

Example 3. Consider a technology comprising of seven hypothetical DMUs, where each uses two inputs to produce four outputs. The input–output data are displayed in Table 5.

The results of the proposed approach have been presented in Table 6. The sets of E, E^+ and E^{++} are as follows:

$$E = \{2, 5, 6, 7\}, E^+ = \{\}, E^{++} = \{2, 5, 6, 7\}$$

Table 4: The results of the proposed approach for Example 2.

DMU	Z^*	Z^{**}	$\lambda_j^* \neq 0$ (step2)	Congestion of I_1	Congestion of I_2
A	0	0	-	0	0
B	0	0	-	0	0
C	9	-	-	0	5
D	9	-	-	5	0
G	9	-	-	2.5	2.5
R	0	0	-	0	0

Table 5: Input–output data for Example 3.

DMU	I_1	I_2	O_1	O_2	O_3	O_4
1	2	2	2	2	2	2
2	2	2	2	3	2	2
3	2	2	2	2	3	2
4	2	2	2	2	2	3
5	1	1	2	2.5	3	2
6	1	3	2	2	2	4
7	2	1	2	2.5	2.25	3

Table 6: The results of the proposed approach for Example 3.

<i>DMU</i>	Z^*	Z^{**}	$\lambda_j^+ \neq 0$ (step2)	<i>Congestion of I_1</i>	<i>Congestion of I_2</i>
1	1.875	0	-	0.5	0
2	0	0	-	0	0
3	0.5	0	-	1	1
4	0.875	0	-	0.5	0
5	0	0	-	0	0
6	0	0	-	0	0
7	0	0	-	0	0

As seen in **table 6**, DMUs 1, 3 and 4 are congested.

5. Conclusion

In this paper, we proposed a new procedure to evaluate congestion in DEA. Our proposed method is able to identify congestion in the performance of DMUs and it can determine the amount of excessive inputs for congested. The numerical examples demonstrated the compatibility of the proposed approach. Numerical results of the proposed model show that the performance of our model same cooper model to evaluate the congestion in DEA.

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