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The Overall Efficiency in the Presence of Imprecise Adaptable Measures

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Abstract

Traditional data envelopment analysis (DEA) models usually evaluate the efficiency scores of decision making units (DMUs) with precise data from an optimistic point of view where the status of each measure (i.e. input/output) is certain. However, there are occasions in real world applications that measures can play both input and output roles in an imprecise environment. In the current study, measures with two roles, input and output, are called "adaptable measures". This paper proposes a DEA-based approach for estimating the performance of DMUs where adaptable and fuzzy data exist. Indeed, efficiency scores are calculated from two aspects, optimistic and pessimistic, when there are adaptable and fuzzy data. Two different efficiency scores are integrated into a geometric average efficiency. Thus, the overall efficiency is calculated and adaptable variables are split into input and output variables in evaluating the efficiency of each DMU. A numerical example is used to illustrate the approach.

Keywords*:* Data envelopment analysis (DEA), Efficiency, Fuzzy data, Adaptable variable.

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1. Introduction

Data envelopment analysis (DEA), initially suggested by Charnes et al. [1], is a nonparametric technique to evaluate the relative efficiency of decision making units (DMUs) that use multiple inputs to produce multiple outputs. In conventional DEA models, the efficiency scores of DMUs with crisp inputs/outputs are usually calculated from an optimistic point of view. Wang et al. [2] measured the performance of DMUs with precise data from different points of view, optimistic and pessimistic, and calculated the overall efficiency by using the geometric average efficiency. However, in the real world, there are situations that imprecise data exist. In the DEA literature, there are methods for assessing the efficiency of firms in the presence of vague inputs and outputs.

One of the popular approaches for the efficiency evaluation in the presence of imprecise information is fuzzy DEA methods. Firstly, Sengupta [3] suggested a fuzzy DEA model to incorporate fuzzy data via the tolerance levels definition. Afterwards, various DEA-based approaches are introduced to measure the performance of DMUs with fuzzy factors. Hatami-Marbini et al. [4] reviewed the fuzzy data envelopment analysis literature over 20 years. Also, Emrouznejad and Tavana [5] classified the application of fuzzy set theory in DEA into six groups: the tolerance approach [3, 6], the possibility approach [7, 8], the α -level based approach [9, 10], the fuzzy arithmetic [11, 12] the fuzzy ranking approach [13], and the fuzzy random/type-2 fuzzy set [14, 15]. Readers can refer to Hatami-Marbini et al. [4] and Emrouznejad & Tavana [5] for more information.

Furthermore, the input/output status of measures has been usually specified in the conventional DEA models. Nonetheless, sometimes the input/output status of a variable is uncertain. It means a variable can be considered as both an input and output. In this study, variables with uncertain status are defined as adaptable variables. In the DEA literature, there are some contexts with studying the subject. Cook and Zhu [16] proposed an approach to classifying flexible measures (i.e. measures that can play either input or output roles). Then, Toloo [17] introduced a modified model to determine the status of flexible measures. Afterwards, Amirteimoori and Emrouznejad [18] defined a new production possibility set (PPS) and a new model for measuring the efficiency of DMUs. Furthermore, Toloo [19], Amirteimoori and Emrouznejad [20], Kordrostami and Jahani [21], Amirteimoori et al. [22], Amirteimoori et al. [23], Toloo [24] are some contexts, that deal with the issue. Amirteimoori et al. [22] proposed a flexible slack-based measure for classifying inputs and outputs. Also, Kordrostami and Noveiri [21] suggested an approach to evaluate the efficiency of DMUs in the presence of flexible and negative measures. In aforementioned papers, flexible measures are finally considered as input or output while all measures are deemed as precise and crisp factors. Kordrostami et al. [25] and Kordrostami and Noveiri [26] proposed approaches for measuring the efficiency in the presence of imprecise and flexible measures. Nonetheless, sometimes a variable can be taken as both an input and an output where fuzzy factors present. Amirteimoori et al. [23] considered precise recyclable outputs into the production process. To illustrate, recyclable outputs are products that some portion of them may be considered as inputs again. Indeed, it seems that adaptable variables can be taken from two aspects. First, they can be deemed as variables same as recyclable outputs. Second, variables with two roles both input and output. In DEA contexts, variables with both input and output roles are so-called dual-role factors [27, 28] and flexible measures. Nevertheless, in those models, finally a dual-role factor can play only one role, input or output while in this study an adaptable measure can play both input and output roles. Indeed, the present models

have some drawbacks. Firstly, they calculate the efficiency from an optimistic viewpoint. Secondly, the majority of them consider precise inputs and outputs. Finally, the role of a variable is determined as an input or an output.
To tackle the aforementioned drawbacks, in the current paper, the overall efficiency of DMUs is calculated where adaptable and fuzzy data present. Actually, the fuzzy expected value models are introduced to estimate the optimistic and pessimistic efficiencies of DMUs. Then, the efficiency scores are integrated as a geometric average efficiency. Moreover, it is indicated how much of adaptable variable is considered as input and how much as output. In sum, fuzzy expected value DEA models are introduced to specify the efficiency of DMUs where fuzzy adaptable measures exist.
The paper is organized as follows. Section 2 reviews some basic concepts of fuzzy variables, the fuzzy expected value and dual-role factors. In Section 3, a DEA-based methodology is developed that is designed to handle situations that adaptable and fuzzy factors present. A numerical example illustrates and clarifies the proposed approach in Section 4. Conclusions appear in Section 5.

2. Basic concepts and fundamentals

Firstly, basic concepts of fuzzy numbers and related issues are provided in this section. It is pointed out that adaptable measures, which are under consideration in this study, have the similar definition of dual-role factors. Actually, a dual-role factor can play both roles, input and output, simultaneously. So, dual-role factors are also described briefly.

2.1. Fuzzy variables

Definition 2.1. A fuzzy set A in X is characterized by a membership function $\mu_A(x)$ which associates with each point in X a real number in the interval $[0,1]$. $\mu_A(x)$ indicates the degree of membership of x in A.[29]

Definition 2.2. A fuzzy subset B of the real numbers R is convex if and only if for $\forall x, y \in R, \forall \lambda \in [0,1]$,
$\mu_B(\lambda x + (1-\lambda)y) \geq \min(\mu_B(x), \mu_B(y))$.

Definition 2.3. Fuzzy numbers are convex normalized fuzzy set of real numbers in which $\mu(x)$ is piecewise continuous.

In this study trapezoidal and triangular fuzzy variables are utilized because of the wide applications of them in practical problems. A trapezoidal fuzzy variable $\alpha = (a,b,c,d)$ is a fuzzy variable with the following membership function

$$\mu_\alpha(x) = \begin{cases} (x-a)/(b-a) & if \quad a \leq x \leq b, \\ 1 & if \quad b \leq x \leq c, \\ (x-d)/(c-d) & if \quad c \leq x \leq d, \\ 0 & otherwise. \end{cases}$$

A triangular fuzzy variable $\beta = (a,b,c)$ is a fuzzy variable with a membership function μ_β as follows:

$$\mu_\beta(x) = \begin{cases} (x-a)/(b-a) & if \quad a \leq x \leq b, \\ (x-c)/(b-c) & if \quad b \leq x \leq c, \\ 0 & otherwise. \end{cases}$$

Definition 2.4. [30] The expected value of a trapezoidal fuzzy variable $\alpha = (a,b,c,d)$ is defined as

$$E(\alpha) = \frac{(a+b+c+d)}{4}$$

Also, the expected value of a triangular fuzzy number $\beta = (a,b,c)$ is shown by

$$E(\alpha) = \frac{(a+2b+c)}{4}.$$

Proposition 1. [30, 31] Assume f and g are fuzzy variables and a and b are real numbers. Thus, we have
$E(af + bg) = aE(f) + bE(g).$

2.3. Dual-role factors

In traditional DEA models, the performance of DMUs with a specified set of inputs and outputs is evaluated. Nevertheless, there are occasions in real-world problems that a measure can play the role of both an input and an output, simultaneously. In the literature, these factors are so-called flexible measures and dual-role factors. Factors like trainees in organizations, awards to scholars, and outages in power plants are deemed as dual-role factors. For instance, according to Cook et al. [27] "the number of nurse trainees on staff in a study of hospital efficiency constitutes an output measure for a hospital, but at the same time it is an important component of the hospital's total staff component, hence it is an input".

3. Efficiency measurement of DMUs with fuzzy adaptable variables

In this section, an approach based on DEA is introduced to estimate the overall efficiency of DMUs when fuzzy adaptable variables exist. For this purpose, the efficiency scores of DMUs are firstly calculated from two aspects, optimistic and pessimistic viewpoints.

Assume, there are n DMUs, DMU_j, $(j=1,...,n)$ with m inputs $\tilde{x}_{ij}$ $(i=1,...,m)$, s outputs $\tilde{y}_{rj}$ $(r=1,...,s)$ and k adaptable variables $\tilde{z}_{tj}$ $(t=1,...,k)$. In this case, inputs, outputs and adaptable variables are considered as trapezoidal fuzzy data, i.e. $\tilde{x}_{ij}=(x_{ij}^L,x_{ij}^M,x_{ij}^N,x_{ij}^U)$, $\tilde{y}_{rj}=(y_{rj}^L,y_{rj}^M,y_{rj}^N,y_{rj}^U)$ and $\tilde{z}_{tj}=(z_{tj}^L,z_{tj}^M,z_{tj}^N,z_{tj}^U)$, respectively. According to Liu and Liu [30] and definition 2.4, we have

$E(\tilde{x}_{ij})=(1/4)(x_{ij}^L+x_{ij}^M+x_{ij}^N+x_{ij}^U)$,

$E(\tilde{y}_{rj})=(1/4)(y_{rj}^L+y_{rj}^M+y_{rj}^N+y_{rj}^U)$, and

$E(\tilde{z}_{tj})=(1/4)(z_{tj}^L+z_{tj}^M+z_{tj}^N+z_{tj}^U)$.

With considering the continuous variable d_t $0\le d_t\le 1$, we define the relative efficiency of DMU_o, the unit under evaluation, as follows:

$$e_o^{Opt}=\frac{E(\sum_{r=1}^{s}u_r\tilde{y}_{ro}+\sum_{t=1}^{k}w_t d_t\tilde{z}_{to})}{E(\sum_{i=1}^{m}v_i\tilde{x}_{io}+\sum_{t=1}^{k}w_t(1-d_t)\tilde{z}_{to})} \quad (1)$$

That v_i, u_r, and w_t are weights of input, output and adaptable variables, respectively. Also, d_t is used to determine the portion of adaptable variable $\tilde{z}_{tj}$ that is taken as output. Due to proposition 1, e_o^{Opt} can be rewritten as follows:

$$e_o^{Opt}=\frac{\sum_{r=1}^{s}u_r E(\tilde{y}_{ro})+\sum_{t=1}^{k}w_t d_t E(\tilde{z}_{to})}{\sum_{i=1}^{m}v_i E(\tilde{x}_{io})+\sum_{t=1}^{k}w_t(1-d_t)E(\tilde{z}_{to})} \quad (2)$$

Thus, the following fractional nonlinear programming is proposed for evaluating the efficiency of DMU_o from optimistic point of view:

$$\begin{aligned} Max\ \ & e_o^{best}=\frac{\sum_{r=1}^{s}u_r E(\tilde{y}_{ro})+\sum_{t=1}^{k}w_t d_t E(\tilde{z}_{to})}{\sum_{i=1}^{m}v_i E(\tilde{x}_{io})+\sum_{t=1}^{k}w_t(1-d_t)E(\tilde{z}_{to})} \\ s.t.\ \ & \frac{\sum_{r=1}^{s}u_r E(\tilde{y}_{rj})+\sum_{t=1}^{k}w_t d_t E(\tilde{z}_{tj})}{\sum_{i=1}^{m}v_i E(\tilde{x}_{ij})+\sum_{t=1}^{k}w_t(1-d_t)E(\tilde{z}_{tj})}\le 1,\ j=1,...,n, \\ & v_i,u_r,w_t\ge\varepsilon,\forall i,r,t, \\ & 0\le d_t\le 1,\forall t. \end{aligned} \quad (3)$$

That ε is the non-Archimedean infinitesimal.

Note that at this stage the continuous variable d_t in (3) is used to indicate the portion of the expected value of adaptable variable, $E(\tilde{z}_{tj})$, that is taken as the expected value of output. Model (3) is transformed into a fractional linear program by using definition 2.4 and the change of variables $w_t d_t=\lambda_t$. Therefore, model (3) is replaced by the following problem:

$$Max\ \ e_o^{best}=\frac{\sum_{r=1}^{s}u_r(y_{ro}^L+y_{ro}^M+y_{ro}^N+y_{ro}^U)+\sum_{t=1}^{k}\lambda_t(z_{to}^L+z_{to}^M+z_{to}^N+z_{to}^U)}{\sum_{i=1}^{m}v_i(x_{io}^L+x_{io}^M+x_{io}^N+x_{io}^U)+\sum_{t=1}^{k}w_t(z_{to}^L+z_{to}^M+z_{to}^N+z_{to}^U)-\sum_{t=1}^{k}\lambda_t(z_{to}^L+z_{to}^M+z_{to}^N+z_{to}^U)} \quad (4)$$

$$s.t.\ \frac{\sum_{r=1}^{s}u_r(y_{rj}^L+y_{rj}^M+y_{rj}^N+y_{rj}^U)+\sum_{t=1}^{k}\lambda_t(z_{tj}^L+z_{tj}^M+z_{tj}^N+z_{tj}^U)}{\sum_{i=1}^{m}v_i(x_{ij}^L+x_{ij}^M+x_{ij}^N+x_{ij}^U)-\sum_{t=1}^{k}\lambda_t(z_{tj}^L+z_{tj}^M+z_{tj}^N+z_{tj}^U)+\sum_{t=1}^{k}w_t(z_{tj}^L+z_{tj}^M+z_{tj}^N+z_{tj}^U)}\le 1,\ j=1,\dots,n,$$

$$v_i,u_r,w_t\ge\varepsilon,\ \forall i,r,t,$$

$$0\le\lambda_t\le w_t,\ \forall t.$$

By utilizing the Charnes and Cooper [32] transformation, i.e.

$$\sum_{i=1}^{m}v_i(x_{io}^L+x_{io}^M+x_{io}^N+x_{io}^U)+\sum_{t=1}^{k}w_t(z_{to}^L+z_{to}^M+z_{to}^N+z_{to}^U)-\sum_{t=1}^{k}\lambda_t(z_{to}^L+z_{to}^M+z_{to}^N+z_{to}^U)=\frac{1}{\rho}$$

$$\bar{u}_r=\rho u_r,\ \bar{v}_i=\rho v_i,\ \bar{w}_t=\rho w_t,\ \bar{\lambda}_t=\rho\lambda_t$$

model (4) changes to the following linear programming:

$$Max\ \ e_o^{best}=\sum_{r=1}^{s}\bar{u}_r(y_{ro}^L+y_{ro}^M+y_{ro}^N+y_{ro}^U)+\sum_{t=1}^{k}\bar{\lambda}_t(z_{to}^L+z_{to}^M+z_{to}^N+z_{to}^U) \quad (5)$$

$$s.t.\ \sum_{i=1}^{m}\bar{v}_i(x_{io}^L+x_{io}^M+x_{io}^N+x_{io}^U)+\sum_{t=1}^{K}\bar{w}_t(z_{to}^L+z_{to}^M+z_{to}^N+z_{to}^U)-\sum_{t=1}^{K}\bar{\lambda}_t(z_{to}^L+z_{to}^M+z_{to}^N+z_{to}^U)=1,$$

$$\sum_{r=1}^{s}\bar{u}_r(y_{rj}^L+y_{rj}^M+y_{rj}^N+y_{rj}^U)+\sum_{t=1}^{k}\bar{\lambda}_t(z_{tj}^L+z_{tj}^M+z_{tj}^N+z_{tj}^U)-\sum_{i=1}^{m}\bar{v}_i(x_{ij}^L+x_{ij}^M+x_{ij}^N+x_{ij}^U)-\sum_{t=1}^{K}\bar{w}_t(z_{tj}^L+z_{tj}^M+z_{tj}^N+z_{tj}^U)+\sum_{t=1}^{K}\bar{\lambda}_t(z_{tj}^L+z_{tj}^M+z_{tj}^N+z_{tj}^U)\le 0,\ j=1,\dots,n,$$

$$\bar{v}_i,\bar{u}_r,\bar{w}_t\ge\varepsilon,\ \forall i,r,t,$$

$$0\le\bar{\lambda}_t\le\bar{w}_t,\ \forall t.$$

Notice that the efficiency obtained from model (5) is between 0 and 1. That is $0<e_o^{best}\le 1$.

Definition 3.1. DMU_o is said optimistic efficient in model (5) if and only if $e_o^{best}=1$.

Now for evaluating the efficiency of DMU_o from pessimistic point of view, model (3) can be substituted by the following program:

$$Min\ \ e_o^{worst}=\frac{\sum_{r=1}^{s}u_rE(\tilde{y}_{ro})+\sum_{t=1}^{k}w_td_tE(\tilde{z}_{to})}{\sum_{i=1}^{m}v_iE(\tilde{x}_{io})+\sum_{t=1}^{k}w_t(1-d_t)E(\tilde{z}_{to})}$$

$$s.t.\ \frac{\sum_{r=1}^{s}u_rE(\tilde{y}_{rj})+\sum_{t=1}^{k}w_td_tE(\tilde{z}_{tj})}{\sum_{i=1}^{m}v_iE(\tilde{x}_{ij})+\sum_{t=1}^{k}w_t(1-d_t)E(\tilde{z}_{tj})}\ge 1,\ j=1,\dots,n, \quad (6)$$

$$v_i,u_r,w_t\ge\varepsilon,\forall i,r,t,$$

$$0\le d_t\le 1,\forall t.$$

As aforementioned in the case of the optimistic viewpoint, model (6) can be transformed to the following linear programming by using definition 2.4, the change of variables $w_td_t=\lambda_t$ and the Charnes and Cooper [32] transformation:

$$Min\ \ e_o^{worst}=\sum_{r=1}^{s}\bar{u}_r(y_{ro}^L+y_{ro}^M+y_{ro}^N+y_{ro}^U)+\sum_{t=1}^{k}\bar{\lambda}_t(z_{to}^L+z_{to}^M+z_{to}^N+z_{to}^U) \quad (7)$$

$$s.t.\ \sum_{i=1}^{m}\bar{v}_i(x_{io}^L+x_{io}^M+x_{io}^N+x_{io}^U)+\sum_{t=1}^{K}\bar{w}_t(z_{to}^L+z_{to}^M+z_{to}^N+z_{to}^U)-\sum_{t=1}^{K}\bar{\lambda}_t(z_{to}^L+z_{to}^M+z_{to}^N+z_{to}^U)=1,$$

$$\sum_{r=1}^{s}\bar{u}_r(y_{rj}^L+y_{rj}^M+y_{rj}^N+y_{rj}^U)+\sum_{t=1}^{k}\bar{\lambda}_t(z_{tj}^L+z_{tj}^M+z_{tj}^N+z_{tj}^U)-\sum_{i=1}^{m}\bar{v}_i(x_{ij}^L+x_{ij}^M+x_{ij}^N+x_{ij}^U)-\sum_{t=1}^{K}\bar{w}_t(z_{tj}^L+z_{tj}^M+z_{tj}^N+z_{tj}^U)+\sum_{t=1}^{K}\bar{\lambda}_t(z_{tj}^L+z_{tj}^M+z_{tj}^N+z_{tj}^U)\ge 0,\ j=1,\dots,n,$$

$$\bar{v}_i,\bar{u}_r,\bar{w}_t\ge\varepsilon,\ \forall i,r,t,$$

$$0\le\bar{\lambda}_t\le\bar{w}_t,\ \forall t.$$

The efficiency resulted from (7) is greater than or equal to 1, i.e. $e_o^{worst}\ge 1$.

Definition 3.2. DMU_o is said pessimistic infficient in model (7) if and only if $e_o^{worst}=1$.

In the existence of triangular fuzzy variables, $\tilde{x}_{ij}=(x_{ij}^L,x_{ij}^M,x_{ij}^U)$, $\tilde{y}_{rj}=(y_{rj}^L,y_{rj}^M,y_{rj}^U)$ and $\tilde{z}_{tj}=(z_{tj}^L,z_{tj}^M,z_{tj}^U)$, models (5) and (7) are substituted by the following models:

$$
\begin{aligned}
Max\ \ & e_o^{best}=\sum_{r=1}^{s}\bar{u}_r(y_{ro}^L+2y_{ro}^M+y_{ro}^U)+\sum_{t=1}^{k}\bar{\lambda}_t(z_{to}^L+2z_{to}^M+z_{to}^U)\\
s.t.\ \ & \sum_{i=1}^{m}\bar{v}_i(x_{io}^L+2x_{io}^M+x_{io}^U)+\sum_{t=1}^{K}\bar{w}_t(z_{to}^L+2z_{to}^M+z_{to}^U)-\\
& \sum_{t=1}^{K}\bar{\lambda}_t(z_{to}^L+2z_{to}^M+z_{to}^U)=1,\\
& \sum_{r=1}^{s}\bar{u}_r(y_{rj}^L+2y_{rj}^M+y_{rj}^U)+\sum_{t=1}^{k}\bar{\lambda}_t(z_{tj}^L+2z_{tj}^M+z_{tj}^U)\\
& -\sum_{i=1}^{m}\bar{v}_i(x_{ij}^L+2x_{ij}^M+x_{ij}^U)-\sum_{t=1}^{K}\bar{w}_t(z_{tj}^L+2z_{tj}^M+z_{tj}^U)\\
& +\sum_{t=1}^{K}\bar{\lambda}_t(z_{tj}^L+2z_{tj}^M+z_{tj}^U)\le 0, j=1,\dots,n,\\
& \bar{v}_i,\bar{u}_r,\bar{w}_t\ge\varepsilon,\ \forall i,r,t,\\
& 0\le\bar{\lambda}_t\le\bar{w}_t,\ \forall t.
\end{aligned}
\tag{8}
$$

and

$$
\begin{aligned}
Min\ \ & e_o^{worst}=\sum_{r=1}^{s}\bar{u}_r(y_{ro}^L+2y_{ro}^M+y_{ro}^U)+\sum_{t=1}^{k}\bar{\lambda}_t(z_{to}^L+2z_{to}^M+z_{to}^U)\\
s.t.\ \ & \sum_{i=1}^{m}\bar{v}_i(x_{io}^L+2x_{io}^M+x_{io}^U)+\sum_{t=1}^{K}\bar{w}_t(z_{to}^L+2z_{to}^M+z_{to}^U)\\
& -\sum_{t=1}^{K}\bar{\lambda}_t(z_{to}^L+2z_{to}^M+z_{to}^U)=1,\\
& \sum_{r=1}^{s}\bar{u}_r(y_{rj}^L+2y_{rj}^M+y_{rj}^U)+\sum_{t=1}^{k}\bar{\lambda}_t(z_{tj}^L+2z_{tj}^M+z_{tj}^U)\\
& -\sum_{i=1}^{m}\bar{v}_i(x_{ij}^L+2x_{ij}^M+x_{ij}^U)-\sum_{t=1}^{K}\bar{w}_t(z_{tj}^L+2z_{tj}^M+z_{tj}^U)\\
& +\sum_{t=1}^{K}\bar{\lambda}_t(z_{tj}^L+2z_{tj}^M+z_{tj}^U)\ge 0, j=1,\dots,n,\\
& \bar{v}_i,\bar{u}_r,\bar{w}_t\ge\varepsilon,\ \forall i,r,t,\\
& 0\le\bar{\lambda}_t\le\bar{w}_t,\ \forall t.
\end{aligned}
\tag{9}
$$

In the next stage, a geometric average of optimistic and pessimistic efficiencies is used to calculate the overall efficiency of DMUs, i.e.

$$e_j^{Overall}=\sqrt{e_j^{best}\times e_j^{worst}},\ j=1,\dots,n. \tag{10}$$

Moreover, portions of adaptable variables are estimated by using the arithmetic average of values that are obtained from both viewpoints. It means

$$
\begin{aligned}
\bar{d}_j^{overall} &= \frac{(1-d_j^{best})+(1-d_j^{worst})}{2},\ j=1,\dots,n\\
d_j^{overall} &= \frac{d_j^{best}+d_j^{worst}}{2},\ j=1,\dots,n.
\end{aligned}
\tag{11}
$$

To explain, for integrating optimistic and pessimistic efficiencies and determining the portions of adaptable measures, the averages of values are calculated.

4. An example

For illustration and clarification the approach, a numerical example is provided. Suppose there are 20 DMUs that each DMU consists of two inputs, one adaptable measure and one output. Data can be seen in Table 1. Data has been given as triangular fuzzy numbers. At first, models (8) and (9) are calculated. The results have been shown in Table 2. The results obtained from model (8) are provided in columns 4-6. As can be found, 6 DMUs, DMU4, DMU5, DMU6, DMU7, DMU16 and DMU17 are optimistic efficient. Also, columns 7-9 show the results obtained from model (9). Column 7 indicates 4 DMUs, DMU1, DMU 11, DMU16 and DMU 20 are pessimistic inefficient. Then, the geometric average efficiencies are computed that are obtained via integrating two different efficiencies. Column 4 of Table 3 indicates them. Afterwards, the arithmetic averages of values of d and $1-d$, which are obtained from both viewpoints, are calculated and shown in columns 6 and 7 of Table 3, respectively. To compare the results, we consider the adaptable measure as an output factor and calculate the optimistic and pessimistic efficiency scores using the fuzzy expected value approach (Wang and Chin's models [33]. Results are shown in columns 2 and 3 of Table 2. Column 2 indicates the optimistic efficiency results when the adaptable measure is considered as an output. The pessimistic efficiency scores when the adaptable measure is deemed as an output are given in column 3. As can be seen, $e_j^{WC.best}\le e_j^{best}$ and $e_j^{WC.worst}\ge e_j^{worst}$. To illustrate, e_j^{best} and e_j^{worst} obtain the results closer than to 1 in contrast to $e_j^{WC.best}$ and $e_j^{WC.worst}$. Column 2 of Table 3 describes the average efficiency, calculated by the geometric average, when the adaptable measure is considered as an output. The average efficiencies comparisons of two approaches do not display an especial pattern. Actually, the varied results have been obtained due to differences between optimistic and pessimistic efficiency scores. Columns 3 and 5 of Table 3 show the ranking of DMUs by $e_j^{WC.average}$ and $e_j^{average}$, respectively. Interestingly, DMU 4 has obtained the best ranking in both approaches.

To summary, incorporating adaptable measures in the efficiency evaluation changes the results.

Table 1. Fuzzy data

#DMU	Input 1	Input 2	Adaptable measure	Output 1
1	(5, 5, 5)	(470, 475, 480)	(34, 35, 36)	(194, 195, 196)
2	(10, 11, 13)	(314, 315, 318)	(48, 50, 52)	(196,197, 200)
3	(4, 6, 9)	(332, 334, 335)	(54, 54, 56)	(221, 223, 223)
4	(5, 7, 10)	(195, 195, 195)	(41, 43, 43)	(190, 190, 190)
5	(4, 5, 9)	(257, 258, 259)	(58, 59, 59)	(207, 208, 209)
6	(2, 4, 5)	(258, 261, 262)	(55, 55, 55)	(192, 196, 196)
7	(7, 9, 10)	(266, 267, 270)	(31, 32, 33)	(204, 205, 206)
8	(9, 13, 15)	(349, 350, 351)	(62, 64, 64)	(198, 199, 200)
9	(9, 11, 12)	(349, 352, 355)	(67, 68, 70)	(222, 223, 225)
10	(5, 6, 8)	(321, 323, 323)	(32, 33, 34)	(185, 185, 187)
11	(20, 20, 20)	(326, 328, 328)	(62, 64, 64)	(169, 170, 173)
12	(15, 17, 21)	(339, 340, 341)	(24, 24, 25)	(217, 218, 218)
13	(15, 17, 19)	(306, 308, 310)	(28, 30, 30)	(168, 170, 171)
14	(16, 17, 18)	(331, 333, 334)	(46,47,48)	(198, 200, 201)
15	(9, 12, 16)	(315, 316, 318)	(39, 41, 41)	(186, 187, 189)
16	(17, 17, 17)	(244, 245, 247)	(14, 14, 14)	(170, 170, 170)
17	(3, 4, 5)	(310, 310, 310)	(19, 20, 21)	(183, 185, 185)
18	(16, 19, 24)	(236, 237, 238)	(36, 38, 38)	(170, 170, 170)
19	(16, 17, 18)	(279, 281, 283)	(25, 25, 27)	(182, 183, 184)
20	(18, 21, 24)	(363, 363, 364)	(22, 23, 25)	(167, 168, 170)

Table 2. Results for an example

#DMU	$e_j^{WC.best}$	$e_j^{WC.worst}$	e_j^{best}	d_j^{best}	$1-d_j^{best}$	e_j^{worst}	d_j^{worst}	$1-d_j^{worst}$
1	0.749537	1	0.920967	0.48915	0.51085	1	1	0
2	0.709686	1.427614	0.954065	0.50165	0.49835	1.050989	0.48705	0.51295
3	0.842279	1.588662	0.974514	0.48388	0.51612	1.070293	0.48576	0.51424
4	1	2.213471	1	0.48385	0.51615	1.098806	0.48964	0.51036
5	1	1.903211	1	0.51206	0.48794	1.080501	0.48516	0.51484
6	1	1.804562	1	0.51204	0.48796	1.068744	0.48613	0.51387
7	0.823915	1.746587	1	0	1	1.083092	0.51502	0.48498
8	0.796623	1.296539	0.941921	0.50213	0.49787	1.026552	0.48315	0.51685
9	0.851391	1.465507	0.955313	0.51492	0.48508	1.04998	0.48184	0.51816
10	0.720766	1.36826	0.965283	0.48147	0.51853	1.049929	0.49567	0.50433
11	0.851219	1.06428	0.936521	0.50212	0.49788	1	0.49141	0.50859
12	0.657184	1.162342	0.984127	0.4872	0.5128	1.034216	0.52498	0.47502
13	0.565594	1.197453	0.953741	0.48826	0.51174	1.026144	0.4961	0.5039
14	0.64147	1.317405	0.945419	0.50157	0.49843	1.041044	0.49368	0.50632
15	0.60767	1.339817	0.950203	0.48371	0.51629	1.043148	0.49465	0.50535
16	0.711293	1	1	0	1	1	1	0
17	0.886777	1.057571	1	0	1	1.022523	0.52038	0.47962
18	0.736174	1.088617	0.970496	0.48545	0.51455	1.047969	0.49517	0.50483
19	0.668304	1.34366	0.979607	0.49001	0.50999	1.048339	0.51216	0.48784
20	0.475297	1	0.937407	0.49053	0.50947	1	1	0

Table 3. Averages of results and ranking

#DMU	$e_j^{WC.overall}$	Rank	$e_j^{overall}$	Rank	$d_j^{overall}$	$\bar{d}_j^{overall}$
1	0.865758	17	0.95967	20	0.744575	0.255425
2	1.006557	8	1.001355	12	0.49435	0.50565
3	1.156761	5	1.021281	5	0.48482	0.51518
4	1.487774	1	1.048239	1	0.486745	0.513255
5	1.379569	2	1.039472	3	0.49861	0.50139
6	1.34334	3	1.033801	4	0.499085	0.500915
7	1.1996	4	1.040717	2	0.25751	0.74249
8	1.016294	7	0.983326	17	0.49264	0.50736
9	1.117014	6	1.001529	11	0.49838	0.50162
10	0.993074	9	1.006717	10	0.48857	0.51143
11	0.951806	11	0.96774	19	0.496765	0.503235
12	0.873998	16	1.008861	8	0.50609	0.49391
13	0.822966	19	0.98928	16	0.49218	0.50782
14	0.91928	13	0.99208	15	0.497625	0.502375
15	0.902312	14	0.995591	14	0.48918	0.51082
16	0.843382	18	1	13	0.5	0.5
17	0.968416	10	1.011199	7	0.26019	0.73981
18	0.895216	15	1.008489	9	0.49031	0.50969
19	0.947615	12	1.01339	6	0.501085	0.498915
20	0.689418	20	0.968198	18	0.745265	0.254735

5. Conclusions

In real world applications, there are situations that the status of imprecise measures is uncertain from input and/or output viewpoints. Also, conventional DEA models usually evaluate the efficiency from an optimistic point of view. In the current paper, the efficiency scores of DMUs from two aspects, optimistic and pessimistic, have been evaluated where these adaptable variables exist in a fuzzy environment. Then, the overall efficiency of DMUs has been calculated by using the geometric average of efficiencies. Actually, the fuzzy expected value has been used to handle fuzzy DEA models introduced herein in order to handle situations that imprecise and adaptable measures exist.

A numerical example has been used to illustrate the approach. Analysis of the results has shown that the average efficiency scores obtained have changed by incorporating fuzzy adaptable measures. Also, the overall efficiency calculation and the consideration of both points of view, optimistic and pessimistic, result in more rational and realistic consequences.

Models developed in the current study have been based on constant return to scale technology. However, the approach can be extended for variable returns to scale technology. Moreover, further research might be concentrated on the investigation of the supplier selection problem where adaptable and imprecise variables present. Also, it seems more research is required in determining the overall efficiency in the existence of fuzzy adaptable measures and undesirable factors.

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