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Data Envelopment Analysis Models In The Presence Of Ratio Data and Non-Discretionary Factors

Azam Ahmadzadeh*

Department of Mathematics, Sari Branch, Islamic Azad University Sari, Iran

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Abstract
The traditional data envelopment analysis (DEA) models for the observations containing ratio data as input or output may result incorrect efficiency scores. To overcome this shortcoming, a set of modified DEA models has been presented in this paper by taking into account the correct convexity of decision making units (DMUs) when a ratio variable is included in the assessment model.

Keywords: Data envelopment analysis; Efficiency; Non-discretionary factors; Ratio data; Convexity.

*. Email: azamahmadzadeh66@yahoo.com

1. Introduction

Data envelopment analysis (DEA) has been proven as a powerful and attractive tool for researchers in several fields, especially operations research, applied mathematics, information management and economics. The standard models of (DEA) assume that all inputs and outputs are controlled by managers or other users. But in real conditions there are also some non-discretionary (ND) inputs and outputs which are not under the control of managers and their efficiency evaluation should be paid attention to. Standard DEA models were originally presented by Charnes et al [1] and Banker et al [2] but they did not pay attention to (ND) inputs and outputs. Banker and Morey (BM) et al [3] suggested that these models be extended to a suitable model including (ND) factors. In addition, they presented constant (CRS) and variable (VRS) returns to scale models in the presence of (ND) and discretionary factor. They assumed that the factors are either fully discretionary or fully (ND). In DEA, the absolute data are discussed mostly, but there are some reports in which the researchers used the ratio data more than absolute data as input (input ratio) and output (output ratio). Hollingsworth and Smith et al [4] illustrate that when data are in the form of ratio, the CCR models of Charnes and Cooper and Rhodes et al [1] have no efficiency; instead its BCC models of Banker, Charnes and Cooper et al [4] should not be used. Emrouznejad and Amin et al [5] demonstrated that using the standard DEA models for the observations containing ratio data as input or output may result incorrect efficiency scores and then a set of modified DEA models taking into account the correct convexity of DMUs when a ratio variable is included in the assessment model, has been proposed by them. This paper develops (ND) and discretionary model of (BM) in input oriented when the one of output is ratio and if there are many output ratios data, this model has no efficiency, but using the convexity consideration, presented by Emrouznejad, the suitable model can be written.

2. ND Factors In DEA

The non-discretionary (ND) variables can be classified into external ND factors, or internal ND factors, depending on whether they are allowed to define the production possibilities set (PPS) or not. Internal ND factors are those factors that can be considered as part of the production process and therefore should be considered in the definition of the PPS. External ND factors are those factors that affect the production process but cannot be considered as a part of it, and therefore they should not be allowed to define the PPS. This section presents the proposed DEA model by Banker and Morey [2] to treat internal ND factors. The Banker and Morey (BM) [2] input-oriented model defined in a variable returns to scale (VRS) technology is shown in (2.1), where D is the set of discretionary inputs, ND is the set of ND inputs, and jo is the unit under assessment.

$$
\begin{aligned}
&\min \theta \\
&\text{s.t.} \sum_{j=1}^{n} \lambda_j \, x_{ij} - \theta \, \mathrm{x_{ijo}} \le 0, \ i \in D \\
&\sum_{j=1}^{n} \lambda_j \, x_{ij} - \mathrm{x_{ijo}} \le 0, \ i \in ND \qquad (2.1) \\
&\sum_{j=1}^{n} \lambda_j \, y_{rj} \ge \mathrm{y_{rjo}} \ , \ \mathrm{r} = 1,2,\dots,\mathrm{s} \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ \ge 0, \quad \mathrm{j} = 1,2,\dots,\mathrm{n}.
\end{aligned}
$$

This model differs from the traditional VRS model of Banker [2] in that the contraction factor θ is associated only with discretionary inputs.

3. ND DEA Model With Ratio Data

In this section, the BM input-oriented model (2.1) is modified for situations that at least one of the output data is in the form of ratio. Assume that for model (2.1), the k−th output of outputs is in the form of ratio, where $1 \leq k \leq s$. Suppose that for unit j contain numerator and denominator of and respectively, i.e. $y_{kj} = \frac{n_{kj}}{d_{kj}}$, $j = 1,2, \dots, n$.

In the case of numerator and denominators of the output-ratio variables as presented separated output and input variables, the BM-VRS input-oriented model (2.1) in the presence of this output ratio should be rewritten as follows:

$$\min \theta$$

$$\text{s.t.} \sum_{j=1}^{n} \lambda_j \; x_{ij} - \theta \, \mathrm{x_{ijo}} \leq 0, \qquad i \in D$$

$$\sum_{j=1}^{n} \lambda_j \; x_{ij} - \mathrm{x_{ijo}} \leq 0, \;\; i \in ND \qquad (3.1)$$

$$\sum_{j=1}^{n} \lambda_j \, y_{rj} \geq \mathrm{y_{rjo}} \;\;, \;\; r = 1,2, \dots, s \;, r \neq k$$

$$\sum_{j=1}^{n} \lambda_j \, n_{kj} \geq \mathrm{n_{kjo}} \quad , r = k$$

$$\sum_{j=1}^{n} \lambda_j \; d_{kj} - \theta \, \mathrm{d_{kjo}} \leq 0,$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \;\; \geq 0, \qquad j = 1,2, \dots, n.$$

It can be seen that the generalized model (3.1) evaluates the efficiency score of the DMU under against n DMUs each of them contains m+1 inputs and s outputs. The problem with this model is that if there are several output-ratios then the number of variables will be increased and may make difficulty when the number of DMUs is not large enough. The following Theorem reveals the validation of model (3.1).

Theorem 3.1. The maximum number of output-ratio variables which is evaluated by model (3.1) is l, when q and p are the number of discretionary outputs and inputs respectively, where

$$l \leq \min \{ \frac{n}{s} - m \,, \frac{n}{3} - (m+s) \}.$$

As it was discussed earlier model (3.1) may not be Suitable if there are too many ratio variables in the assessment model. To overcome this drawback, the correct convexity for the ratio variables should be defined as ratio of convex combination of numerator to the convex combination of denominator rather than a simple convex combination of ratio variable. In this case, the convexity assumption (when assessing unit jo) should be taken into the model as follow:

$$\frac{\sum_{j=1}^{n} \lambda_j \, n_{kj}}{\sum_{j=1}^{n} \lambda_j \; d_{kj}} \geq \frac{\mathrm{n_{kjo}}}{\mathrm{d_{kjo}}} = \mathrm{y_{kjo}}$$

Hence model (2.1) should be reformulated in the following form:

$$\min \theta$$

$$S.T. \sum_{j=1}^{n} \lambda_j \; x_{ij} - \theta \, \mathrm{x_{ijo}} \leq 0, \qquad i \in D$$

$$\sum_{j=1}^{n} \lambda_j \; x_{ij} - \mathrm{x_{ijo}} \leq 0, \;\; i \in ND \qquad (3.2)$$

$$\sum_{j=1}^{n} \lambda_j \, y_{rj} \geq \mathrm{y_{rjo}} \;\;, \;\; r = 1,2, \dots, s \;, r \neq k$$

$$\sum_{j=1}^{n} \lambda_j \, n_{kj} - \mathrm{y_{kjo}} \sum_{j=1}^{n} \lambda_j \; d_{kj} \geq 0, \quad r = k$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \;\; \geq 0, \qquad j = 1,2, \dots, n.$$

Theorem 3.2. $S_1 \subseteq S_2$ where S_1 and S_2 are the feasible region of model (3.1) and (3.2), respectively.

4. Numerical Example

Consider a case where ten universities have used one discretionary input (x_1= total expenditure) and one non-discretionary input (x_2= area) to produce two outputs, y_1 = % degree awarded and y_2 = amounts of research income. Assume that % degree awarded is in the form of output-ratio and can be considered as number of degree awarded (n) to number of student (d). Data set is listed in Table 1.

Table 1: Data set

DMU	x_1	x_2	y_1	y_2	n	d
U_1	163	49	0.020	25	1200	6000
U_2	281	176	0.29	16	2500	8500
U_3	393	277	0.41	41	6000	14500
U_4	84	267	0.14	65	290	2050
U_5	127	356	0.15	69	700	4500
U_6	118	503	0.15	28	600	4000
U_7	120	6227	0.13	30	550	4200
U_8	242	660	0.87	55	10500	12000
U_9	202	880	0.80	48	8500	10500
U_{10}	96	330	0.16	16	340	2100

The efficiency scores obtained by applying three models are given in Table 2.

Table 1: Efficiency comparison in models (2.1), (3.1), and (3.2)

DMU	Model (2.1) Input: $x_1,\ x_2$ Output: $y_1,\ y_2$	Model (3.1) Input: $x_1,\ x_2, d$ Output: $n,\ y_2$	Model (3.2) Input: $x_1,\ x_2$ Output: $y_1,\ y_2$
U_1	100.00	100.00	100.00
U_2	57.57	74.53	47.87
U_3	100.00	100.00	28.61
U_4	100.00	100.00	100.00
U_5	100.00	100.00	100.00
U_6	72.70	74.96	71.44
U_7	70.00	73.11	70.00
U_8	100.00	100.00	88.51
U_9	100.00	100.00	88.51
U_{10}	91.22	99.94	88.18

As it is expected the higher efficiency scores in model (3.1) as compared to model (3.2) is due to the higher number of variables in the DEA models which is proved by the Theorem 3.2.

5. Conclusions

DEA is usually undertaken with absolute data, but there are some cases reported that the researchers use ratio data. This paper suggested that the Banker and Morey models be extended to suitable models including ratio data. The modification set of these models was presented according to the convexity consideration of units when a ratio data is included in the assessment model. The properties and efficiency of proposed model were discussed.

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