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Introducing a Relational Network DEA Model with Stochastic Intermediate measures for Portfolio Optimization

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Abstract
Conflict intermediate measures in DEA models, especially in constraint and open the black box, is the main difference between traditional DEA and network DEA models. Furthermore, from the application's perspective, intermediate measures aren't deterministic. So, for measuring the efficiency more precisely, they can be considered as imprecise data. The aim of this paper is introducing a stochastic relational model for measuring overall efficiency that deals with intermediate and outputs as stochastic data. The proposed model is applied for portfolio optimization. An actual data set of 27 Iranian stock industries is applied as numerical example. The result shows that SR-NDEA has better discriminant power than R-NDEA model.

Keywords: Data envelopment analysis, Asset allocation, Network structure, stochastic DEA.

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1. Introduction

The portfolio selection problem is a famous challenge in financial real market. While there are a big data set of assets the issue could be Portfolio selection is an investment where a given resource must be divided among different stock industries. This decision making could be considered as a two stage process. The issue is considered in the first stage, is to select the industries for allocation the sources and the second stage is selection stocks among those industries. In designing a portfolio optimization, the utility of investors and their preferences must be considered. Two core problems for decision-making are about how to increase profit and how to reduce risk. Seminal answering to both of these questions were started with the work of Markowitz (1952) [27]. One of Markowitz recognized the two main components of portfolio performance mean reward and risk specified by variance and by applying a simple parametric optimization model. He clarified the optimal trade-off between these two factors. This vital work was quadratic form. His main idea was widely extended by many researchers [28]. But despite of advantages, his model has disadvantages, too. One of the main limitation of his work was about the utility function of investors which they aren't quadratic. The other limitation of Markowitz model was about the increasing the number of assets. When they are increased, the covariance matrix of asset returns will become huge and we encounter with more complex calculation. But, the coherent risks are introduced as risk in literature [1].

Data envelopment analysis (DEA) is a mathematical approach that has been applied as a useful tool for assessing the efficiency of portfolio. This seminal approach is one of the best practice approach for finding the efficient frontier decision making units (DMUs) that employing multiple inputs to produce outputs Charnes et al. (1978) [3,9]. However, most of studies has been focused on assessing efficiency of portfolio for more study in this field see [4,10,11, 13, 14, 16, 18, 19, 20, 21, 26]. Considering multi-stage production processes, that first time introduced by Färe et al (2000) [14] where outputs from foregoing stages, called as intermediate products, are supplied as inputs to the other stages or as exogenous outputs of the other process or both stages [5,6]. In practice in such an asset allocation problem, the exact amount of data can't be determined. In several literature such, Ziemba (2003) [30] using stochastic linear programming and construction of the scenario that describe the uncertainly in the decision variables. This approach has been applied to various types of practical relevance.

Stochastic DEA that in this paper for simplified we called it SDEA First time introduced by Land et al(1993) [23] and subsequently, Cooper et al [7,8], Khodabakhshi [22], Behzadi et al [2], Cooper et al [7], present more stochastic DEA models.

As we know portfolio choice problem steel remains a challenge for realistic problems and it has been a research area in finance for decades. In this paper, we propose a new approach based on a linear version of the relational network DEA model that is called stochastic relational network DEA (SR-NDEA). In our presented model inputs are fixed but intermediate and outputs are random. Although, we use the fundamental analysis information as the set of inputs, intermediate and outputs. As it was mentioned before, two phase for asset allocation is considered. First phase is investment and the output is return. The second phase is assumed as profitability and the target output is liquidity ratio. The contribution of our propose model is considering random variables in our modelling for more accurate prediction. For practical part we apply SR-NDEA in Iranian stock market. In such a structure, we need to considered a suitable intermediate and output that have random variable. Most of the previous researches focused on parametric methods. But, in this paper we introduced a linear version of a non-parameteric method. For verify SR-NDEA we use relational network DEA (R-NDEA) and stoch-CCR.

Rest of the paper is organized as follows: Section 2 preliminaries of NDEA and SDEA and efficiency definition are provided. In Section 3, our SNDEA model is presented. Using a real numerical data set will be employed for verifying our proposed approach

Section 5 conclusions are given.

2. Preliminaries

2.1. DEA and NDEA

Suppose n homogeneous DMUs $(j=1,...,n)$, each utilizing m external inputs x_{ij}, $(i=1,...,m)$ are supplied to produce final output y_{rj}, $(r=1,...,s)$. Let us consider the input oriented CCR model which could be written as follows:

$$\begin{aligned} &\min\ \theta \\ &s.t.\ \sum_{j=1}^{n} x_{ij}\lambda_j \le \theta x_{io} \quad \text{i} = 1,...,m \\ &\sum_{j=1}^{n} y_{rj}\lambda_j \ge y_{ro} \qquad \text{r=1,...,s} \\ &\lambda_j \ge 0 \qquad\qquad \text{j=1,...,n} \end{aligned} \tag{1}$$

If DMUs have a generic network structure, then the first stage produce an intermediate Z is output of the second stage. A relational structure model could be formulated as:

$$\begin{aligned} &\max\ \frac{\sum_{r=1}^{s} u_r y_{ro} + u_I z_o}{\sum_{r=1}^{s} v_i x_{io}} \\ &s.t.\ \frac{\sum_{r=1}^{s} u_r y_{rj} + u_I z_j}{\sum_{r=1}^{s} v_i x_{ij}} \le 1 \qquad \text{j=1,...,n} \\ &\text{u}_r \ge 0\ \forall r, u_I > 0\ , v_i \ge 0 \quad \forall \text{i} \end{aligned} \tag{2}$$

In network literature have proposed two kind of efficiency overall and divisional. If the network has merely one intermediate value with weight u_I it is called relational model. Model (2) uses for measuring the overall efficiency. If we consider a DMU without internal structure the efficiency score can obtain as:

$$\begin{aligned} &\max\ \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{r=1}^{s} v_i x_{io}} \\ &s.t.\ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{r=1}^{s} v_i x_{ij}} \le 1 \qquad \text{j=1,...,n} \\ &\text{u}_r \ge 0\ \forall r\ , v_i \ge 0 \quad \forall \text{i} \end{aligned} \tag{2}$$

Model (2) can be written as:

$$\begin{aligned} &\max\ \sum_{r=1}^{s} u_r y_{ro} \\ &s.t. \sum_{r=1}^{s} v_i x_{io} = 1 \\ &\sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{s} v_i x_{ij} \le 0 \\ &v_i > 0\ \ \forall \text{i},\ u_r > 0\ \ \forall \text{r} \end{aligned} \tag{3}$$

So, due to Model (2) and (3) the overall efficiency score for a DMU with internal structure could be measure by following model:

$$\begin{aligned} &\max \frac{1}{2}\left[\frac{u_I z_o}{\sum_{i=1}^{m} v_i x_{io}} + \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io} + u_I z_o} \right] \\ &\text{s.t.}\ \frac{u_I z_j}{\sum_{i=1}^{m} v_i x_{ij}} \le 1,\ \ \text{j=1,...,n,} \\ &\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij} + u_I z_j} \le 1,\ \text{j=1,...,n,} \\ &\text{u}_r \ge 0\ \forall r\ \ , \text{v}_i \ge 0\ \ , \ \forall \text{i}\ , \text{u}_I \ge 0 \end{aligned} \tag{4}$$

Model (4) can be written as the follower linear programming:

$$\begin{aligned} &\max \frac{1}{2}\left[u_I z_o + \sum_{r=1}^{s} u_r y_{ro} \right] \\ &s.t. \\ &\sum_{i=1}^{m} v_i x_{io} = 1 \end{aligned}$$

$$
\begin{aligned}
& u_I z_j - \sum_{r=1}^{s} v_i x_{ij} \le 0 \qquad \text{j=1,...,n} \\
& \sum_{i=1}^{m} v_i x_{ij} + u_I z_o = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{s} v_i x_{ij} + u_I z_{jw} \le 0 \qquad \text{j=1,...,n} \\
& u_r \ge 0 \ \forall r \ , v_i \ge 0 \ , \ \forall i, u_I \ge 0
\end{aligned} \tag{5}
$$

For writing our model we need to convert Model (5) to envelopment form as:

$$
\begin{aligned}
& \min \ \theta_1 + \theta_2 \\
& s.t. \ \textstyle\sum_{j=1}^{n} (\lambda_j + \mu_j) x_{ij} \le (\theta_1 + \theta_2) x_{io} \\
& \left(\lambda_j + \mu_j\right) \ge \left(\frac{1}{2} - \theta_2\right) z_o \\
& \textstyle\sum_{j=1}^{n} \mu_j y_{rj} \ge \frac{1}{2} y_{ro} \\
& \lambda_j, \mu_j \ge 0 \quad j = 1,...,n \\
& \theta_1, \theta_2 \ \ free
\end{aligned} \tag{6}
$$

Model (6) will be used in next section to proposed our SR-NDEA model.

2.2. Stochastic DEA

In this part, first let us consider Model (1). In Model (1) all input and output vectors are assumed deterministic. But stochastic data envelopment analysis (SDEA) approach was developed by assuming the value of inputs and outputs as random variables that first time introduced by Cooper et al [7,8]. Present SDEA approaches lead to chance constrained optimization problem.

Since these data sets ‘are random variables $\tilde{X}_j = \left(\tilde{x}_{1j},...,\tilde{x}_{mj}\right) \in R^{m+}$ and $\tilde{Y}_j = \left(\tilde{y}_{1j},...,\tilde{y}_{rj}\right) \in R^{s+}$ are random input and output vectors of DMU_j. All input and output components have been regarded to be normally distributed:

$$
\begin{aligned}
& \tilde{x}_{ij} \sim N\left(x_{ij}, \sigma_{ij}^2\right), \quad i = 1,...,m \\
& \tilde{y}_{rj} \sim N\left(y_{rj}, \sigma_{rj}'^2\right), \quad r = 1,...,s
\end{aligned}
$$

Chance constrained model related to input-oriented stochastic CCR model for evaluating DMUo

Could be as follower moedel:

$$
\begin{aligned}
& \min \ \theta \\
& s.t. \ \text{p}\left(\left\{\sum_{j=1}^{n} x_{ij}\lambda_j \le \theta x_{io}\right\}\right) \ge 1\text{-}\alpha \quad \text{i} = 1,...,m \\
& \text{p}\left(\left\{\sum_{j=1}^{n} y_{rj}\lambda_j \ge y_{ro}\right\}\right) \ge 1\text{-}\alpha \qquad \text{r=1,...,s} \\
& \lambda_j \ge 0 \qquad\qquad\qquad \text{j=1,...,n.}
\end{aligned} \tag{11}
$$

Where in Model (11), p mean probability and α is a level of error between 0 and 1 which is predetermined number. The linear deterministic equivalent of Model (11) which is obtain by Cooper et al. [] is as:

Where Φ is the cumulative distribution function of the standard normal distribution and $\Phi^{-1}(\alpha)$, is its inverse in level of α. Model (12) is quadratic and nonlinear programming model.

Definition1. DMU_o is stochastic efficient in level α if and only if $\theta^* = 1$ in the optimal solution of Model (12).

In stochastic CCR model there is symmetric error structure for random inputs and outputs. Assume related inputs and outputs of DMU_j, $j = 1,...,n$ are as following structure [2]

$$
\begin{aligned}
& \tilde{x}_{ij} = x_{ij} + a_{ij}\tilde{\varepsilon}_{ij} \quad i = 1,...,m \\
& \tilde{y}_{ij} = y_{ij} + b_{rj}\tilde{\xi}_{rj} \quad r = 1,...,s
\end{aligned} \tag{12, 13}
$$

Where a_{ij} and b_{rj} are non negative and real value. However, $\tilde{\varepsilon}_{ij}$ and $\tilde{\xi}_{rj}$ are random variable with normal distributions, $\tilde{\varepsilon}_{ij} \sim N\left(0, \bar{\sigma}^2\right)$ and $\tilde{\xi}_{rj} \sim N\left(0, \bar{\sigma}^2\right)$ and are errors of inputs and outputs in contraints to mean values, respectively. If the normal distributions is symmetric (13) is called symmetric error structure, relations (14) concluded from (13).

$$
\begin{aligned}
& \tilde{x}_{ij} \sim N\left(x_{ij}, \bar{\sigma}_{ij}^2 {a_{ij}}^2\right) \\
& \tilde{y}_{rj} \sim N\left(y_{rj}, \bar{\sigma}_{rj}^2 {b_{rj}}^2\right)
\end{aligned} \tag{14}
$$

It is noted that every variable with normal distribution can be stated as symmetric error structure. Sofar let us assume that *i*th input and *r* the of every DMUs are uncorrelated. For

every $j \neq k$

$$\begin{aligned} &\operatorname{cov}(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}) = 0 \quad i = 1,...,m \\ &\operatorname{cov}\left(\tilde{\xi}_{rj}, \tilde{\xi}_{rk}\right) = 0 \quad r = 1,...,s \end{aligned} \tag{15}$$

Due to (13) and (15) it can be assumed a same error for all DMUs. $\tilde{\varepsilon}_i = \tilde{\varepsilon}_{ij}$ ∀i∀j and $\tilde{\xi}_r = \tilde{\xi}_{rj}$ ∀r∀j.

The linear form of deterministic equivalent of Model (11) is

$$\begin{aligned} &\theta^*(\alpha) = \min \quad \theta \\ &s.t. \sum_{j=1}^{n} \lambda_j x_{ij} - \Phi^{-1}(\alpha)\bar{\sigma}(p_i^+ + p_i^-) \leq \theta x_{io} \\ &\sum_{j=1}^{n} \lambda_j y_{rj} - \Phi^{-1}(\alpha)\bar{\sigma}(q_r^+ + q_r^-) \geq y_{ro} \\ &\sum_{j=1}^{n} \lambda_j a_{ij} - \theta a_{io} = p_i^+ - p_i^- \qquad i = 1,...,m \\ &\sum_{j=1}^{n} \lambda_j b_{rj} - b_{ro} = q_r^+ - q_r^- \qquad r = 1,...,s \\ &\lambda_j, p_i^+, p_i^-, q_r^+, q_r^- \geq 0, \\ &j = 1,...,n, \quad i = 1,...,m, \quad r = 1,...,s \end{aligned} \tag{16}$$

For seeing theorems and process see Behzadi et al. [2].

3. Investment Allocation Problem and our Proposed SR-NDEA for Portfolio Choice

In this section we first define the investment problem. Let us consider $r = (r_1,...,r_m)$ be random returns of assets 1, 2, . . . , m. Our aim is to invest our capital in this asset in order to obtain some desirable characteristics of the total return on the investment. Denoting by $(x_1,...,x_m)$. The fractions of the initial capital invested in assets $1,...,N$, we can easily derive the formula for the total return: $r(x) = r_1(x_1) + ... + r_m(x_m)$ Clearly, the set of possible asset allocations can be defined as follows: $X = \left\{ x \in \mathbb{R}^m \,|\, x_1 + ... + x_m = 1, x \geq 0 \right\}$. The main challenge in formulating a meaningful portfolio optimization problem is the definition of the preference structure among feasible portfolios. If we use only the mean return. then the resulting optimization problem has a trivial and meaningless solution: invest everything in assets that have the maximum expected return. For these reasons the practice of portfolio optimization resorts usually to two approaches that maximize return and minimize risk. Several approaches were proposed. Let us we present a two- stage investment that illustrate in Fig 1.

Generally one of the disadvantages of portfolio selection methods are in this kind of models that the decision maker is allowed not to make a decision. This means that this decision maker "decided" not to decide. But DEA approach models don't have this challenge. SR-NDEA allows to decision maker considering stochastic data for calculating the overall efficiency. In our modelling mean returns considered as intermediate and liquidity ratio as target output which aren't deterministic.

All intermediate and output components have been considered to be normally distributed ;

$$\tilde{z}_j \sim N\left(z_j, \sigma_j^2\right)$$

$$\tilde{y}_{rj} \sim N\left(y_{rj}, \sigma_{rj}'^2\right), \quad r = 1,...,s$$

Now we use Model (10) and write the Chance constrained model related to input- oriented stochastic CCR model for evaluating DMUo Could be written as follow:

$$\begin{aligned} &\min \ (\theta_1 + \theta_2) \\ &s.t. \sum_{j=1}^{n} \mu_j x_{ij} \leq (\theta_1 + \theta_2) x_{io} \qquad i = 1,...,m \\ &p\left\{ \left(\sum_{j=1}^{n} \mu_j y_{rj} \right) \geq \frac{1}{2} y_{ro} \right\} \geq 1\text{-}\alpha \qquad r = 1,...,s \\ &p\left\{ \left(\lambda_j + \mu_j\right) z_j - \left(\frac{1}{2} - \theta_2\right) z_o \right\} \geq 1 - \alpha \\ &\mu_j \geq 0, \quad \lambda_j \geq 0 \end{aligned} \tag{18}$$

That α is a level of error and its predetermined. Although we could substitution.

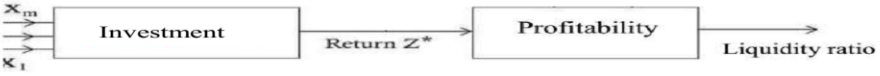

Fig1. Two stage investment. Our goal is maximized the target output using SNDEA approach. For this purpose initial asset is considered as shared input for two stage. In our investment model, target output and intermediate product are random variables

The symmetric error has been calculated as:

$$\tilde{y}_{rj} = \tilde{\varepsilon}_r + a_{rj} \qquad z_j = \tilde{\xi} + b_j$$

The deterministic equivalent of Model (18) is follows:

$$\begin{aligned}
&\varphi^*(\alpha) = \min\ (\theta_1 + \theta_2) \\
&s.t. \sum_{j=1}^{n} \mu_j x_{ij} \le (\theta_1 + \theta_2) x_{io} \qquad i = 1,...,m \\
&\sum_{j=1}^{n} \mu_j y_{rj} + \phi^{-1}(\alpha)\sigma\left(p_r^+ + p_r^-\right) \ge \frac{1}{2} y_{ro} \quad r = 1,...,s \\
&\sum_{j=1}^{n} \mu_j a_{rj} - \frac{1}{2} a_{ro} = p_r^+ - p_r^- \\
&\left(\lambda_j + \mu_j\right) z_j - \phi^{-1}(\alpha)\sigma(q_r^+ + q_r^-) \ge \left(\frac{1}{2} - \theta_2\right) z_o \\
&\left(\lambda_j + \mu_j\right) b_j - \left(\frac{1}{2} - \theta_2\right) b_o = q_r^+ - q_r^- \\
&\mu_j \ge 0, \quad \lambda_j \ge 0 \\
&\theta_1, \theta_2 \quad free \\
&p_r^+, p_r^-, q_r^+, q_r^- \ge 0
\end{aligned} \tag{19}$$

where Φ is the cumulative distribution function of the standard normal distribution and $\Phi^{-1}(\alpha)$, is its inverse in level of α. Model(18) is quadratic and non linear programming model.

We can calculate the symmetric error for CCR model according substitution the random variables *Y* and *Z* in Models (13)-(16).

4. Numerical example

In this part for our proposed approach is used for the information of a real data set of Iranian stock market in 2013. Each company assume as a DMU. We've been eliminated the information that have negative inputs or incomplete data. Finally, the information of fundamental analysis of 27 companies with four outputs, one intermediate and one output are applied. The information of data set is presented in Table 1.

The result obtained from our proposed model compared with R-NDEA model and Stoch-CCR version scores. The efficiency scores and ranking of units is presented in Table 2.

Table 1. The information of inputs, intermediate and output of 27 company of Iranian stock industry

Companies	Input1	Input2	Input3	Input4	Intermediate	Output
	P\E	Quick ratio	Debt to equity ratio	Sigma Index	Return	Liquidity ratio
DMU1	7.43	1.18	1.22	2.44	2.44	157.67
DMU2	13.38	0.49	3.78	3.05	3.05	183.48
DMU3	11.58	0.59	2.85	2.46	2.46	110.28
DMU4	7.7	0.86	2.27	4.54	4.54	122.76
DMU5	6.58	1.16	1	2.09	2.09	166.99
DMU6	8.7	0.87	3.91	2.53	2.53	156.08
DMU7	7.76	1.07	1.84	2.64	2.64	164.07
DMU8	8.96	0.97	1.36	2.88	2.88	228.88
DMU9	7.93	7.07	0.1	3.45	3.45	187.63
DMU10	9.03	0.86	3.44	2.78	2.78	143.68
DMU11	7.91	0.96	1.72	2.52	2.52	167.43
DMU12	18.43	1.06	1.23	3.59	3.59	169.7
DMU13	6.41	0.9	5.95	3.41	3.41	227.86
DMU14	7.47	0.72	3	3.49	3.49	187.99
DMU15	7.46	1.28	1.21	3.19	3.19	205.22
DMU16	8.39	1.35	0.86	3.64	3.64	155.82
DMU17	6.82	0.79	4.43	5.01	5.01	177.08
DMU18	6.94	1.21	0.94	2.99	2.99	219.36
DMU19	6.81	0.73	2.28	21.32	21.32	147.27
DMU20	8.2	0.82	5.16	3.27	3.27	198.36
DMU21	7.52	1.21	0.84	3.12	3.12	174.39
DMU22	8.84	1.01	0.95	7.25	7.25	26.37
DMU23	5.73	0.97	1.45	4.05	4.05	163.71
DMU24	5.4	0.95	2.83	7.14	7.14	44.51
DMU25	10.22	0.6	4.81	3.02	3.02	181.41
DMU26	6.9	1	1.93	2.55	2.55	228.42
DMU27	6.75	1.06	1.57	3.43	3.43	221.73

Table2. The efficiency scores obtained from SR-NDEA, R-NDEA and Stoch-CCR

Companies			
	SR-NDEA	**R-NDEA**	**Stoch-CCR**
DMU1	0.779827	0.401062	1.000000
DMU2	1.000000	0.500000	1.000000
DMU3	0.571486	0.319207	1.000000
DMU4	0.471987	0.292616	0.722986
DMU5	0.913427	0.500000	1.000000
DMU6	0.602052	0.369674	0.936984
DMU7	0.591139	0.355236	0.995421
DMU8	0.977637	0.500000	1.000000
DMU9	1.000000	0.500000	1.000000
DMU10	0.546933	0.327787	0.855829
DMU11	0.639068	0.382100	1.000000
DMU12	0.733919	0.372615	1.000000
DMU13	0.865140	0.500000	1.000000
DMU14	0.847689	0.500000	1.000000
DMU15	0.822391	0.431161	1.000000
DMU16	0.754788	0.377394	1.000000
DMU17	0.744259	0.435348	1.000000
DMU18	1.000000	0.500000	1.000000
DMU19	0.663824	0.405180	1.000000
DMU20	0.789140	0.451982	1.000000
DMU21	0.876021	0.438010	1.000000
DMU22	0.132519	0.066738	1.000000
DMU23	0.719428	0.433417	1.000000
DMU24	0.200604	0.121126	1.000000
DMU25	0.904865	0.483409	1.000000
DMU26	0.849085	0.500000	1.000000
DMU27	0.857821	0.500000	1.000000

Due to the information indicated in Table 1 it is clear that the discriminant power of SR-NDEA is better than Stoch-CCR.

5. conclusion

In this paper the novel SR-NDEA model was proposed. Furthur more, it is indicated how a SR-NDEA structure could be used for solving a portfolio selection problem. However, this paper aims to use DEA for solving asset allocation problem, but classic DEA models have limitation for this purpose, because of it's a kind of practical problem that managers deal with units with random data. In our modeling we consider asset as shared input as deterministic data. In our presented model, we assume mean return and total profit as random variables. We assume the mean return of investment of th first stage as input in the second stage. Using numerical example, we illustrate in practice application of our model. By introducing this model, using the capability the various type of DEA-network structure and extention them to multi-stage structure in diversification for covering the risk management in investment problem can be considered as the future work. We compared our proposed model with R-NDEA and stoch-CCR. In some investment cases such stock the data sets don't have normal distribution. So, we need to transform data to the normal distribution. So, it could be one of the extension of this paper.

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