



An Alternative Secondary Goal Approach to Modify Cross Efficiency Evaluation in Data Envelopment Analysis

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Abstract

The cross efficiency evaluation is used to performance measurement of decision making units in data envelopment analysis concept. One of the most important shortcoming of this method is existing alternative optimal solution and therefore, the efficiency scores are not unique. We are going to summarize the pervious models proposed by researchers and suggest an alternative secondary goal approach to modify them to remove the shortcomings and difficulties of basic cross efficiency method. Also we tried the presented model to rank the efficient units.

Keywords: Data Envelopment Analysis, Efficiency, Cross efficiency

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1. Introduction

Now days, Data Envelopment Analysis is one of the most applied science in performance evaluation of homogeneous decision making unit such as chain stores, commercial banks, schools, hospitals,...etc. First time Charnes et al. [4], used the non parametric method for evaluating educational centers in USA and they named it CCR model. Then Banker et al.[1], developed their proposal, namely BCC model. So far, many models are presented to develop DEA science in different versions. To review these models see Cook et al.[5].

The units under evaluation are divided into efficient and inefficient units by using DEA models. Fundamental DEA models such as CCR, BCC,...etc give score one to efficient units. When the number of efficient units be more than one, distinction is essential between these units. So, researchers presented ranking efficient units. In the past years this branch of DEA developed noticeably. One of the methods is cross efficiency ranking method that is presented by Sexton et al.[15]. The main idea of cross method is based on equalized evaluation of units which are obtained by solving multiplier forms (for example CCR model). This method presents a unique ranking of units. Also, it eliminates unrealistic weight schemes without requiring the elicitation of weight restriction from application area experts. Of course, there are some important shortcoming in this method.

In next sections we will show that the purposed mean value in evaluation method

cannot be suitable criterion to rank units. There is an important point here; it is possible that obtained mean for inefficient unit be more than obtained number for efficient unit. Also, the method cannot present suitable ranking of inefficient units. It means that inefficient unit with higher efficiency score may has lower cross efficiency score mean value than worse performing unit than itself.

There is an important shortcoming in cross evaluation; that is existing alternative optimum solution in multiplier form. Thus, there are different cross efficiency matrix and there is no reason for equal ranking. Cross evaluation method has been used in many applications. For first time, Sexton et al [15], used it to determine nursing homes efficiency. Another application of this method is Research and Development project selection, preference voting by Green et al. [10]. Some studies on other DEA issues are Nicole et al. [14], Beasley [2], and Marrotas et al. [13], Sexton [15], Doyle et al. [7], expressed that cross evaluation method because of existing alternative solution is not so valid. Because multiplier form of CCR model cannot uniquely determine weights, so cross efficiency matrix does not uniquely determine and distinguish. Then ranking is not unique and there isn't any unit rank stability. They have used secondary goal method to solve this ambiguity. Also, Liang et al.[12], used secondary goal and constructed different secondary objective function to remove ambiguity of non-uniqueness of weights in cross evaluation. The

main idea of this paper is based on the proposal of Doyel et al.[7], and Liang et al.[12]. We are going to try in this paper to present more better and prefect-performing ranking than previously presented methods. We do it with changing secondary goal function of the papers and changing constraint secondary goal problem in evaluation. This paper is organized as follows:

In the second section, we review DEA models and presented models in cross evaluation methods. In third section, the proposed model will be presented. In fourth section, we show a comparison between previous models and proposaled models with data sets. Conclusion are given in the last section.

2. DEA models of cross- efficiency evaluation method

Suppose we have n decision making unit (DMU), $DMU_j, j = 1, \dots, n$ uses different m inputs to produce different s outputs. i th input and r th output $DMU_j, j = 1, \dots, n$ are $x_{ij}, (i = 1, \dots, m), y_{rj}, (i = 1, \dots, m)$. Some times cross method is named two phase method. In the first phase we should use multiplier form of one of main DEA models. For this purpose the following problem is solve to evaluate DMU_p :

$$\begin{aligned} \max \quad & zz_{pp} = \sum_{r=1}^s u_{rp} y_{rp} / \sum_{i=1}^m v_{ip} x_{ip} \\ \text{St} \quad & \sum_{r=1}^s u_{rp} y_{rj} / \sum_{i=1}^m v_{ip} x_{ij} \leq 1, \\ & j=1, \dots, n \\ & u_{rp} \geq \varepsilon, \quad r = 1, \dots, s \end{aligned} \tag{1}$$

$$v_{ip} \geq \varepsilon, \quad i = 1, \dots, m$$

In the above model, v_{ip} and u_{rp} shows the i th input and the r th output of DMU_p . In the second phase, we use obtained weights of DMU_p to evaluate cross efficiency of DMU_j , then we have:

$$z_{pj}^* = \sum_{r=1}^s u_{rp}^* y_{rj} / \sum_{i=1}^m v_{ip}^* x_{ij}, \quad j=1, \dots, n \tag{2}$$

In the above formulation * shows optimal solution obtained form model (1). Now cross efficiency value of DMU_j is the mean of all z_{pj} s ($p=1, \dots, n$) that is writen as follows:

$$CRE_j = \frac{1}{n} \sum_{p=1}^n z_{pj}, \quad j = 1, \dots, n.$$

By using Charnes-Cooper transformation[3] model become changed into the following liner programming:

$$\begin{aligned} \max \quad & \sum_{r=1}^s u_{rp} y_{rp} \\ \text{St} \quad & \sum_{i=1}^m v_{ip} x_{ip} = 1 \\ & \sum_{r=1}^s u_{rp} y_{rj} - \sum_{i=1}^m v_{ip} x_{ij} \leq 0, \\ & j=1, \dots, n \quad \quad \quad j \neq 0 \\ & u_{rp} \geq \varepsilon, \quad r = 1, \dots, s \\ & v_{ip} \geq \varepsilon, \quad i = 1, \dots, m \end{aligned} \tag{3}$$

The above model also is named multiplier form of CCR model that is presented equivalently in the following deviation variables form:

$$\begin{aligned} \min \quad & \alpha_p \\ \text{St} \quad & \sum_{i=1}^m v_{ip} x_{ip} = 1 \end{aligned}$$

$$\sum_{r=1}^s u_{rp} y_{rj} - \sum_{i=1}^m v_{ip} x_{ij} + \alpha_j = 0, \quad j=1, \dots, n \quad (4)$$

$$u_{rp} \geq \varepsilon, \quad r = 1, \dots, s$$

$$v_{ip} \geq \varepsilon, \quad i = 1, \dots, m$$

$$\alpha_j \geq 0, \quad j = 1, \dots, n$$

where α_j is the deviation variable for the j th DMU. In this problem DMU_p is efficient, if and only if $\alpha_p^* = 0$. In fact, α_p^* shows deviation from efficiency of DMU_p . Thus the measure of $1 - \alpha_p^*$ is equivalent to efficiency of DMU_p . As mentioned in introduction, the obtained weights from model (3) or model (4) are not unique. As Despoits [6] expressed, the weights are different, depending on the particular software in use. Thus, the obtained measure in formulation (2) is not unique and therefore, the ranking will not be presented unique. For first time Green and Doyle [7] tried to remove this ambiguity. They used a model that the mean value of the efficiency scores of other DMUs are minimized instead of maximizing the ratio of weighted sum of outputs to weighted sum of inputs. So, they used inputs and outputs of $n-1$ remaining unit (all units except unit under evaluation) to make a new unit named composite DMU that its inputs and outputs obtained from aggregating inputs and outputs for $n-1$ remaining units. Their proposed model minimized the ratio of weighted sum of outputs to weighted sum of inputs for the composite unit. While the efficiency of unit under evaluation had been

maximized. This method is introduced as secondary goal method. Also Liang et al. [12], presented a method that is continuing of secondary goal method. They used minimizing the sum of evaluation variable from ideal points. In fact, ideal points are defined as that its equivalent set weight for every DMU is efficient. In the absence of ideal point, the amount of β_j is deviation from efficiency of DMU_j .

$$\min \sum_{j=1}^n \beta_j$$

$$St \sum_{r=1}^m v_{ip} x_{ip} = 1$$

$$\sum_{r=1}^s u_{rp} y_{rj} - \sum_{i=1}^m v_{ip} x_{ij} + \beta_j = 0, \quad j=1, \dots, n \quad (5)$$

$$\sum_{r=1}^s u_{rp} y_{rp} = 1 - \alpha_p^*$$

$$u_{rp} \geq \varepsilon, \quad r = 1, \dots, s$$

$$v_{ip} \geq \varepsilon, \quad i = 1, \dots, m$$

$$\beta_j \geq 0, \quad j = 1, \dots, n$$

In the past model, the sum of deviational variable from efficiency of remaining DMUs is minimized. Thus model (5) restricted the optimal weights of problem to the weights where $1 - \alpha_p^*$ score efficiency is obtained. Of course the optimal weights of problem are equivalent to DMU_p . In this procedure they considered the development of DEA that Troutt [16] presented. Then they organized the cross evaluation based on minimizing the maximum of deviational variable from defined

efficiency. Their proposed model is as follows: $\min \max \beta_j$

$$\begin{aligned}
 \text{St } & \sum_{r=1}^m v_{ip} x_{ip} = 1 \\
 & \sum_{r=1}^s u_{rp} y_{rj} - \sum_{r=1}^m v_{ip} x_{ij} + \beta_j = 0, \\
 & j=1, \dots, n \\
 & \sum_{r=1}^s u_{rp} y_{rp} = 1 - \alpha_p^* \\
 & u_{rp} \geq \varepsilon, \quad r = 1, \dots, s \\
 & v_{ip} \geq \varepsilon, \quad i = 1, \dots, m \\
 & \beta_j \geq 0, \quad j = 1, \dots, n
 \end{aligned} \tag{6}$$

In model (6), when efficiency of DMU_p under evaluation from model (4) is considered in constraints, the maximum of deviations from efficiency of all DMUs is minimized. According to the structure of constraints of problems (5) and (6) efficiency score of DMUs may be worse performing than model (4) with increasing the number of constraints. The minmax model is changed into a linear programming problem as follows:

$$\begin{aligned}
 \min & \theta \\
 \text{St } & \sum_{r=1}^m v_{ip} x_{ip} = 1 \\
 & \sum_{r=1}^s u_{rp} y_{rj} - \sum_{r=1}^m v_{ip} x_{ij} + \beta_j = 0, \\
 & j=1, \dots, n \\
 & \sum_{r=1}^s u_{rp} y_{rp} = 1 - \alpha_p^* \\
 & \theta - \beta_j \geq 0, \quad j = 1, \dots, n \\
 & u_{rp} \geq \varepsilon, \quad r = 1, \dots, s \\
 & v_{ip} \geq \varepsilon, \quad i = 1, \dots, m \\
 & \beta_j \geq 0, \quad j = 1, \dots, n
 \end{aligned} \tag{7}$$

From sets of constraints $\sum_{r=1}^s u_{rp} y_{rj} - \sum_{r=1}^m v_{ip} x_{ij} + \beta_j = 0, \quad j = 1, \dots, n$ and $\theta - \beta_j \geq 0, \quad j = 1, \dots, n$ can be changed into $\theta \geq \sum_{r=1}^s u_{rp} y_{rj} - \sum_{r=1}^m v_{ip} x_{ij}, \quad j = 1, \dots, n$. Thus, it seems that model (6) can simply be presented. In the new manner n constraints and n variables of problem are decreased and problem is changed as follows:

$$\begin{aligned}
 \min & \theta \\
 \text{St } & \sum_{r=1}^m v_{ip} x_{ip} = 1 \\
 & \theta \geq \sum_{r=1}^s u_{rp} y_{rj} - \sum_{r=1}^m v_{ip} x_{ij}, \\
 & j=1, \dots, n \\
 & \sum_{r=1}^s u_{rp} y_{rp} = 1 - \alpha_p^* \\
 & u_{rp} \geq \varepsilon, \quad r = 1, \dots, s \\
 & v_{ip} \geq \varepsilon, \quad i = 1, \dots, m \\
 & \beta_j \geq 0, \quad j = 1, \dots, n
 \end{aligned} \tag{8}$$

Now suppose $W_p^* = (v_{1p}^*, \dots, v_{mp}^*, u_{1p}^*, \dots, u_{sp}^*)$ be equivalent to the optimal weights of DMU_p obtained from models (5) or (7). So, to calculate cross efficiency score of DMU_j the next formulation is used: $z_j^*(w_p) = \frac{\sum_{r=1}^s u_{rp}^* y_{rj}}{\sum_{r=1}^m v_{ip}^* x_{ij}}, \quad j = 1, \dots, n$. Then the mean value of $z_j(w_p)$ is used for cross efficiency estimation:

$$CRE_j = \frac{1}{n} \sum_{p=1}^n z_{pj}, \quad j = 1, \dots, n.$$

3 Proposed model: Modified cross efficiency method

In this section, we try to present a new objective function. The objective function is

minimizing the real efficiency deviations of decision making units from their obtained efficiency by using the weights of DMU_p under evaluation. In this regards, consider the following model:

$$\begin{aligned} \min \max & \left\{ z_{jj}^* - \sum_{r=1}^s u_{rp}^* y_{rj} \right\} \\ \text{St} & \sum_{r=1}^m v_{ip} x_{ip} = 1 \\ & \sum_{r=1}^s u_{rp} y_{rj} - \sum_{r=1}^m v_{ip} x_{ij} \leq 0, \\ & j=1, \dots, n \\ & \sum_{r=1}^s u_{rp} y_{rp} = 1 - \alpha_p^* \\ & u_{rp} \geq \varepsilon, \quad r = 1, \dots, s \\ & v_{ip} \geq \varepsilon, \quad i = 1, \dots, m \end{aligned} \tag{9}$$

where E is a set of efficient units obtained from models (3) and (4). The set of last constraints related to efficient units. Because, it is possible, there are some inefficient units in the previous methods that have higher cross score than efficient units. To solve this ambiguity, the model is only used for efficient units. Principally, the ranking of efficient unit is important and the ranking of inefficient unit is not important. In presented objective function of model (9), the maximum of real efficiency deviation of DMUs obtained by the weights of DMU_p is minimized. In fact, we want to minimized difference among real efficiency and obtained efficiency by weights of DMU_p under evaluation. Linear programming of above model problems are as

follows:

$$\begin{aligned} \min & \theta \\ \text{St} & \sum_{r=1}^m v_{ip} x_{ip} = 1 \\ & \sum_{r=1}^s u_{rp} y_{rj} - \sum_{r=1}^m v_{ip} x_{ij} \leq 0, \\ & j=1, \dots, n \\ & \sum_{r=1}^s u_{rp} y_{rp} = 1 - \alpha_p^* \\ & \theta - z_{jj}^* + \sum_{r=1}^s u_{rp}^* y_{rj} \geq 0 \\ & u_{rp} \geq \varepsilon, \quad r = 1, \dots, s \\ & v_{ip} \geq \varepsilon, \quad i = 1, \dots, m \end{aligned} \tag{10}$$

3.1 Illustration

Example1: Consider 18 chines cities which produce 3 outputs by using 2 inputs to illustrate the pervious models. Table of inputs and outputs is as follows:

where:

Input 1: Investment in fixed assets by stateowned enterprizes (10,000 RMB), where RMB is the Chinese monetary unit;

Input 2: Foreign funds actually used (10,000 US dolor);

Output 1: Total industrial output value (based on fixed prices of 1980) (10,000 RMB);

Output 2: Total value of retail sales (10,000 RMB);

Output 3: Handling capacity of coastal ports (10,000 tones).

The following table shows the results of ranking:

Table 1. 18 chines cities

DMUs	Input ₁	Input ₂	Output ₁	Output ₂	Output ₃
1	2874.8	16738	160.89	80800	5092
2	946.3	691	21.14	18172	6563
3	6854.0	43024	375.25	144530	2437
4	2305.1	10815	176.68	70318	3145
5	1010.3	2099	102.12	55419	1225
6	282.3	757	59.17	27422	246
7	17478.6	116900	1029.09	351390	14604
8	661.8	2024	30.07	23550	1126
9	1544.2	3218	160.58	59406	2230
10	428.4	574	53.69	47504	430
11	6228.1	29842	258.09	151356	4649
12	697.7	3394	38.02	45336	1555
13	106.4	367	7.07	8236	121
14	4539.3	45809	116.46	56135	956
15	957.8	16947	29.20	17554	231
16	1209.2	15741	65.36	62341	618
17	972.4	23822	54.52	25203	513
18	2192.0	10943	25.24	40267	895

Table 2: Results for 18 chines cities

DMUs	ccr efficiency	sexton	model 4	model 6	model 8	model 10
1	0.46790048	0.3269	0.394	0.394	0.394	0.4157
2	1	0.6036	1	1	1	0.9046
3	0.27791237	0.2217	0.2389	0.2389	0.2389	0.2318
4	0.50222003	0.3686	0.4365	0.4365	0.4365	0.4244
5	0.63107667	0.5376	0.5795	0.5795	0.5795	0.58
6	1	0.9352	0.969	0.969	0.969	0.9396
7	0.35803618	0.2541	0.3029	0.3029	0.3029	0.2854
8	0.49594494	0.3372	0.3687	0.3687	0.3687	0.4288
9	0.65766276	0.5012	0.6176	0.6176	0.6176	0.5381
10	1	0.8276	0.7363	0.7363	0.7363	0.8705
11	0.30096994	0.2251	0.2365	0.2365	0.2365	0.2648
12	0.78660649	0.4737	0.4424	0.4424	0.4424	0.6265
13	0.75144399	0.496	0.3869	0.38697	0.3869	0.593
14	0.13819726	0.1113	0.1126	0.1126	0.1126	0.1226
15	0.18671071	0.1399	0.1294	0.1294	0.1294	0.1585
16	0.47036807	0.3148	0.241	0.241	0.241	0.3751
17	0.30594473	0.2354	0.2442	0.2442	0.2442	0.2646
18	0.19525863	0.1116	0.0867	0.0867	0.0867	0.1462

It is clear that units 2, 6 and 10 are efficient in ccr model. Liang Liang et al. [12] used this data set to cross evaluation. They stated that the cross evaluation scores obtained by sexton method and models 4, 6 and 8 are the same. Therefore, "the crosse efficiency scores are

unique or stable". Of course it is not true, there is no guarantee about the uniqueness or stability of crosse efficiency scores. The readers can easily see this problem in the above table. Moreover, it can be easily seen that maybe a worth unit than the other has

higher cross efficiency score, for example see unit 15 and unit 18. the ccr efficiency score of unit 18 is grater than unit 15, but its cross efficiency score is less than. This problem may be occurred for efficient units; an inefficient unit has grater cross efficiency score than the efficient one. We presented this problem in the next example. It should be noted that model 10 is used for all data set to show the drawbacks. Using this model only for efficient units decrease the computational process of ranking. The following table shows ranking efficient units by mentioned model:

Example2: Now, consider 20 Iranian cement factory which each of them produces tow outputs by using four inputs. The following table shows these data set:

where:

Input1: fixed assets (billion rials), where Rials is the Iranian money unit

Input2: number of shareholders

Input3: current assets (billion rials)

Input4: number staff

Output1: sales (billion rials)

Output2: cover for dividend (billion rials)

Table 3: Results of Ranking

DMUs	sexton	model 4	model 6	model 8	model 10	AP
2	3	1	1	1	2	1
6	1	2	2	2	1	3
10	2	3	3	3	3	2

Table 4: 20 Iranian Cement Factory

DMUs	Output1	Output2	Input1	Input2	Input3	Input4
1	22750.06	88.04	1532.61	616.81	262.26	49833
2	742.74	11.76	401.44	201.72	78.21	15600
3	645.76	12.77	422.56	210.63	136.51	18501
4	519.38	30.34	364.73	155.47	71.34	7471
5	463.38	12.6	336.56	185.92	67.19	8035
6	294.78	6.42	181.96	53.72	41.5	15000
7	153.85	0.88	79.4	15.2	25	1732
8	131.85	3.36	104.23	38.09	29.83	6823
9	128.21	5.54	130.59	69.76	46.53	7435
10	125.43	0.72	111.03	41.62	22.11	8500
11	120.16	1	62.03	19.8	20.5	549
12	118.09	7.87	117.7	74.8	40.16	5314
13	116.95	6.98	77.36	43.88	23.14	3600
14	115.49	10.24	72.48	38.81	12.02	4298
15	105.8	12.95	127.96	56.36	22.41	1940
16	95.89	2.99	58.59	19.91	11.92	270
17	87.37	37.97	159.49	109.11	28.37	13000
18	84.6	2.87	31.2	15.95	11.13	500
19	83.12	5.04	85.74	42.76	25.97	233
20	83.72	0.79	30.15	2.07	1	140

Clearly, units 1, 15, 16, 17, 18, 19 and 20 are efficient. There are some important drawbacks in sexton methods and models 4, 6, 8. For example, unit 4 is an inefficient one, but its cross efficiency score is grater than unit 19 which it is efficient; and its cross score is grater than units 16 and 19 in models 4, 6, 8. It is clear that this problem is not occurred in model 10. Results of ranking is summarized in the following table for efficient units:

4 Conclusions

In this research a modified cross efficiency model presented to evaluate decision making

units. A secondary goal approach based on cross evaluation suggested in this work. Two data set are added to illustrate and explain the mentioned models.

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Table 5. Results of 20 Iranian Cement Factory

DMUs	ccr efficiency	sexton	model 4	model 6	model 8	model 10
1	1	0.9413	1	1	0.9027	0.9727
2	0.25773097	0.2236	0.2335	0.2217	0.1969	0.2081
3	0.23766062	0.2076	0.2069	0.2074	0.1616	0.1858
4	0.80618489	0.6694	0.6469	0.5811	0.4859	0.5723
5	0.36567526	0.2934	0.2864	0.2648	0.2299	0.267
6	0.39039864	0.2212	0.2374	0.2496	0.1926	0.1991
7	0.28009630	0.2101	0.2004	0.209	0.1458	0.1923
8	0.28907265	0.1977	0.2056	0.2107	0.1611	0.1742
9	0.25768790	0.2214	0.224	0.2242	0.1728	0.1927
10	0.09572299	0.0616	0.0705	0.0756	0.0642	0.0623
11	0.43715019	0.2813	0.2423	0.2545	0.181	0.2782
12	0.43425873	0.3471	0.3429	0.3324	0.2627	0.3001
13	0.58148001	0.4854	0.4875	0.4677	0.3769	0.419
14	0.79832436	0.6785	0.7484	0.6765	0.6427	0.6009
15	1	0.8369	0.7951	0.7023	0.596	0.7154
16	1	0.6756	0.5803	0.563	0.4305	0.653
17	1	0.8578	0.9832	0.907	0.8846	0.781
18	1	0.8004	0.7122	0.6981	0.5101	0.7033
19	1	0.6669	0.5277	0.5424	0.385	0.6771
20	1	0.7835	0.8339	0.6847	0.7766	0.7777

Table 6. Results of ranking

DMUs	sexton	model 4	model 6	model 8	model 10	AP
1	1	1	1	1	1	1
15	3	4	3	4	4	5
16	6	6	6	6	7	7
17	2	2	2	2	2	4
18	4	5	4	5	5	6
19	7	7	7	7	6	3
20	5	3	5	3	3	2

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