Available online at http://ijdea.srbiau.ac.ir

Int. J. Data Envelopment Analysis (ISSN 2345-458X)
Vol. 1, No. 4, Year 2013 Article ID IJDEA-00144, 11 pages
Research Article

International Journal of Data Envelopment Analysis

Science and Research Branch (IAU)

Using Non-Archimedean DEA Models for Classification of DMUs: A New Algorithm

S. Mehrabian[a*]

(*a*) *Department of Math., Faculty of Mathematical Sci. & Computer, Kharazmi University, Karaj, Iran.*

Received 23 October2013, Accepted 30 December 2013

Abstract

A new algorithm for classification of DMUs to efficient and inefficient units in data envelopment analysis is presented. This algorithm uses the non-Archimedean Charnes-Cooper-Rhodes[1] (CCR) model. Also, it applies an assurance value for the non-Archimedean ε using only simple computations on inputs and outputs of DMUs (see [18]). The convergence and efficiency of the new algorithm show the advantage of this algorithm compared to the Thrall's algorithm (see [23]).

Keywords: Data Envelopment Analysis, Classification, Efficiency, Non-Archimedean.

1. Introduction

Data envelopment analysis (DEA) is a mathematical programming technique that evaluates the relative efficiency of a group of decision making units (DMUs). This method was originated in 1978 by the paper of Charnes, Cooper and Rhodes and the first DEA model was called CCR model. Since the seminal paper, a variety of DEA models has appeared in the literature as have numerous studies employing the technique. Each of the various models for DEA seeks to determine which of n DMUs determine an *efficient frontier*. Units that lie on the frontier are deemed *efficient* in DEA terminology. Units that do not lie on the frontier are termed *inefficient* and the analysis provides measure of their relative efficiency.

The classification and characterization of the DMUs plays an important role in DEA. Charnes, Cooper and Thrall (where, we will refer to as CCT, see [11, 12]) presented a structure for this characterization using two linear program (the envelopment and multiplier) problems. They presented a number of

* Corresponding author. E-mail address: saeid_mehrabian@tmu.ac.ir

[1] See [10].

theorems, which not only extended the CCR model theoretically, but also provided a tool in the classification and characterization of DMUs.
In this work, a new algorithm (where, we will refer to as M-algorithm) is presented for classification of DMUs by the non-Archimedean CCR models. The new procedure is based on some theoretical results of CCT and Arnold et al. ([5]). M-algorithm is compared with Thrall's algorithm (where, we will refer to as T-algorithm) and its advantage is that it needs less computational effort compared to T-algorithm. In other words, M-algorithm achieves the same results of the T-algorithm by solving with fewer linear program problems.
This paper is organized as follows. In the next section, we give some basis concepts of the CCR model. Section 3 contains slacks and some of concepts for the proposed algorithm. In section 4, we introduce M-algorithm. Comparison of two algorithms from a computational point of view on real data domains is included in section 5. Finally, section 6 gives some concluding remarks.

2. Fundamental and Concepts

Consider a collection of n DMUs to be evaluated. Each DMU is a vector $\begin{pmatrix} y \\ -x \end{pmatrix} \in \mathbf{R}^{s+m}$, where $y \in \mathbf{R}^s$, $x \in \mathbf{R}^m$, $(y,x) \geq 0$ and neither of the vectors y or x is zero. The amounts of output r and input i for DMU_j are represented, respectively by y_{rj} and x_{ij}.
A DEA data domain D considers a set of n decision making units $\text{DMU}_1,\dots,\text{DMU}_n$ and a collection of technologies which is characterized by a data matrix $P=(P_1,\dots,P_n)=\begin{pmatrix} Y \\ -X \end{pmatrix} \in \mathbf{R}^{(s+m)\times n}$, with $P_j=\text{DMU}_\text{j}$, $Y=(y_1,\dots,y_n)\in \mathbf{R}^{s\times n}$ and $X=(x_1,\dots,x_n)\in \mathbf{R}^{m\times n}$. A standard assumption for DEA is that no two columns of P are proportional.
The production possibility set associated to the data matrix P is defined as:

$$K(P)=\left\{ p=\begin{pmatrix} y \\ -x \end{pmatrix} \in \mathbf{R}^{s+m} : p \leq P\lambda \text{ for some } \lambda \geq 0,\ \lambda \in \mathbf{R}^n \text{ and } (y,x)\geq 0 \right\}.$$

Consider the linear programming problem associated to DMU_p ($p=1,\dots,n$):

Envelopment Problem:

$$\begin{aligned} &\min\ \theta \\ &\text{s.t.}\quad S = P\lambda - \begin{pmatrix} y_p \\ -\theta x_p \end{pmatrix} \geq \mathbf{0}, \\ &\qquad\ \lambda \geq \mathbf{0}, \end{aligned} \qquad (1)$$

where $\theta \in \mathbf{R}$, $\lambda \in \mathbf{R}^n$ and $S \in \mathbf{R}^{s+m}$.

Problem (1) is the CCR model and is referred to as the envelopment (or primal) program for DMU_p. Let $\left(\theta_p^*, \lambda_P^*, S_p^*\right)$ be a solution of (1). The DEA-radial-efficiency for DMU_p is defined as the optimal value θ_p^*.

The DMU_p is said to be DEA-radial-efficient if $\theta_p^* = 1$ and the DMU_p is said to be DEA-radial-inefficient if $\theta_p^* < 1$. This definition is equivalent to saying that a DMU is DEA-radial-efficient if it lies in the frontier of the production possibility set.

The definition of efficient used in DEA is based on the idea that to remain in the production possibility set, a DMU is technical efficient if the inputs and outputs, corresponding to the DMU, cannot be, respectively, decreased or increased. Therefore, a DMU_p can be a boundary point ($\theta_p^* = 1$) and not be efficient. For this reason, a given DMU_p is called DEA-efficient if $\theta_p^* = 1$ and $S_p^* = 0$ for all solutions $\left(\theta_p^*, \lambda_P^*, S_p^*\right)$ of problem (1).

There is an alternative dual approach to DEA-radial-efficiency for DMU_p using the multiplier space W_p defined as:

$$W_p = \left\{ w = \begin{pmatrix} u \\ v \end{pmatrix} \in \mathbf{R}^{s+m} : h_p(w) \ge h_j(w),\ j = 1,\ldots,n,\ w \ge \mathbf{0} \text{ and } u \ne \mathbf{0},\ v \ne \mathbf{0} \right\}.$$

Here, the functions h_j are defined as $h_j(w) = \dfrac{u^T y_j}{v^T x_j},\ j = 1,\ldots,n,$ for each $w = \begin{pmatrix} u \\ v \end{pmatrix},\ u \in \mathbf{R}^s,\ v \in \mathbf{R}^m,\ w \ge \mathbf{0}$ such that $v^T x_j > 0$.

Let $W_p^m = \left\{ w \in W_p : v^T x_p = u^T y_p = 1 \right\}$. The set W_p^m is called the normalized multiplier set for DMU_p. Observe that $W_p^m = \varnothing$ if and only if $W_p = \varnothing$.

Consider the linear programming problem:

Multiplier Problem:

$$\begin{aligned} \max\ & u^T y_p \\ \text{s.t.}\ & v^T x_p = 1, \\ & -t_j = u^T y_j - v^T x_j = w^T P_j \le 0,\ j = 1,\ldots,n \\ & w = \begin{pmatrix} u \\ v \end{pmatrix} \ge \mathbf{0}, \end{aligned} \tag{2}$$

where $u \in \mathbf{R}^s$, $v \in \mathbf{R}^m$ and $T \in \mathbf{R}^n$.

Problem (2) is the dual problem (1) and is referred to as the multiplier (or dual) program for DMU_p.

It is easy to see that $(w_p^*,\ t_p^*)$ is a solution of problem (2) if and only if $w_p^* \in W_p^m$. Therefore, by duality theory, $W_p \neq \varnothing$ if and only if $\theta_p^* = 1$. Hence, DMU_p is DEA-radial-efficient if $W_p \neq \varnothing$ and is DEA-radial-inefficient if $W_p = \varnothing$.

Let $\boldsymbol{RE}$ be the set of all DEA-radial-efficient DMU_j and $\boldsymbol{N}$ be the set of all the DEA-radial-inefficient DMU_j, for $j = 1, \ldots, n$.

The DEA-radial-efficient can be computed using either of the two linear programs (1) and (2). Hence, the solutions of these problems answer the question of DEA-radial-efficient. However, among those DMUs that are DEA-radial-efficient (or DEA-radial-inefficient), there are important differences depending on their multiplier sets. Thus, the set of DMUs may be partitioned into the following six classes (see [11, 12] for more details):

$$
\begin{aligned}
E \quad &= \left\{ \mathrm{DMU}_j \in \boldsymbol{RE} : \dim W_j = s + m \right\}, \\
E' \quad &= \left\{ \mathrm{DMU}_j \in \boldsymbol{RE} : \dim W_j < s + m \text{ and there exist } w > 0,\ w \in W_j \right\}, \\
F \quad &= \left\{ \mathrm{DMU}_j \in \boldsymbol{RE} : \text{ every } w \in W_j \text{ has at least one zero component} \right\}, \\
NE \quad &= \left\{ \mathrm{DMU}_j \in \boldsymbol{N} : \mathrm{DMU}'_j \in E \right\}, \\
NE' \quad &= \left\{ \mathrm{DMU}_j \in \boldsymbol{N} : \mathrm{DMU}'_j \in E' \right\} \\
NF \quad &= \left\{ \mathrm{DMU}_j \in \boldsymbol{N} : \mathrm{DMU}'_j \in F \right\}
\end{aligned}
$$

Here, $\mathrm{DMU}'_j = P_j(\theta_j^*) = \begin{pmatrix} y_j \\ -\theta_j^* x_j \end{pmatrix}$.

Note that $(E,\ E',\ F)$ and $(NE,\ NE',\ NF)$ are partitions of set $\boldsymbol{RE}$ and the set of $\boldsymbol{N}$, respectively. The elements of $E,\ E'$ and F are called, respectively, DEA-extreme efficient, DEA-non-extreme efficient, and DEA-weak efficient. It can be seen (see [12]) that any given DMU_p is DEA-efficient if it belongs to $E \cup E'$, or equivalently, if there exist a positive multiplier vector $w \in W_j$. Figure (1) shows classification of DMUs for two inputs and one output.

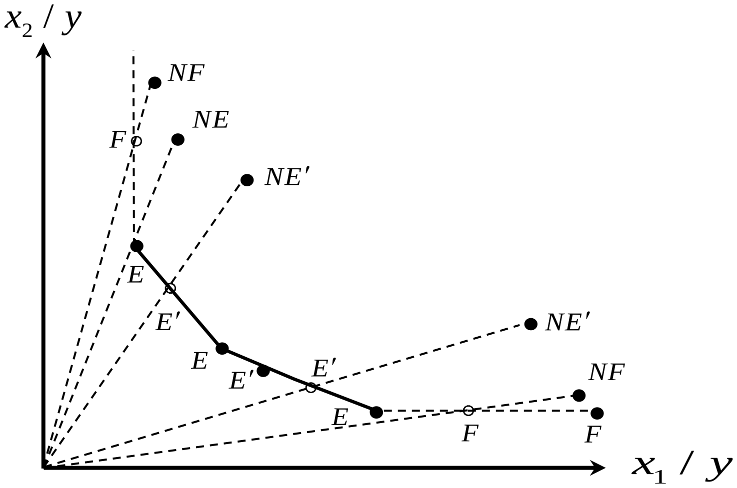

Figure 1: Classification of DMUs with two inputs and one output.

3. Slacks

The envelopment slacks play an important role in efficiency analysis. Because of the importance of slacks, we shall describe how to modify the radial efficiency model in order to identify the existence of any positive slacks and calculate radial efficiency. In order to achieve this, we use the non-Archimedean CCR model for DMU_k as follows:

Envelopment Problem:

$$
\begin{aligned}
E(P_k):\min\ & \theta-\varepsilon\mathbf{1}^T S\\
\text{s.t.}\ & P\lambda - S=\begin{pmatrix} y_k\\ -\theta x_k\end{pmatrix},\\
& \lambda\geq \mathbf{0},\\
& S\geq \mathbf{0},
\end{aligned}
$$

Multiplier Problem:

$$
\begin{aligned}
M(P_k):\max\ & u^T y_k\\
\text{s.t.}\ & v^T x_k=1,\\
& w^T P+T^T\leq \mathbf{0},\\
& w=\begin{pmatrix} u\\ v\end{pmatrix}\geq \varepsilon\mathbf{1},\ T\geq \mathbf{0},
\end{aligned} \tag{3}
$$

where ε is a small positive non-Archimedean infinitesimal. This non-Archimedean approach ensures that the sum of slacks will be maximal without worsening the optimal value of the radial-efficiency. The problem $E(P_k)$ can be solved in two phases, first using the problem (1) to determine the optimal θ_k^* and then solve the following problem to determine S_k^*:

$$
\begin{aligned}
SE(P_k):\min\ & \mathbf{1}^T S\\
\text{s.t.}\ & P\lambda - S=\begin{pmatrix} y_k\\ -\theta_k^* x_k\end{pmatrix},\\
& \lambda\geq \mathbf{0},\\
& S\geq \mathbf{0}.
\end{aligned} \tag{4}
$$

Definition 1: Suppose that $(\lambda^*,\ S^*)$ and $(T^*,\ w^*)$ are optimal solution of $E(P_k)$ and $M(P_k)$, respectively. It is said that the *strong complementary slackness conditions (SCSC)* holds, if $\lambda^{*T}T^*=0,\ S^{*T}w^*=0$ and $\lambda^*+T^*>0,\ S^*+w^*>0$. Then $(\lambda^*,\ w^*)$ is an SCSC solution and we call $(\lambda^*,\ w^*)$, an SCSC pair for the DMU_k.

Definition 2: Suppose that $U_m \in \boldsymbol{N}$. Then $\mathrm{DMU}_k \in U_m$, if the optimal basic solution for $E(P_k)$ is unique.

4. A New Algorithm for DMUs Classification: M-Algorithm

In order to determining the efficiency classes of each DMUs, we introduce a new algorithm (M-algorithm). This algorithm uses the non-Archimedean CCR (3), based on the following theorem (see [5]):

Theorem (1): Using the problems first in (1) and then in (4) produces a solution that is optimal for the envelopment problem in (3).

This theorem declares $\hat{\theta}_k-\varepsilon\mathbf{1}^T\hat{S}_k=\theta_k^*-\varepsilon\mathbf{1}^T S_k^*$, where '^' designates an optimal value for the problem $E(P_k)$ in (3) and '*' designates an optimal value for the problem $SE(P_k)$ in (4) with θ_k^*, an optimal value for the problem (1) In M-algorithm, we calculate an assurance value for ε using only simple computations on inputs and outputs of DMUs ([18]) .

Suppose $\bar{\varepsilon}$, an assurance value for non-Archimedean, is known. Solving the problem (3) for DMU_k with $\bar{\varepsilon}$ substituted for ε lead us to an optimal solution $(\theta_k,\ \lambda_k,\ S_k)$ to the problem $E(P_k)$ with dual solution $(T_k,\ w_k)$.

Assume $\theta_k=1$. If $\mathbf{1}^T S_k=0$ then DMU_k belong to E or E', according to λ_k^k is positive or zero. Otherwise it belongs to F (see [23]).

Now, suppose $\theta_k<1$. If the envelopment and multiplier problems (3) have unique optimal solutions, then $(\lambda_k,\ w_k)$ is an SCSC pair Therefore, $\mathrm{DMU}_k \in U_M$. The uniqueness of the optimal envelopment and multiplier solutions can be obtained, respectively, from equations[1] $ze(\lambda_k)+ze(S_k)=n+1$ and $ze(T_k)+ze(w_k)=m+s-1$ that is a result of duality theorem in LP. If $\mathbf{1}^T S_k\neq 0$, $\mathrm{DMU}_k \in \boldsymbol{NF}$. Otherwise DMU_k belong to $\boldsymbol{NE}$ or $\boldsymbol{NE'}$ according to $(\lambda_k^k>0,\ \lambda_k^j=0 \text{ for } j\neq k)$ be established or not.

If $(\lambda_k,\ w_k)$ is an SCSC pair ($\mathrm{DMU}_k \in U_M$) and $(\lambda_k^k>0,\ \lambda_k^\ell=0 \text{ for some } \ell\neq k)$, then, $\mathrm{DMU}_\ell \in E$ (see [23]). This means, we have classified the DMU_ℓ.

---

[1] $ze(\mathbf{x})$ is the number of zeros in vector $\mathbf{x}$

Convergence of M-algorithm is guaranteed by introduced theorems in [12, 23]. We outline M-algorithm as pseudo code in Table (1).

5. Computational Results and Comparison of M-Algorithm and T-Algorithm

In this section, we compare two algorithms M and T from a computational point of view. These two algorithms have been applied to 25 real data domains. The computational work is carried out using the software GAMS for model representation with LP optimizer XA (see [9]). The results are summarized in Table (2) and sources of the real data domains are reported in Table (3).

Table 1:
Pseudo code of M-algorithm

Step 0: Input problem data, $k = 1$. $flag(j) = 1$ if DMU_j classified, else 0. $flag(j) = 0,\ j = 1, \dots, n$. Compute an assurance value $\bar{\varepsilon},\ \varepsilon \leftarrow \bar{\varepsilon}$.

Step 1:
Solve problem $E(P_k)$.
case ($\theta_k < 1$)
    if (($\lambda_k,\ w_k$) is an SCSC pair), $\text{DMU}_k \in U_M$.
    if ($\mathbf{1}^T S_k = 0$),
        if ($\lambda_k^k > 0,\ \lambda_j^\ell = 0$ for $j \neq k$), $\text{DMU}_k \in \boldsymbol{NE}$.
        else,
            $\text{DMU}_k \in \boldsymbol{NE}'$,
            if (($\theta_k,\ \lambda_k,\ S_k$) is unique & ($\lambda_k^k > 0,\ \lambda_k^\ell = 0$ for some $\ell \neq k$),
                $\text{DMU}_\ell \in E,\ flag(\ell) = 1$
        endif
    else,
        $\text{DMU}_k \in \boldsymbol{NF}$,
        if (($\theta_k,\ \lambda_k,\ S_k$) is unique & ($\lambda_k^k > 0,\ \lambda_k^\ell = 0$ for some $\ell \neq k$),
            $\text{DMU}_\ell \in E,\ flag(\ell) = 1$.
    endif
case ($\theta_k = 1$)
    if ($\mathbf{1}^T S_k = 0$),
      if ($\lambda_k^k > 0$), $\text{DMU}_k \in E$.
      else $\text{DMU}_k \in E'$.
      endif

else, $\text{DMU}_k \in F$.
endif

Step 2:

$k \leftarrow k+1$
if ($k > n$), stop.
else, if ($flag(k)=1$), go to Step 2
else, go to Step 1.

Table 2:
Computational results of real data

Domains	$i,\ o$	n	n_T	n_M	$\|E\|$	$\|E'\|$	$\|F\|$	$\|N\|$	$\|NE\|$	$\|NE'\|$	$\|NF\|$	$\|U_M\|$
R1	2, 1	13	13	9	6	0	0	7	0	7	0	7
R2	2, 1	13	13	11	3	0	0	10	0	9	1	10
R3	2, 1	13	13	11	2	0	0	11	0	8	3	11
R4	2, 1	14	14	13	5	0	0	9	0	0	9	5
R5	3, 2	15	15	10	7	0	0	8	0	0	8	8
R6	2, 3	18	18	15	3	0	0	15	0	0	15	15
R7	2, 2	19	19	17	6	0	0	13	0	6	7	12
R8	2, 3	23	23	16	8	0	0	15	0	4	11	3
R9	3, 3	28	28	26	8	0	0	20	0	0	20	3
R10	3, 3	28	28	24	10	0	0	18	0	0	18	8
R11	4, 4	30	30	19	17	0	0	13	0	0	13	13
R12	4, 2	30	30	22	9	0	0	21	0	5	16	21
R13	2, 8	30	30	27	16	0	0	14	0	0	14	11
R14	1, 5	37	37	35	2	0	0	35	0	0	35	35
R15	3, 1	41	41	36	7	0	0	34	0	1	33	34
R16	4, 3	42	42	32	19	0	0	23	0	0	23	23
R17	2, 3	44	44	40	6	0	1	37	0	0	37	37
R18	3, 3	50	50	42	13	0	0	37	0	3	34	32
R19	3, 3	52	52	43	14	0	0	38	0	0	38	37
R20	4, 4	55	55	37	20	0	0	35	0	1	34	35
R21	4, 2	69	69	51	21	0	0	48	0	6	42	43
R22	4, 3	74	74	59	22	0	0	52	0	0	52	52
R23	5, 2	108	108	102	19	0	1	78	0	8	70	11
R24	3, 3	1282	1289	1267	15	0	0	1267	0	100	1167	920
R25	6, 12	1282	1282	1119	254	0	1	1027	0	0	1027	440
	Total	3410	3417	3083	512	0	3	2885	0	158	2727	1826
	% of Total	**%100**	%100	%90.4	%15.0	%0	%0.1	%84.6	%0	%4.6	%80	%53.5
							$\|N\|$ **% of**	**100**	**%0**	%5.5	%94.5	%63.3

These results consist of:

- Cardinality for the sets E, E', F and NE, NE', NF produced by partitioning each data domain in both algorithms.
- n_T and n_M , the number of the solved LPs in T and M algorithms, respectively.

A quick review of these results shows that:

1. As expected, the cardinality of the sets E, E', F and NE, NE', NF are the same for all data domains in both T and M algorithms. Note that in domain R3, which is associated to 13 Mexican banks in 1991, DMU 13 is considered as a member of the set NF by these two algorithms while it is reported as an NE' member by [24]
2. Set E' is empty for all domains. Also, set F is empty for all domains, except domains R17 (44 power plant data in Iran), R23 (108 water supply services in Japan) and R25 (1282 Canadian banks branch) in which $|F|=1$. These domains represent only $\%0.1$ of the DMUs population.
3. Set NE is empty in all domains.
4. In all real domains, $|NE'|$ is much smaller than $|NF|$, and overall they cover only $\%5.5$ of set $\boldsymbol{N}$.
5. The most interesting part of these results is that the number of solved LPs in M-algorithm is less than those solved in T-algorithm. This shows that in M-algorithm, solving $\%90.4$ of the LPs is enough to complete the computation work, while in T-algorithm; we need to solve at least one LP for each DMUs.
6. In M-algorithm, the minimum number of the problems that we need to solve, is $n-|E|$. This means the more efficient DMUs involve less computational work. See real data domains R3, R6, R14 and R24 in this regard.

6. Conclusion

In this paper, we introduced a new algorithm based on a set of theorems. This algorithm uses the non-Archimedean CCR model to classify DMUs. We showed how M-algorithm can classify DMUs successfully without solving all related problems.

Table 3:
The real data domains references.

Domain	Source	Domain	Source
R1, R2, R3	[24]	R13	[21]
R4	[25]	R14	[19]
R5.R20	[20]	R15	[22]
R6	[27]	R16, R22	[16]
R7	[15]	R17	[26]
R8	[13]	R18, R19	[8]
R9, R10	[14]	R21	[7]
R11	[17]	R23	[1]
R12	[4]	R24, R25	[2,3]

References

[1] Aida, K., W. W. Cooper, J. T. Pastor, and T. Sueyyoshi (1998), Evaluating Water Supply Service in Japan with RAM: A Range Adjusted Measure of Inefficiency, Omega, Vol. 26, No. 2, pp. 207-232.

[2] Alirezaee, M. R., M. Howland, C. van de Panne (1995), A Large Scale Study of Bank Branch Efficiency, Paper Presented at the 37^{th} National Conference of the Canadian Operational Research Society, May 23-25, 1995 in Calgary.

[3] Alirezaee, M. R., M. Howland, C. van de Panne (1998), Sampling Size and Efficiency Bias in Data Envelopment Analysis, Journal of Applied Mathematics & Decision Science, Vol. 2, No. 1, pp. 51-64.

[4] Allen, R., A. Athanassopoulos, R. G. Dyson and E. Thanassoulis (1997), Weights Restrictions and Value Judgments in Data Envelopment Analysis: Evolution Development and Future Directions, Annals of Operations Research, Vol. 73, pp. 13-34.

[5] Arnold, V., I. Bardhan, W. W. Cooper, and A. Gallegos (1998), Primal and Dual Optimality in Computer Codes Using Two-Stage Solution Procedures in DEA, in Operations Research: Methods, Models, and Applications, J. E. Aronson and S. Zionts (editors), Westport, CT: Greenwood Publishing Co., pp. 57-96.

[6] Banker, R. D., A. Charnes, and W. W. Cooper (1984), Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis, Management Sciences, Vol. 30, No. 9, pp. 1078-1092.

[7] Banker, R. D. and R. C. Morey (1986), The Use of Categorical Variable in Data Envelopment Analysis, Management Science, Vol. 32, No. 12, pp. 1613-1627.

[8] Beasley, J. E. (1990), Comparing University Departments, Omega, Vol. 18, No. 2, pp. 171-183.

[9] Brooke, A., D. Kendrick, and A. Meeraus (1996), GAMS: A User's Guide, Release 2.50, The Scientific Press.

[10] Charnes, A., W. W. Cooper and E. L. Rhodes (1978), Measuring the Efficiency of Decision Making Units, European Journal of Operational Research, Vol. 2, No. 6, pp. 429-444.

[11] Charnes, A., W. W. Cooper and R. M. Thrall (1986),Classifying and Characterizing Efficiencies and Inefficiencies in Data Envelopment Analysis, Operations Research Letters, Vol. 5, No. 3, pp. 105-110.

[12] Charnes, A., W. W. Cooper and R. M. Thrall (1991), A Structure for Classifying and Characterizing Efficiencies and Inefficiencies in Data Envelopment Analysis, The Journal of Productivity Analysis, Vol. 2, pp. 197-237.

[13] Charnes, A., W. W. Cooper and E. L. Rhodes (1981), Evaluating Program and Managerial Efficiency: An Application of Data Envelopment Analysis to Program Follow Through, Management Science, Vol. 27, No. 6, pp. 668-697.

[14] Charnes, A., W. W. Cooper, and S. Li (1998), Using DEA to Evaluate the Efficiency of Economic Performance by Chinese Cities, Socio-Economic Planning Sciences, Vol. 23, No. 6, pp. 325-344.

[15] Jahanshahloo, G. R., and M. R. Alirezaee (1994), Measuring the Efficiency of Academic Units at the Teacher Training University, Journal of Economic and Management, Quarterly Journal of the Islamic Azad University, Vol. 20, pp.1-13.

[16] Jahanshahloo, G. R., M. R. Alirezaee, F. Hossein-zadeh lotfi and C. van de Panne (1997), Determining the Comparative Efficiency of High Schools; An Application of DEA, Proceeding: the 28th Annual Iranian Mathematics Conference in Tabriz, March 28-31, pp. 219-230.

[17] Martić, M., G. Savić (1998), An Application of DEA for Comparative Analysis and Ranking of region in Serbia with Regards to Social-Economic Development, Working Paper, Faculty of Organizational Sciences, Belgrade, Yugosla.

[18] Mehrabian Saeid (2007), The Uniqueness of the Overall Assurance Interval for Epsilon in DEA Models by the Direction Method, Applied Mathematical Sciences, for coming.

[19] Oral, M. O. Kettani, and P. Lang (1991), A Methodology for Collective Evaluation and Selection of Industrial R&D Projects, Management Science, Vol. 37, pp. 871- 885.

[20] Seiford, L. M. and J.Zhu (1997), Stability Rrgions for Maintaining Efficiency in Data Envelopment Analysis, European Journal of Operational Research, Vol. 1o8, No. 1, pp. 127-139.

[21] Sueyoshi, T., K. Ohnishi, and Kinse (1999), A Benchmark Approach for Baseball Evaluation, European Journal of Operational Research, Vol. 115, No. 3, pp. 429-448.

[22] Sueyoshi, T. (1998), Privatization of Nippon Telegraph and Telephone: Was it is a Good Policy Decision?, European Journal of Operational Research, Vol. 107, No. 1, pp. 45-61.

[23] Thrall, R. M. (1996), Duality, Classification, and Slack in DEA,'' Annals Operations Research, Vol. 66, pp. 109-138.

[24] Taylor, W. M., R. G. Thompson, R. M. Thrall, and P. S. Dharmapala (1997), DEA/AR Efficiency and Profitability of Mexican Banks: A Total Income Model, European Journal of Operational Research, Vol. 98, No. 2, pp. 346-363.

[25] Tone, K., A Simple Characterization of Returns to Scale in DEA, Research Report, Journal of Operational Society of Japan, Vol. 39, No. 4.

[26] Zhu, J. (1996), Robustness of the Efficient DMUs in Data Envelopment Analysis, European Journal of Operational Research, Vol. 90., No. 3, pp. 451-460.

[27] Zhu, J. (1998), Data Envelopment Analysis Vs. Principal Component Analysis: An Illustrative Study OF economic Performance of Chinese cities, European Journal of Operational Research, Vol. 111, No. 1, pp. 50-61.