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The calculation of unit's efficiency by using the Interval Balance Index and the Interval TOPSIS

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Abstract

Data envelopment analysis (DEA) is a technique for measuring the efficiency of decision making units. In all models of the DEA, for each unit under assessment, the numerical efficiency is obtained which may be less than or equal to one. Given the possible large number of functional units, we use various ranking methods for evaluating units. One of the rating methods is Balance index and Topsis. This method has been used for categorical data. In this paper, we assume data as interval, introduce the interval Balance index and the interval Topsis and run it on a single example.

Keyword: DEA, Ranking, Interval Data, Balance Index, Topsis

1. Introduction

DEA, which was developed by Charnes et al [3] to evaluate the relative efficiency of decision-making units in 1978, is a non-parametric method and is based on linear programming. in1957, Farrell [5] was the first to construct the production possibility set in a non-parametric method. Charnes et al developed Farrell approach and presented a model called CCR. Then Banker et al [2] offered BCC model in 1984.

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Cooper et al [4] (1999) added Data envelopment analysis to the uncertain data. in2007, Alirezaee and Afsharian [1], ranked DMUs by the Balance index method. In 2008, Jahanshahlooet al [6] ranked DMUs by the interval Topsis method. In this paper, we intend to obtain the efficiency of a range of intervals and calculate the efficiency of units by the interval Topsis and interval Balance index method. Considering the rating is not completely specified in the interval efficiency, we attempt to rank DMUs as well as interval data by Jahanshahloo et al [7] method and determine the actual position of the data in comparison with each other.

Furthermore, this paper will be as follows: InSection2, the necessary introductions for the next sections will be presented. In Section 3, ranking interval data by the interval Balance index will be introduced. InSection4, ranking interval data by the interval Topsis method will be presented. In Section5, a numerical example will be presented to illustrate the method and in the final section we will have conclusions.

2. Background

One of the models that is used to obtain the efficiency of DMUs is the CCR model in the Input Orientation. The amount of the θ of the CCR envelopment model in the Input Orientation produces the desirable DMU efficiency.

$$\begin{aligned} \min \quad & \theta \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j X_{ij} \le \theta X_{io} \qquad i = 1,\dots, m \\ & \sum_{j=1}^{n} \lambda_j Y_{rj} \ge Y_{ro} \qquad r = 1,\dots, s \\ & \lambda_j \ge 0, \qquad j = 1,\dots,n \end{aligned} \tag{1}$$

This information is not sufficient to the complete ranking of DMU. Because there may be several efficient DMUs. Namely, They may have$\theta^* = 1$.

Here we rank DMUs with constant DEA models and Balance index methods are used for this purpose.

The Balance index is obtained from the CCR Multiplier model in the Input Orientation:

We write the CCR Multiplier model in Input Orientation which is the Dual Envelopment model of CCR in the Input Orientation.

$$\begin{aligned} \max \quad & \sum_{r=1}^{s} u_r y_{rp} \\ s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad (j = 1,\dots,n) \\ & \sum_{i=1}^{m} v_i x_{ip} = 1 \\ & u_r \ge 0 \qquad (r = 1,\dots,s) \\ & v_i \ge 0 \qquad (i = 1,\dots,m) \end{aligned} \tag{2}$$

The optimal value of the objective function of Model 2, is the θ (The optimal value of the objective function) obtained from Model 1.

So, in the first step, to obtain θ^* we can solve Model 2 and obtain $\sum_{r=1}^{s} u_r y_{rp} = \theta_p^*$. By the solution of Model 2 for the assessment of DMU_P, (u_p^*, v_p^*) results.

Now. Consider in the second stage of constraint $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0$. Clearly, if (u_p^*, v_p^*), DMU_p are efficient, then $\sum_{r=1}^{s} u_{p_r}^* y_{rj} - \sum_{i=1}^{m} v_{p_i}^* x_{ij} = 0$ ، But if $\sum_{r=1}^{s} u_{p_r}^* y_{rj} - \sum_{i=1}^{m} v_{p_i}^* x_{ij} < 0$, this means that (u_p^*, v_p^*), does not make DMU_j efficient. and the more negative and distant the value, (u_p^*, v_p^*), Puts DMU_jin a worse situation.

For DMUs that have equal $\theta^*(\sum_{r=1}^{s} u_r y_{rp})$, is done in this way:

We calculate $\sum_{r=1}^{s} u_{p_r}^* y_{rj} - \sum_{i=1}^{m} v_{p_i}^* x_{ij}$ for each DMU_j (j=1,…,p,…,n) and calculate all values together,

If the values of the balance index for DMU_P was smaller (more negative), DMU_P among those DMUs with DMU_P, have the sameθ^*would have worse ranking. Then DMUs will be ranked alphabetically.

3. Interval Balance Index method

After the presentation of a certain mode of Balance Index, we want to offer its interval mode. Multiplier model CCR in Input Oriented that DMU under evaluation (DMU_P)at best State, and other units (DMU_j)at worstareisas follows:

$$\begin{aligned} \theta_p^U &= \max \quad u_p y_p^u \\ s.t. \quad & v_p x_p^l = 1 \\ & u_p y_j^l - v_p x_j^u \leq 0 \quad \forall_{j \neq p} \\ & u_p y_p^u - v_p x_p^l \leq 0 \\ & u_p, v_p \geq 0 \end{aligned} \tag{3}$$

In this case, the balance index rate for DMUs under evaluation and the rest of the units can be defined as follows:

$BI_J^L = u_p^* y_j^l - v_p^* x_j^u$ For other DMUs

$BI_P^U = u_p^* y_p^u - v_p^* x_p^l$ For the DMU under evaluation

And, in the same vein, if the DMU under evaluation (DMU_P) is in its worst state, and other units (DMU_j) are in their best state, is defined as follows:

$$
\begin{aligned}
\theta_p^L = \max \quad & u_p y_p^l \\
s.t. \quad & v_p x_p^u = 1 \\
& u_p y_j^u - v_p x_j^l \le 0 \quad \forall\, j \ne p \\
& u_p y_p^l - v_p x_p^u \le 0 \\
& u_p, v_p \ge 0
\end{aligned}
\tag{4}
$$

In this case, the balance index rate for DMUs under evaluation and the rest of the units can be defined as follows:

$BI_J^U = u_p^* y_j^u - v_p^* x_j^l$ For other DMUs

$BI_P^L = u_p^* y_p^l - v_p^* x_p^u$ For DMUs under evaluation

As a result, the upper bound and lower balance index are defined as follows:

The lower bound of the Balance index DMU_P: $\sum_{\substack{j=1 \\ j \ne p}}^{n} (u_j^* y_p^u - v_j^* x_p^L) + u_p^* y_p^l - v_p^* x_p^u$

The upper bound of the balance index DMU_P: $\sum_{\substack{j=1 \\ j \ne p}}^{n} (u_j^* y_p^l - v_j^* x_p^u) + u_p^* y_p^u - v_p^* x_p^l$

4. Interval TOPSIS method:

TOPSIS method was presented by Chen and Hwang, The basic principle is that the chosen alternative should have the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution. The procedure of interval TOPSIS can be expressed in a series of steps:

The normalized values n_{ij}^l and n_{ij}^u are calculated as:

$n_{ij}^{l} = \frac{x_{ij}^{l}}{\sqrt{\sum_{i=1}^{m}\left[\left(x_{ij}^{l}\right)^{2}+\left(x_{ij}^{u}\right)^{2}\right]}}$ and $n_{ij}^{u} = \frac{x_{ij}^{u}}{\sqrt{\sum_{i=1}^{m}\left[\left(x_{ij}^{l}\right)^{2}+\left(x_{ij}^{u}\right)^{2}\right]}}$ for (i=1,…,m) (j=1,…,n)

Then the interval $\left[n_{ij}^{l}, n_{ij}^{u}\right]$ is the normalized form of interval $\left[x_{ij}^{l}, x_{ij}^{u}\right]$. If the criteria have different importance, we can construct the weighted normalized decision matrix as $v_{ij}^{l} = w_i * n_{ij}^{l}$ and $v_{ij}^{u} = w_i * n_{ij}^{u}$ for i=1,…,m and j=1,…,n, where w_i is the weight of ith criteria and $\sum_{i=1}^{n} w_i = 1$.

Now suppose Alternative k to define that the ideals follow these steps:

(1) First set A_k (Alternative k) in its best situation (the lower bounds of all cost indexes and upper bounds for all benefit indexes) and set other alternatives in their best situation, too. Then, define A_k^{+u} in this form:

$A_k^{+u} = \{v_1^{+u}, v_2^{+u}, \dots, v_n^{+u}\} = \left\{(\, max\ v_{ij}^{u} \middle|\ i \in O\ ), (\, min\ v_{ij}^{l} \middle|\ i \in I\right\}$ where O is associated with benefit criteria and I with cost criteria.

(2) Set A_k in the worst case (upper bounds for inputs and lower bounds for outputs) and set other alternatives in their best situation. So we have:

$A_k^{+l} = \{(v_1^{+l}, v_2^{+l}, \dots, v_n^{+l})\} = \{(\max_{j \neq k}\{v_{ij}^{u}, v_{ik}^{l}\} \mid i \in O), (\min_{j \neq k}\{v_{ij}^{l}, v_{ik}^{u}\} \mid i \in I)\}$

By this approach we can make an interval ideal to evaluate A_k. So the ideal and the negative-ideal are changed for each alternative. This is logically true because of the property that all rates are nondeterministic.

(3) Define A_k^{-u} in this form:

$A_k^{-u} = \{(v_1^{-u}, v_2^{-u}, \dots, v_n^{-u})\} = \{(\min_{j \neq k}\{v_{ij}^{l}, v_{ik}^{u}\} \mid i \in O), (\max_{j \neq k}\{v_{ij}^{u}, v_{ik}^{l}\} \mid i \in I)\}$

(4) Define A_k^{-1}in this form :

$A_k^{-1} = \left\{v_1^{-1}, v_2^{-1}, \dots, v_n^{-1}\right\} = \left\{(min\ v_{ij}^{l} \middle|\ i \in O\ ), (\, max\ v_{ij}^{u} \middle|\ i \in I\right\}$

We define d_k^{+u} as the distance between the worst case of $A_k$and d_k^{+u}. So we have

$$d_k^{+u} = \sqrt{\sum_{i \in 1}(v_i^{+u} - v_{ik}^{u})^2 + \sum_{i \in O}(v_i^{+u} - v_{ik}^{l})^2}$$

and define θ* in the form of

$$d_k^{+l} = \sqrt{\sum_{i \in 1}(v_i^{+l} - v_{ik}^{l})^2 + \sum_{i \in O}(v_i^{+l} - v_{ik}^{u})^2}$$

Now we can define the other two distances using the same procedure.

d_k^{-u}: The distance between the best situation of A_k and A_k^{-l}.

$$d_k^{-u} = \sqrt{\sum_{i\in 1}\left(v_i^{-u} - v_{ik}^{l}\right)^2 + \sum_{i\in O}\left(v_i^{-u} - v_{ik}^{u}\right)^2}$$

d_k^{-l}: The distance between the worst situation of A_k and A_k^{-u}

$$d_k^{-l} = \sqrt{\sum_{i\in 1}(v_i^{-l} - v_{ik}^{u})^2 + \sum_{i\in O}(v_i^{-l} - v_{ik}^{l})^2}$$

After these definitions, we can let R_k be in this interval:

$$\frac{d_k^{-l}}{d_k^{-u} + d_k^{+u}} \le R_K \le \frac{d_k^{-u}}{d_k^{-l} + d_k^{+l}}$$

The final step is to rank the options in order of preference

5. Numerical Example:

In this paper, the performance of electronic services in 30 branches of the Refah bank in 1389 will be assessed. Variables will be introduced in terms of two inputs and five outputs and then using the interval Balance index method and interval topsis, the data will be solved in the form of gams software. And then after obtaining the upper and lower limits for the offered model, using the method mentioned in Jahanshahloo and colleagues, paper in 2009, Bank branches will be ranked.

The following table illustrates the input and output data.

Table 1:

The data of the inputs and outputs

	INPUT DATA				OUTPUT DATA							
	INPUT1		INPUT2		OUT PUT 1	OUTPUT2		OUT PUT 3	OUTPUT4		OUTPUT5	
	L	U	L	U		L	U		L	U	L	U
DMU1	1870202871	2122932989	10674980	12117545	404	25018.5	28399.4	6	115.63	131.3	39660.3	45019.8
DMU2	2150449338	2441050600	3958581	4493525	765	28996	32914.4	42	209.98	238.4	40848.9	46369.1
DMU3	2528052626	2869681359	28325497	32153268	981	64583.5	73311	45	1489.3	1691	33457.3	37978.5
DMU4	1136253828	1289801643	32373066	36747805	829	16934.9	19223.4	96	23.125	26.25	397.75	451.5
DMU5	1046802099	1188261842	10766374	12221290	220	20883.7	23705.9	154	294.15	333.9	306.175	347.6
DMU6	1924404970	2184459695	97047882	110162462	261	67329.8	76428.5	134	202.58	230	44435.2	50439.9
DMU7	2603221081	2955007713	67028730	76086667	682	75386.6	85574	2	71.225	80.85	265.475	301.4
DMU8	1099459016	1248034559	51392589	58337534	717	34205.6	38828	87	185	210	16812.8	19084.8
DMU9	1579341699	1792766253	9843792	11174034	1,975	24821.5	28175.7	52	220.15	249.9	46799.5	53123.7
DMU10	1295118608	1470134636	17920821	20342554	1,707	32097.5	36435	78	22.2	25.2	1296.85	1472.1
DMU11	1531862830	1738871320	31820945	36121073	192	43953.2	49892.9	121	229.4	260.4	283.05	321.3
DMU12	988324309	1121881648	9099579	10329252	246	59022.4	66998.4	63	8.325	9.45	458.8	520.8
DMU13	3548842299	4028415582	306866303	348334725	4,107	77140.4	87564.8	75	42.55	48.3	91.575	104.0
DMU14	2115803803	2401723236	26299366	29853335	2,444	66706.4	75720.8	36	345.03	391.7	1012.88	1149.8
DMU15	1559981157	1770789422	8815641	10006944	1,648	37125.8	42142.8	22	333	378	21695.9	24627.8
DMU16	1272494916	1444453688	131986299	149822287	329	44741.3	50787.5	210	537.43	610.1	28838.7	32735.9
DMU17	1372412324	1557873449	18177087	20633451	284	55944.9	63505.1	64	226.63	257.3	148	168.0
DMU18	2467588738	2801046676	16028088	18194046	1,460	39616.8	44970.5	553	430.13	488.3	97423.8	110589.2
DMU19	849153943	963904476	74054238	84061568	436	15921.1	18072.6	394	27.75	31.5	35829	40670.7
DMU20	952866320	1081632039	75507987	85711769	159	30848.8	35017.5	62	41.625	47.25	48200.8	54714.5
DMU21	903175476	1025226216	37216645	42245922	229	30656.4	34799.1	197	562.4	638.4	58879	66835.7
DMU22	1489083972	1690311536	40863258	46385320	2,169	13070.3	14836.5	18	130.43	148.1	182.225	206.9
DMU23	2025705705	2299449719	58968501	66937218	555	27537.3	31258.5	135	43.475	49.35	81688.6	92727.6
DMU24	2454634125	2786341439	21296048	24173892	789	54987.6	62418.3	22	15.725	17.85	428.275	486.2
DMU25	1407453992	1597650477	147775663	167745349	1,384	28132	31933.7	174	91.575	104	40771.2	46280.9
DMU26	1159940803	1316689560	282492102	320666714	1,586	50354.2	57158.9	334	87.875	99.75	80704.4	91610.4
DMU27	1235117793	1402025603	28806079	32698793	838	10878	12348	292	42.55	48.3	69375.9	78751.1
DMU28	1616696124	1835168573	26900139	30535293	922	4600.95	5222.7	523	80.475	91.35	138157	156827.0
DMU29	1676040114	1902532021	108372683	123017642	1349	23381.2	26540.9	110	41.625	47.25	175.75	199.5
DMU30	1258483488	1428548825	38474167	43673379	475	6518.48	7399.35	90	76.775	87.15	327.45	371.7

Finally, the upper and lower bounds obtained from solving model and the final Ranking have been presented.

Table 2:
Final ranking of DMUs

	Balance index		**Topsis**		**Ranking**	
	EFF(u)	**EFF(l)**	**EFF(u)**	**EFF(l)**	**Balance index**	**Topsis**
DMU1	-24.439	-33.095	0.339982	0.325745	14	28
DMU2	-22.235	-30.491	0.364284	0.348981	12	17
DMU3	-25.491	-32.404	0.514418	0.489673	15	3
DMU4	-24.27	-31.211	0.336416	0.32534	13	29
DMU5	-12.366	-17.116	0.364505	0.35513	4	25
DMU6	-57.777	-71.223	0.35185	0.31227	25	10
DMU7	-56.791	-72.059	0.321782	0.292154	24	16
DMU8	-28.68	-35.867	0.347261	0.328445	18	19
DMU9	-9.884	-15.185	0.418275	0.40159	2	9
DMU10	-13.507	-19.29	0.374551	0.363649	5	22
DMU11	-27.558	-35.854	0.346494	0.329636	16	23
DMU12	-8.502	-12.616	0.349082	0.336647	1	27
DMU13	-167.113	-199.055	0.402269	0.381346	29	12
DMU14	-19.933	-28.208	0.431719	0.410607	11	6
DMU15	-10.501	-15.511	0.400536	0.385787	3	15
DMU16	-65.122	-76.663	0.37579	0.340152	27	8
DMU17	-17.252	-23.874	0.353494	0.337067	10	21
DMU18	-13.543	-22.734	0.583642	0.550646	8	1
DMU19	-34.478	-42.249	0.4039	0.386467	21	13
DMU20	-40.516	-48.856	0.325745	0.301592	22	20
DMU21	-15.122	-18.454	0.429406	0.405193	6	5
DMU22	-28.403	-35.939	0.372382	0.3596	17	18
DMU23	-44.162	-56.071	0.369615	0.336583	23	11
DMU24	-35.201	-47.364	0.332272	0.315879	20	26
DMU25	-74.507	-88.478	0.344995	0.309913	28	14
DMU26	-125.476	-144.895	0.406682	0.363802	30	4
DMU27	-16.721	-23.307	0.422121	0.401296	9	7
DMU28	-13.611	-20.694	0.52713	0.493685	7	2
DMU29	-64.977	-78.478	0.313871	0.28914	26	24
DMU30	-32.301	-40.191	0.321665	0.309686	19	30

6. Conclusion

In this paper, we have proposed the ranking results by using 2 methods. As the following methods have the different indexes for ranking, the results are different.

For example, when we use TOPSIS Method, the selected alternative should have the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution but in Balance Index Method, each of the DMUs are evaluating according to their own weights and the others'.

So, the results are different because of the differences of the methods.

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