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Methods of Optimization in Imprecise Data Envelopment Analysis

N. Malekmohammadi[*a], H. Alimohammadizadeh Noughi[b]

(a) *Department of Mathematics, Faculty of Science, Islamic Azad University, South Tehran Branch, Iran.*

(b) *Department of Mathematics, Faculty of Science, Islamic Azad University, South Tehran Branch, Iran.*

Abstract

In this paper imprecise target models has been proposed to investigate the relation between imprecise data envelopment analysis (IDEA) and mini-max reference point formulations. Through these models, the decision makers' preferences are involved in interactive trade-off analysis procedures in multiple objective linear programming with imprecise data. In addition, the gradient projection type method can be suggested to determine a normal vector at a given efficient solution on the efficient frontier and to establish an interactive procedure for searching for the most preferred solution (MPS) that maximizes the decision maker implicit utility function.

Keywords: Data envelopment analysis, gradient projection method, multiple objective linear programming, imprecise data.

1 Introduction

In recent years the relation between data envelopment analysis (DEA) and multiple objective linear programming (MOLP) has received great deal of attentions for researchers. The structures of these two types of model have much in common but DEA is directed to assessing past performances as part of management control function and MOLP to planning future performances [1].
There exist some studies about the similarities between DEA and multiple criteria decision analysis (MCDA) generally and MOLP in particular. Doyle and Green [2] indicate that DEA is an MCDA method itself. Golany [5] firstly established an interactive model involving both DEA and MOLP approaches where the decision makers (DMs) can allocate a set of input levels as resources and to choose the most preferred set of output levels from a set of alternative points on the efficient frontier. Therefore, the effective integration of assessing past performances and planning future targets with the decision makers' preferences taken into account which increase interests to support both management control and planning.
Yang et al. [4] established models (during the investigation of the relations between the output-oriented dual DEA model and the mini-max reference point formulations), named the super-ideal point model,

[*]Corresponding author, Email address: n.malekmohammadi@gmail.com

the ideal point model and the shortest distance model. Yang et al. method considered a model which is a radial model with output oriented projects all DMUs to efficient frontier by solving n linear programming. Malekmohammadi et al. [7] proposed model which is non-radial and also all of the inputs and outputs will be projected to the efficient frontier, simultaneously. Instead of solving n mathematical programming problem, it is needed to solve one.

On the other hand, considering the importance of imprecise data such as fuzzy, ordinal and interval data in organizations the DEA models with exact data have been extended to imprecise data. Cooper et al. [2] introduced applications of DEA whose data was imprecise. In imprecise data envelopment analysis (IDEA), the data can be ordinal, interval or fuzzy, which results in a non-linear DEA model. Despotis and Smirlis [4] also studied the problem of IDEA and developed an alternative approach to deal with imprecise data in DEA. They converted a nonlinear DEA model to linear programming equivalent by applying transformations only to the variables, resulting in efficiency scores which were intervals. According to their approach, Wang et al. [8] developed a new pair of interval DEA models that resulted in the best lower bound efficiency and the best upper bound efficiency of each DMU. They used a fixed and uniformed production frontier to determine the efficiencies of decision-making units (DMUs) with interval input and output data.

In this paper, a super-ideal model is suggested which considers both the decrease of total input consumption and the increase of total output production with imprecise data. Thereby, Yang et al. method is improved by the suggested method which determines the most preferred solution for DMs, mostly, for imprecise data.

2 Interactive MOLP Methods for Evaluating Efficiency and Target Setting with Imprecise Data

In this section, the equivalence between a DEA model and the mini-max reference point formulations is found by the following model. So, efficiency analysis can be conducted and interactive MOLP techniques will be used to locate the most preferred solution or set target value for all DMUs, simultaneously.

By the proposed model (1) [7] and the gradient projection method a normal vector can be determined at a given efficient solution on the efficient frontier and to establish an interactive procedure for searching for the most preferred solution that maximizes the decision makers implicit utility function, as follows:

Let $j, r = 1,...,n,$ be the indices for decision-making units (DMUs). Consider the index sets of inputs $, I = \{1,...,m\}$ and outputs $, O = \{1,...s\},$ and their subsets $I \equiv I_f \cup \bar{I}_f$ and $O \equiv O_f \cup \overline{O}_f$ where I_f and O_f are used to indicate inputs and outputs which have limited resources. The vector $(\lambda_{1r}, \lambda_{2r}, \ldots, \lambda_{nr})$, such that $\sum_{j=1}^{n} \lambda_{jr} = 1, r = 1,...,m$ is imposed for convex combination between inputs or outputs for n DMUs.

In our models, the best possible relative efficiency intervals $[\theta_i^L, \theta_i^U]$ and $[Z_k^L, Z_k^U]$ are considered where θ_i^U and Z_k^U stand for the upper bound of the best possible relative efficiency achieved by all DMUs under evaluation and the DMUs are in the state of the best production activity, while θ_i^L and Z_k^L stand for the lower bound of the best possible relative efficiency of all DMUs under evaluation. Also, in our models $[G_i^L, G_i^U]$ and $[G'^L_k, G'^U_k]$ indicate the interval of existing resources for total interval input i and total interval output k for all DMUs.

s'_i, s''_k are considered for the permissibility of total input reduction and total output production, respectively and M in the objective function is a penalty factor that has to be considered by DM.

$$
\begin{aligned}
\max_{\lambda_j, Z_k, \theta_i} \quad & \sum_{k \in O} P_k^+ Z_k^U - \sum_{i \in I} P_i^- \theta_i^U - M \sum_{i \in I_f} s'_i - M \sum_{k \in O_f} s''_k \\
\text{s.t.} \quad & \sum_{r=1}^{n} \sum_{j=1}^{n} \gamma_{jr} x_{ij}^L = \theta_i^U \sum_{j=1}^{n} x_{ij}^L, && i \in I, \\
& \sum_{r=1}^{n} \sum_{j=1}^{n} \gamma_{jr} y_{kj}^U = Z_k^U \sum_{r=1}^{n} y_{kr}^U, && k \in O, \\
& \sum_{j=1}^{n} \theta_i^U x_{ij}^L \geq G_i^L - s'_i, && i \in I_f, \\
& \sum_{r=1}^{n} Z_k^U y_{kr}^U \leq G'^U_k + s''_k, && k \in O_f, \\
& \sum_{j=1}^{n} \gamma_{jr} = 1, \quad r = 1, \ldots, n, \\
& \gamma_{jr} \geq 0, \; j = 1, \ldots, n, \; \theta_i^U \text{ free}, \; i \in I, \; Z_k^U \text{ free}, \; k \in O, \\
& s'_i \geq 0, \; \forall i \in I_f, \; s''_k \geq 0, \; \forall k \in O_f.
\end{aligned}
$$

Where G_i^L and G'^U_k, indicate the bounds for total consumption of input x_{ij}^L and total production of output y_{kr}^U, respectively.

After solving the mentioned model, gradient projection method can be used as follows,

Step 1: Generate a total output and input payoff table.

Optimize each of the upper bounds of total composite outputs and the lower bounds of total inputs of the observed DMU. For each total composite output and input, set target for total output and input values as $Y_k'^*$, $X_i''^*$ respectively, from the decision maker (DM) as initial point.

Step 2: Generate initial efficient solution

Set the initial weighting parameters for all DMUs, and reach the initial solution of the decision variables and the initial objective value and the initial dual variable values for the first k and i constraints on the outputs and inputs.

Step 3: Compute the normal vector and check optimality condition

At interaction t, compute the normal vectors. Collect reference composite total output and input respectively. By collecting the optimal indifference trade-off between the collected composite input and output each of the other can be done. If the DM agrees on such optimal indifference trade-offs between each of the other composite total outputs and inputs, then the current solutions are most preferred solution (MPS) and the interactive process will be finished. Otherwise, the DM would provide new indifference trade-offs.

Step 4: Determine the trade-off direction

The projection of the DMs indifference trade-offs onto the tangent plane of the efficient frontier can be obtained, or the new trade-off direction can be estimated.

Step 5: Determine the trade-off step size and calculate the new weighting vector.

The trade-off step sizes are to be determined by the largest and smallest permissible step.

3 Results and discussions

In this section, the proposed MOLP interactive procedure is applied to search for MPSs along the efficient frontier to the United Kingdom (UK) retail bank industry. The data set collected from Wong and Yang (2004) through a study on data envelopment analysis and multiple criteria decision making based on the evidential reasoning approach-performance measurement of UK retail banks (Yang (2001);Yang and Xu (2002)). It is mentionable that the data has been changed from exact to interval to be more suitable for the research in this paper.

Table 1
Data Set

DMU	Bank	Input	Input	Input	Input	Output	Output
		No.of Branches (000)	No.of ATMs ('000)	No.of staff ('0,000)	Asset size (£'00 bn)	Customer satisfaction	Total revenue (£ m)
1	Abbey	0.77	[2.18,2.2]	[2.96, 3]	[2.96,3]	6.79	[9,10.57]
2	Barclas	1.95	[3.19,3.20]	[3.53,4]	[3.53,3.6]	2.55	13.35
3	Halifax	0.80	[2.10,2.2]	[2.41,2.5]	[2.41,2.5]	9.17	[8,8.14]
4	HSBC	1.75	[4.00, 4.1]	[4.85, 5]	[4.85,5]	5.82	23.67
5	Lloyds TSB	2.50	[4.30,4.7]	[2.40,2.45]	[2.40,2.43]	6.57	14.01
6	Nat West	1.73	[3.30,3.40]	[3.09,3.2]	[3.09,3.1]	4.86	12.04
7	RBS	0.65	[1.73,1.80]	[1.34,1.36]	[1.34,1.4]	7.28	[7,7.36]

Adapted from:Yang et al. (2009).

Table 1 consist of seven DMUs (retail banks), and four inputs and two outputs are considered. The DMUs are comparable major banks in the UK including Abbey National, Barclays, Halifax, HSBC, Lloyds TSB, NatWest and RBS (Royal Bank of Scotland). The four inputs are namely number of branches, number of ATMs, number of staff and asset size. The two outputs are customer satisfaction and total revenue.
Model (1) is run (solved by LINGO) to reach the efficiency score and after, the input consumption and output production are calculated. By the gradient projection method we are able to reach the most preferred solution. By the proposed method the more decrease of total input consumption has been achieved than Yang et al. method. The main difference of the suggested method in this paper was the consideration of imprecise data.

4 Conclusions

In this paper, Yang et al. method is improved via proposed imprecise model. This approach results in decreasing total input consumption and increasing total output production at the same time. Instead of solving n independent linear programming (LP) models just one LP model is solved, thereby, yields in saving time in calculation of the method. The proposed method results in the more decrease of total input consumption than Yang et al. method.

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