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A Ranking DEA model based on cross-weights evaluation

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Abstract
The cross-efficiency method in data envelopment analysis (DEA) has widely been used as a suitable utility for ranking decision-making units (DMUs). In cross-efficiency, the average of n efficiency values for each DMU is considered as the overall efficiency score. Another method based on the concept of cross-efficiency for calculating the efficiency score for each DMU is to use the average comparable weights to calculate the efficiency, which is called the cross-weight evaluation. In the cross-weight method, as in the cross-efficiency method, there is the issue of multiple optimal solutions. In this paper, for overcoming this issue, we use the neutral strategy for cross-weight evaluation. Unlike the aggressive and benevolent formulations, the neutral way is only concerned with its own interests and is indifferent to other DMUs. This study proposes a new cross-weight evaluation by maximizing the minimum of the output weights while keeping the efficiency of the unit under evaluation unchanged. Numerical examples are provided to illustrate the potential application of this new model and its effectiveness in ranking DMUs.

Keywords: Data envelopment analysis, Cross-weight evaluation, Ranking.

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1. Introduction

Data envelopment analysis introduced to assess the relative efficiency of a homogeneous group of decision-making units, such as banks, police stations, hospitals, tax offices, schools and university departments [1]. The cross-evaluation method is a ranking method in DEA that involves self-evaluation efficiency and peer-evaluation efficiency [2]. Traditional DEA models are constructed based on linear programming and usually have alternate optimal solutions, so the cross efficiency scores obtained from these models are not unique[3]. For overcoming this issue, Doyle and Green [3] proposed the aggressive and benevolent models which these ideas are widely used in cross-efficiency evaluation. Both models attempt that maximize the efficiency of the DMU under evaluation, but simultaneously the benevolent model maximizes the average efficiency of other DMUs and the aggressive model minimizes the average efficiency of other DMUs. Liang, et al. [4] proposed the benevolent game cross efficiency. In this model, a unique set of weights is determined based on the Nash equilibrium and the benevolent strategy. Using the symmetric weight assignment technique, Jahanshahloo, et al. [5] suggested a new secondary goal for evaluation cross-efficiency score. Another strategy for cross-efficiency evaluation is the neutral strategy that first proposed by Wang and Chin [6]. Unlike aggressive or benevolent models, the neutral cross-efficiency method attempts to specify a set of weights for inputs and outputs of each DMU from its own profit perspective. Wang, et al. [7] proposed a neutral method for cross-efficiency evaluation based on the distance of each DMU from the best DMU (IDMU) or the worst DMU (ADMU. Based upon method in Wang, et al. [7], Carrillo, et al. [8] proposed a neutral model that determines an optimal set of weights that maximize the efficiency score of the IDMU and minimize the efficiency score of the ADMU simultaneously while keeping the efficiency of the evaluated unit unchanged. Shi, et al. [9] utilized an ideal frontier and anti-ideal frontier as evaluation criteria and proposed a new method for evaluating cross-efficiency scores. Using IDMU and ADMU, Liu, et al. [10] introduced a prospect value based on prospect theory and proposed a new secondary goal based on neutral strategy for evaluating cross-efficiency scores.

Örkcü, et al. [11] proposed an extended model for cross-efficiency evaluation of the basic two-stage network systems. This model can be ranked each DMU based on the efficiency score of sub-stages and overall efficiency score, that the overall efficiency is the product of those of the stages. Shi, et al. [12] proposed a neutral cross efficiency evaluation method based on the prospect theory, which reflects the bounded rationality of DMUs when facing gain and loss, as secondary goals.

Cross-weight evaluation is another method based on concept of cross efficiency that can be used for calculating the efficiency score for ranking each DMU. In this method the average comparable weights are used to calculate the efficiency. In the cross-weight method, as in the cross-efficiency method, there is the issue of multiple optimal solutions. In this paper, for overcoming this issue, we propose a neutral secondary goal for cross-weight evaluation. The proposed model can guarantee not only the maximum self-evaluation efficiency of the DMU being evaluated but also maximize the minimum of the output weights. The reminder of this paper is arranged as follows. In section 2, we address the cross-efficiency evaluation approach. The new model for evaluating the efficiency score is introduced in Section 3. In section 4, using two data sets, we compare results of the proposed model with others models and demonstrate effectiveness of the proposed model. Concluding is discussed in Section 5.

2. Cross-efficiency evaluation

We consider n DMUs that each $DMU_j\,(j=1,2,...,n)$ produces s different outputs indexes $Y_j=(y_{1j},y_{2j},...,y_{sj})\in\mathbb{R}_+^s$ from m different inputs indexes $X_j=(x_{1j},x_{2j},...,x_{mj})\in\mathbb{R}_+^m$, where $\mathbb{R}_+^s$ and $\mathbb{R}_+^m$ are two sets of nonnegative numbers. The efficiency of DMU_j is as follows:

$$\frac{\sum_{r=1}^{s}u_r y_{rj}}{\sum_{i=1}^{m}v_i x_{ij}} \qquad (1)$$

Where $u_r\,(r=1,2,...,s)$ and $v_i\,(i=1,2,...,m)$ are the rth output and ith input weights respectively.

The cross-efficiency evaluation process is often two-step that is called self-evaluation and peer-evaluation which the efficiency score each DMU is evaluated against the weights of all DMUs, not just its own. Suppose $DMU_p=\left(X_p,Y_p\right)$ be the DMU under evaluation. In the first phase of cross-efficiency process, self-evaluation, the relative efficiency of DMU_p other DMUs can be calculated using the traditional DEA model such as the CCR model [1] that has the following form:

$$\begin{aligned}
E_{pp}^{*} &= Max\sum_{r=1}^{s}u_r^p y_{rp}\\
s.t.\quad & \sum_{r=1}^{s}u_r^p y_{rj}-\sum_{i=1}^{m}v_i^p x_{ij}\le 0,\ j=1,2,...,n\\
& \sum_{i=1}^{m}v_i^p x_{ip}=1, \qquad\qquad (2)\\
& u_r^p\ge 0, r=1,2,...,m\\
& v_i^p\ge 0, i=1,2,...,s
\end{aligned}$$

Where E_{pp}^{*} is referred the relative efficiency of DMU_p. Let $u_r^{p*}(r=1,2,...,s)$ and $v_i^{p*}(i=1,2,...,m)$ be optimal solution of model (2) for evaluation DMU_p, then in the second phase of cross-efficiency method, peer-evaluation, the cross efficiency score of DMU_j, using $u_r^{p*}(r=1,2,...,s)$ and $v_i^{p*}(i=1,2,...,m)$, calculates as follows:

$$E_{pj}=\frac{\sum_{r=1}^{s}u_r^{p*}y_{rj}}{\sum_{i=1}^{m}v_r^{p*}x_{ij}},\ \left(j=1,2...,n;\,j\ne p\right) \quad (3)$$

In this case, $\bar{E}_j={}^{1}\!/_{n}\sum_{p=1}^{n}E_{pj}$ is final score of $DMU_j\,(j=1,2,...,n)$ for ranking.

Model (2) usually generates alternative optimal solutions, so we have different cross-efficiency scores for each DMU. For overcoming this vagueness, a secondary goal in cross-efficiency evaluation is introduced by Sexton, et al. [2]. In this regard, Doyle and Green [3] present new secondary goals that are called benevolent and aggressive model. As mentioned in last section these models maximize the efficiency of DMU_p while maximize (minimize) the average the cross efficiency of other DMUs. The benevolent and aggressive formulations are as follows:

$$Min \quad \sum_{r=1}^{s} u_r^p \left(\sum_{j=1, j\neq p}^{n} y_{rj} \right) \qquad (a)$$

$$Max \quad \sum_{r=1}^{s} u_r^p \left(\sum_{j=1, j\neq p}^{n} y_{rj} \right) \qquad (b)$$

$$\begin{aligned} s.t. \quad & \sum_{i=1}^{m} v_i^p \left(\sum_{j=1, j\neq p}^{n} x_{ij} \right) = 1, \\ & \sum_{r=1}^{s} u_r^p y_{rp} - E_{pp}^* \times \sum_{i=1}^{m} v_i^p x_{ip} = 0, \quad (4) \\ & \sum_{r=1}^{s} u_r^p y_{rj} - \sum_{i=1}^{m} v_i^p x_{ij} \le 0, \ j = 1,2,\dots,n; \ j \neq p \\ & u_r^p \ge 0, r = 1,2,\dots,m \\ & v_i^p \ge 0, i = 1,2,\dots,s \end{aligned}$$

Where the maximization corresponds with the benevolent strategy and the minimization stands for the aggressive formulation.

Due to the different nature of the two models, two models provide different weights. As a result, the two methods usually will produce different rankings. Aiming to avoid aggressive and benevolent strategy in evaluating cross-efficiency score performance, Wang and Chin [6] proposed the following neutral model as a secondary goal to evaluate cross-efficiency in DEA, which intends to increase the efficiency of each output of DMU_p, $\frac{u_r^p y_{rp}}{\sum_{i=1}^{m} v_i^p x_{ip}} (r = 1,2,\dots,r)$, as possible as:

$$Max\, \delta = \underset{r=1,2,\dots,s}{Min} \left\{ \frac{u_r^p y_{rp}}{\sum_{i=1}^{m} v_i^p x_{ip}} \right\}$$

$$\begin{aligned} s.t. \quad & E_{pp}^* = \frac{\sum_{r=1}^{s} u_r^p y_{rp}}{\sum_{i=1}^{m} v_i^p x_{ip}} \qquad (5) \\ & \frac{\sum_{r=1}^{s} u_r^p y_{rj}}{\sum_{i=1}^{m} v_i^p x_{ij}} \le 1, \quad j = 1,2,\dots,n; \ j \neq p \\ & u_r^p \ge 0, \ r = 1,2,\dots,s \\ & v_i^p \ge 0, \ i = 1,2,\dots,m \end{aligned}$$

For DMU_p, Model (5) finds an optimal set of weights to maximize its each of outputs efficiency while its relative efficiency is kept. In this paper, we proposed a new secondary goal based on the neutral cross-efficiency evaluation idea.

3. Proposed model for the cross-weight evaluation

As can be seen from the traditional cross-efficiency evaluation model, each DMU determines a set of input and output weights. Therefore, n DMUs have n sets of input and output weights. The cross-efficiency evaluation is to use these n sets of weights to calculate n efficiency values for each DMU and then average them into an overall efficiency score. This is only one of the possible ways for assessing and ranking DMUs. Another way for assessing and ranking DMUs is to use the n sets of weights to generate an average set of input and output weights for the n DMUs which this way is referred to as cross-weight evaluation. For this, the n sets of weights have to be comparable; otherwise, they cannot be averaged. Accordingly we proposed the following model for the cross-efficiency evaluation of

$DMU_p\,(p=1,2,...,n)$, based on which n sets of generated weights are comparable.

$$
\begin{aligned}
& Z_{pu}^{*} = Max\ \delta \\
& s.t.\quad \sum_{r=1}^{s} u_r^p y_{rj} - E_{jj}^{*} \sum_{i=1}^{m} v_i^p x_{ij} \le 0,\ j=1,2,...,n \\
& \qquad \sum_{r=1}^{s} u_r^p y_{rp} - E_{pp}^{*} \sum_{i=1}^{m} v_i^p x_{ip} = 0, \\
& \qquad \sum_{j=1}^{n} \sum_{i=1}^{m} v_i^p x_{ij} = 1 \qquad\qquad (6) \\
& \qquad \delta \le u_r^p,\ r=1,2,...,m \\
& \qquad v_i^p \ge 0,\ i=1,2,...,s
\end{aligned}
$$

Where E_{pp}^{*} is obtained from model (2) for evaluating DMU_p. In this model we maximize the minimum of output weights while the efficiency of DMU_p is kept. Advantage of the model (6) is that it can effectively reduce the number of zero weights for outputs and reduces influences of the existence of multiple optimal solutions.

Let $\left(\bar{u}_r^p\,(r=1,2,...,s),\bar{v}_i^p\,(i=1,2,...,m)\right)$ be optimal solution of model (6) for the cross-weight evaluation of $DMU_p\,(p=1,2,...,n)$, thus the cross weight matrix is as follows:

According information in Table 1, the final values of input and output weights are as follows:

$$
\begin{aligned}
& v_i^{*} = \frac{1}{n}\sum_{j=1}^{n} \bar{v}_i^{j},\ i=1,2,...,m \\
& u_r^{*} = \frac{1}{n}\sum_{j=1}^{n} \bar{u}_r^{j},\ r=1,2,...,s \qquad (7)
\end{aligned}
$$

Then we calculate the final score of $DMU_j\,(j=1,2,...,n)$ using (7) and (1). Model (6) has nothing to do with the efficiency of other DMUs, so it can be categorized as a neutral model rather than aggressive and benevolent.

Table1: Cross-weight evaluation for n DMUs.

DMU		Input weights				Output Weights			
		$\bar{v}_1^p$	$\bar{v}_2^p$	$\ldots$	$\bar{v}_m^p$	$\bar{u}_1^p$	$\bar{u}_2^p$	$\ldots$	$\bar{u}_s^p$
Target DMU	1	$\bar{v}_1^1$	$\bar{v}_2^1$	$\ldots$	$\bar{v}_m^1$	$\bar{u}_1^1$	$\bar{u}_2^1$	$\ldots$	$\bar{u}_s^1$
	2	$\bar{v}_1^2$	$\bar{v}_2^2$	$\ldots$	$\bar{v}_m^2$	$\bar{u}_1^2$	$\bar{u}_2^2$	$\ldots$	$\bar{u}_s^2$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$			$\vdots$
	n	$\bar{v}_1^n$	$\bar{v}_2^n$	$\ldots$	$\bar{v}_m^n$	$\bar{u}_1^n$	$\bar{u}_2^n$	$\ldots$	$\bar{u}_s^n$

4. Numerical Example

In this section, to evaluate the performance of the proposed model, we consider two numerical examples that used in previous studies in the DEA literature. In all of the numerical examples below, models (4a), (4b), (5) and the proposed model are applied to rank all of DMUs and compare their performance.

Example 1. Consider the case of 10 top-level universities in first-tier cities of China in 2013 as DMUs with two inputs and two outputs which is adapted from [10]. The universities data are summarized in Table 2.

The CCR efficiency scores for 10 universities are revealed in the second column of Table 3. According to the results of the CCR model, 4 of 10 universities are identified as efficient DMUs. In this case, it is not possible to recognize their superiority over each other; so, we use results of models (4a), (4b), (5) and proposed model to rank them that are shown in the third through the tenth columns of Table 3, respectively.

Table 2: Data for 10 universities in 2013.

Universities (DMU)	Inputs		Outputs	
	Research and development fund (x1)	**Research and development staff (x2)**	**technology transfer revenue (y1)**	**Publication papers (y2)**
1	2159718	1868	32441	2174
2	4351036	2762	741905	6570
3	983639	642	4316	1782
4	2053945	1234	35819	2540
5	1754690	1141	1960	5078
6	2342515	836	2460	2992
7	2876203	1882	159634	7189
8	429710	594	1855	1465
9	1016343	1829	790	4515
10	1388514	975	75736	2267

Table 3: Results of models (4-5) and proposed model in Example 1

DMU	**CCR Model (2)**	**Aggressive Model(4)**	**Rank**	**Benevolent Model(4)**	**Rank**	**Neutral Model (5)**	**Rank**	**Proposed model**	**rank**
1	0.3501	0.2824	10	0.3380	10	0.3250	10	0.3355	10
2	1	0.8721	2	0.9816	3	0.9847	1	0.9816	2
3	0.6346	0.5484	8	0.6301	7	0.6084	7	0.6218	7
4	0.5105	0.4302	9	0.4857	9	0.4743	9	0.4801	9
5	1	0.8639	3	0.9924	2	0.9571	3	0.9789	3
6	0.8068	0.5559	7	0.5658	8	0.5726	8	0.5563	8
7	1	0.8737	1	1	1	0.9759	2	0.9908	1
8	0.8853	0.6544	5	0.8183	5	0.7750	5	0.8151	5
9	1	0.7218	4	0.9108	4	0.8661	4	0.9108	4
10	0.6905	0.5882	6	0.6805	6	0.6653	6	0.6760	6
sum		6.390891		7.403190		7.204322		7.346835	

Table4: Ranking models correlation test in example 1

	Spearmen's rho	**Model(4a)**	**Model(4b)**	**Model(5)**	**Proposed model**
Model(4a)	**Correlation**	1.0000	0.9758	0.9758	0.9879
	Sig.(bilateral)		0	0	0
Model(4b)	**Correlation**	0.9758	1.0000	0.9636	0.9879
	Sig.(bilateral)	0		0	0
Model(5)	**Correlation**	0.9758	0.9636	1.0000	0.9879
	Sig.(bilateral)	0	0		0
Proposed model	**Correlation**	0.9879	0.9879	0.9879	1.0000
	Sig.(bilateral)	0	0	0	

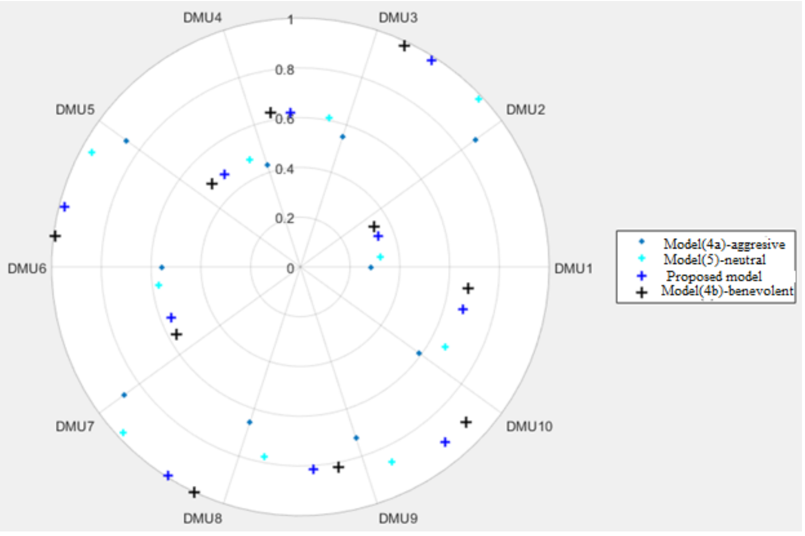

Figure 1. Illustrative comparison between the efficiency scores of models (4a), (4b), (5) and proposed model in Example 1.

Fig. 1 provides an illustrative comparison between models (4a), (4b), (5) and the proposed model in example 1 according to efficiency scores that are shown in Table 3. As can be seen n table 3, DMU_7 took first place in Model (4a), (4b) and proposed model whereas it is gained the second rank in Model (5). Note that DMU_1 has the worst performance in all methods.

Table 4 shows the correlation among the ranking values of the four models in table 2. After the Spearman test, the test values of correlations among models are all above 0.9.

Example 2. This example is taken from [14] and is about ranking of 29 companies (DMUs) of China's metal manufacturing industry with four inputs and five outputs described below and the data set is provided in Table 5:

Inputs: total assets(x1); asset-liability ratio(x_2); financial cost ratio(x_3); financial expense(x_4)

Outputs: net assets (y_1); revenue growth rate (y_2); total assets turnover ratio (y_3); earnings per share (y_4); current ratio (y_5)

The CCR efficiency scores and their rankings for 29 DMUs in the second column in table 6 show that 7 of 29 DMUs are efficient, so we cannot find any difference between them for ranking. Thus we use cross-efficiency for further distinction. The results of model 4-6 for evaluations of 29 enterprises are shown in the third and the tenth columns in Table 6.

Fig. 2 provides an illustrative comparison between the results of the models (4a), (4b), (5) and the proposed model for ranking of DMUs in example 2 according to ranking that are shown in Table 6.

As can be seen in Fig2, in all models, DMU1 and DMU3 that are non-efficient DMUs are ranked 29th and 28th, respectively. Moreover, the DMU10 have the first rank in four models.

Table 5: Data for 29 companies

DMUs	Inputs				Outputs				
	x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4	y_5
1	1.0000	0.8615	0.3778	0.6999	0.5401	0.1636	0.3994	0.5530	0.1115
2	0.1325	0.9033	0.4627	0.1934	0.4929	0.3918	0.4147	0.1805	0.1232
3	0.5260	0.8275	0.4582	1.0000	0.3989	0.1566	0.6249	0.1963	0.1138
4	0.1199	0.7958	0.6509	0.1992	0.3455	0.2025	0.3143	0.1000	0.1147
5	0.1653	0.5995	0.5093	0.2311	0.3936	0.1955	0.3140	0.1558	0.1240
6	0.1118	0.2704	0.1000	0.1348	0.4850	0.2541	0.2797	0.2383	0.2412
7	0.1574	0.4337	0.2593	0.1287	0.2352	0.3100	1.0000	1.0000	0.1832
8	0.1103	0.5008	0.3008	0.1525	0.5701	0.2599	0.6326	0.1614	0.1656
9	0.1185	0.4528	0.6124	0.2038	0.4761	0.2053	0.3702	0.1292	0.1595
10	0.1023	0.3989	0.1882	0.1408	0.6753	0.2155	0.6769	0.2640	0.1675
11	0.1256	0.4072	0.3392	0.1645	0.4029	0.2114	0.4979	0.1870	0.1523
12	0.1276	0.5038	0.7720	0.2113	0.4390	0.1738	0.2313	0.1099	0.1267
13	0.1022	0.2820	0.3069	0.1507	0.7561	0.1564	0.4748	0.2319	0.2255
14	0.1065	0.4393	0.4164	0.1575	0.6612	0.2986	0.3056	0.1082	0.1515
15	0.1269	0.6168	0.2408	0.1440	0.4388	0.6869	0.2748	0.3282	0.1115
16	0.1533	0.7862	0.4799	0.2251	0.4298	0.3047	0.4377	0.1163	0.1175
17	0.1179	0.5188	0.2765	0.1484	0.4556	0.2488	0.5382	0.3218	0.1702
18	0.1179	0.3204	0.3163	0.1583	0.4300	0.1782	0.6388	0.2062	0.1769
19	0.1415	0.3499	0.1166	0.1000	0.2643	0.2928	0.3847	0.3925	0.1831
20	0.1257	0.5399	0.5380	0.1764	0.4225	0.5014	0.2477	0.2126	0.1306
21	0.1281	0.3000	0.2011	0.1273	0.3879	0.1333	0.4839	0.4246	0.1924
22	0.1151	0.4305	0.2607	0.1475	0.4632	1.0000	0.2472	0.1998	0.2056
23	0.1066	0.3780	0.3058	0.1502	0.6645	0.3584	0.2691	0.1998	0.1363
24	0.1547	0.8324	0.6759	0.2691	0.3882	0.1245	0.2970	0.5145	0.1226
25	0.1291	0.4833	0.5727	0.1820	0.4311	0.3336	0.2041	0.1403	0.1357
26	0.1244	0.6089	0.3394	0.1611	0.4079	0.2191	0.4133	0.2062	0.1262
27	0.1022	0.3801	0.2214	0.1448	0.7779	0.2486	0.5813	0.2191	0.1490
28	0.1100	0.8030	0.6977	0.2066	0.7037	0.2658	0.6461	0.1741	0.1161
29	0.1835	0.7399	0.4916	0.2834	0.1000	0.3468	0.5062	0.7585	0.1381

Table 6: Results of models (4-6) and proposed model in Example 2

DMU	CCR Model(2)	Aggressive Model(4a)	Rank	Benevolent Model(4b)	Rank	Neutral Model(5)	Rank	Proposed model	Rank
1	0.4761	0.1507	29	0.1436	29	0.1487	29	0.1313	29
2	0.6452	0.4348	19	0.4899	20	0.4910	18	0.5172	19
3	0.3862	0.1631	28	0.1893	28	0.1675	28	0.1577	28
4	0.5170	0.3185	25	0.3675	25	0.3583	24	0.3871	24
5	0.3910	0.2910	27	0.3419	26	0.3266	27	0.3426	27
6	1	0.7589	6	0.7744	6	0.7946	6	0.7940	6
7	1	0.7743	4	0.8762	5	0.8712	4	0.8675	5
8	0.9115	0.6503	7	0.7586	7	0.7249	7	0.7618	7
9	0.6624	0.4376	18	0.5263	18	0.4898	19	0.5231	18
10	1	0.8333	1	0.9380	1	0.9105	1	0.9352	1
11	0.7084	0.4917	16	0.5906	16	0.5514	16	0.5749	16
12	0.4955	0.3195	24	0.3792	24	0.3578	25	0.3850	25
13	1	0.7860	3	0.9198	3	0.8676	5	0.9106	2
14	0.8787	0.5568	15	0.6527	13	0.6247	15	0.6631	13
15	0.9433	0.5797	13	0.6388	14	0.6603	13	0.6602	14
16	0.5225	0.3508	23	0.4097	23	0.3945	23	0.4135	23
17	0.8375	0.5978	12	0.6737	11	0.6678	11	0.6961	11

18	0.9678	0.6125	10	0.7537	8	0.6841	10	0.7105	8
19	1	0.6134	9	0.6310	15	0.6613	12	0.6463	15
20	0.6891	0.4476	17	0.5328	17	0.5151	17	0.5379	17
21	0.9522	0.6217	8	0.6966	10	0.6875	8	0.6990	10
22	1	0.7672	5	0.9057	4	0.8880	2	0.8957	4
23	0.9200	0.6115	11	0.7070	9	0.6867	9	0.7101	9
24	0.6845	0.3120	26	0.3387	27	0.3510	26	0.3693	26
25	0.5536	0.3748	21	0.4465	21	0.4272	21	0.4516	21
26	0.5988	0.4322	20	0.4909	19	0.4846	20	0.5088	20
27	1	0.8032	2	0.9203	2	0.8832	3	0.9094	3
28	0.9539	0.5657	14	0.6571	12	0.6319	14	0.6766	12
29	0.7049	0.3673	22	0.4124	22	0.4207	22	0.4224	22
sum		15.023981		**17.163034**		**16.728394**		**17.258686**	

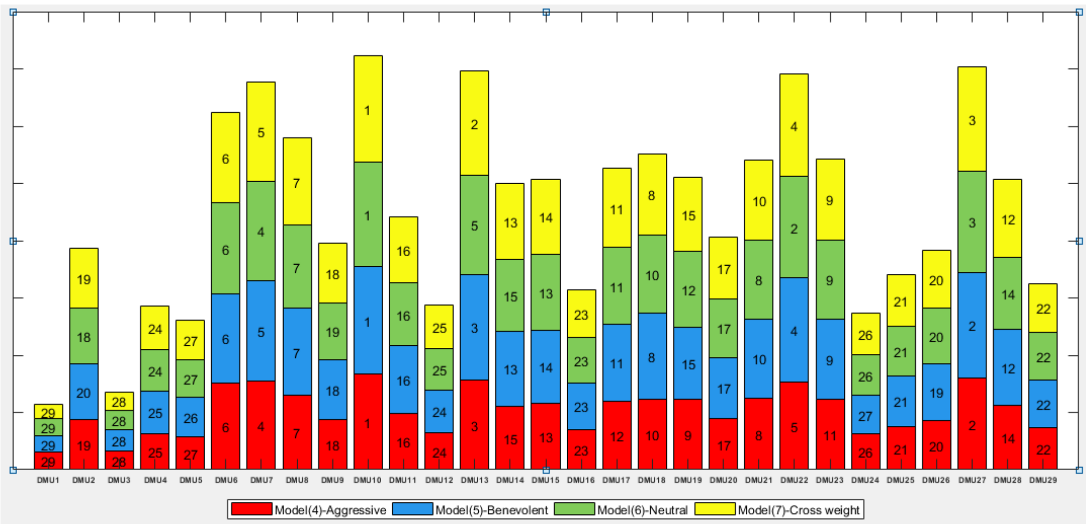

Figure 2. Illustrative comparison between the ranking results of models (4a), (4b), (5) and the proposed model in Example 2.

Table7: Ranking models correlation test in example 2

	Spearmen's rho	Model(4a)	Model(4b)	Model(5)	Proposed model
Model(4a)	**Correlation**	1.0000	0.9842	0.9921	0.9882
	Sig.(bilateral)		0	0	0
Model(4b)	**Correlation**	0.9842	1.0000	0.9887	0.9946
	Sig.(bilateral)	0		0	0
Model(5)	**Correlation**	0.9921	0.9887	1.0000	0.9946
	Sig.(bilateral)	0	0		0
Proposed model	**Correlation**	0.9882	0.9946	0.9946	1.0000
	Sig.(bilateral)	0	0	0	

Table 7 shows the correlation among the ranking values of the four models in table 6. After the Spearman test, the test values of correlations among models (4a), (4b), (5) and the proposed model are all above 0.9.

5. Conclusion

Cross-weights method is a method for ranking DMUs in data envelopment analysis. The structure of this method is similar to cross-efficiency method, with the difference that instead of using average efficiencies, comparable weights are used. There is also the issue of optimal multiple solutions in data envelopment analysis models for cross-weight models. This can decline the benefit of cross-weight assessments. For overcoming this problem, several different secondary goals are proposed based on benevolent, aggressive and neutral point of view. In this paper, we proposed a model based on neutral strategy in DEA, which seeks input and output weights that not only undertakes the maximum self-assessment efficiency of DMU under evaluation, but also the maximize the minimum of output weights. Finally, two numerical examples are illustrated and validated our proposed method.

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