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Identification of congestion in two-stage data envelopment analysis

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Abstract

This paper aims at examining congestion in two-stage decision making units. Providing an example, it will be proved that presence or absence of congestion in the whole process of a two-stage decision making unit has nothing to do with presence or absence of it in each of stages. In other words, it is likely that the first stage to be weak efficient and the second one will have congestion, while the whole process lacks congestion. It is also possible that each stage has congestion, but it doesn't mean that the whole process should have congestion. Then, to identify congestion in two-stage decision making units, modified Cooper model is developed.

Keywords: decision making units, network, efficiency, congestion

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1. Introduction

Data Envelopment Analysis (DEA), as a method, is utilized to evaluate the efficiency of multi-input and multi-output, congruent decision-making units (DMUs). In DEA, however, in addition to efficiency measurement, other factors such as Return to Scale (RTS), congestion, and efficient units ranking are taken into consideration. Due to its importance, many studies have been conducted on congestion and it has been widely discussed by many researchers. Congestion is an economic phenomenon, which is likely to occur in production. The main focus of most companies is on profits and costs and mainly ignore the imposed costs on the consumers. In the 1980s, Far and Groskovef introduced the concept of congestion and presented a method to identify it using DEA [1]. Following that, in order to study congestion, another model was developed by Cooper, Thampson, and Therall (CTT) [2]. Cooper's studies were developed by Brockett and finally led to a model called BCSW [3]. Congestion in Chinese industries was re-examined by Jahanshahloo and Khodabakhshi [4] and led to a modified one-model approach, developed by Khodabakhshi to calculate congestion. Jahanshahloo, Hossein Zadeh, Noura, Rashidi, and Parker [5] proposed a new method to calculate congestion of DMU, which remarkably simplifies the computation process. Tone and Sahoo [6] proposed an approach to measure the degree of scale economics and congestion. Also, NEW approach was proposed by Wei and Yan. Sometimes, abundance and excess of resources have negative impact on efficiency. In fact, the accumulation of resources leads to money wasting because we cannot allocate money to other sections. In such cases, it is common to say congestion has occurred at input.

One of the most basic methods in evaluating multi-stage units is to apply DEA models for the first stage, second stage, and the whole process. This was proposed by Seiford and Zhou [7] (1999). However, in this approach, a DMU might generally be efficient, while neither of stages one and two perform efficiently. After that, Chen and Zhou [8] (2004) presented a DEA, in which the efficiency of each stage is defined based on production possibility set. Then, the two stages are connected using mediating variable measures. Kao and Hwang [9] (2008) proposed another approach aiming at analyzing the overall efficiency of such processes and making comparison of stages one and two possible. However, the model at RTS condition turns to a non-linear model. The model is also not able to present an efficient image. This model was modified by Chen et al. [10] (2009) and in the modified one the image is efficient. In another approach, Chen et al [11] (2009), based on convex combination suggested the first and second stages of their model at constant and variable returns to scale conditions. This model was generalized by Cook et al. [12] (2010) for multistep processes with parallel structures. Also, using SBM in DEA, Ton et al. [13] (2009) evaluated networks performances.

As in the real world many under review units are multiple steps, network is widely used by many researchers to evaluate the performance of many organizations. As an illustration, Paradi [14] (2011) used two-stage processes to evaluate the performance of commercial bank branches. Amado et al. [15] (2011) started network integration and BSC. Chen et al. [16] (2011) used network to evaluate supply chains.

The remainder of this paper is organized as follows. In Section 2, BCC model and congestion model are discussed. In Section 3, our proposed two- stage congestion model is introduced. Numerical examples are used to illustrate the proposed approaches. Section 5 concludes the paper.

2. Background

Definition1: Suppose **n** decision making units with the input vector $x_j = (x_{1j}, \dots, x_{mj})$ and output vector $y_j = (y_{1j}, \dots, y_{sj})$ providing that $x_j \geq 0$ ، $x_j \neq 0$ & $y_j \geq 0$ ،$y_j \neq 0$. So, set of all possible activities is called production possibility set, which is expressed as:

$$T = \{(x,y) | \text{vector x can produce vector } y\}$$

According to production technology, this definition identifies production possibility set. To form set T, we accept major principles which are the bases of theory and construction of different models of DEA.

Property 1) inclusion of observations: all observed activities belong to T. In other words:

$$(x_j, y_j) \in T,\ j = 1, \dots, n$$

Property 2) Convexity: If $(x\ ,\ y\ ) \in T$ and $(\bar{x}\ ,\ \bar{y}\ ) \in T$, *then* for each $\lambda \in [0,1]$, $(\lambda x + (1-\lambda)\bar{x}, \lambda y + (1-\lambda)\bar{y}) \in T$. In other words, T is a convex set.

Property 3) Ray immensity or constant return to scale: for each $(x,y) \in T\ and\ \lambda \geq 0$, we have $(\lambda x, \lambda y) \in T$.

Property 4) Possibility: If $(\bar{x}, \bar{y}) \in T\ and\ x \geq \bar{x}, then, (x, \bar{y}) \in T. If\ y \leq \bar{y}, then, (\bar{x}, y) \in T, suggesting\ that$ if output $\bar{y}\ is\ produced\ by\ \bar{x}$, then the very output could also be produced by any input greater than $\bar{x}$. Any output smaller than $\bar{y}\ could\ be\ produced\ by\ input\ \bar{x}\ as\ well.$

Property 5) Minimum interpolation: Here, we consider T as the smallest set so that can be applied to properties 1-4 above.

A. BCC

T_V is a production possibility set which is obtained accepting properties 1-5, except for property 3 (constant return to scale):

$$T_V = \left\{ \begin{array}{l} (x,y) : \sum_{j=1}^{n} x_j \lambda_j \leq x, \sum_{j=1}^{n} y_j \lambda_j \geq y, \\ \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0\ j = 1, 2 \dots, n \end{array} \right\}$$

Where, V means variable return to scale. This set was first introduced by Banker et al (1985) and the model which evaluates DMUs is known as BCC and shown as follows:

$$\begin{aligned} Min \quad & \theta - \varepsilon \left(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^- \right) \qquad (1) \\ s.t \quad & \sum_{j=1}^{n} x_{ij} \lambda_j + s_i^- = \theta x_{io}, \quad i = 1, 2, \dots, m \\ & \sum_{j=1}^{n} y_{rj} \lambda_j - s_r^+ = y_{ro}, \quad r = 1, 2, \dots, s \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j, s_r^+, s_i^- \geq 0, r = 1, 2, \dots, s\ , i = 1, 2, \dots, m. \end{aligned}$$

Definition 2 (Technical efficiency): The performance of DMUo is to be characterized as technically efficient if and only if the evidence shows that it is not possible to improve some of its inputs or outputs without worsening some of its other inputs or outputs.

Definition 3 (Technical inefficiency): Inefficiency is present in the performance of DMUo if the evidence shows that it is possible to improve some of its inputs or outputs without worsening any of its other inputs or outputs.

Definition 4 (BCC efficiency): DMU_o belonging to production possibility set T_V is technical efficiency if and only if $\theta^*_{BCC} = 1$. Otherwise DMU_o is inefficient and $(1 - \theta^*_{BCC})$ is the value of technical inefficiency in input-oriented model.

B. Congestion

Sometimes, abundance and excess of resources have negative impact on efficiency. That is, the accumulation of resources will cause energy loss and in fact allocating money to other sections would be difficult. In such cases, it is common to say congestion has occurred at input.

Congestion is a special case of inefficiency, in which increase in input leads to decrease in output. Now, following Cooper et al. (2000), (2001c) and (2002), we define the congestion of inputs as below:

Definition 5 (Congestion):

Input congestion occurs when increasing one or more inputs decreases some outputs without improving other inputs or outputs. Conversely, congestion occurs when decreasing some inputs increases some outputs without worsening other inputs or outputs.

There is distinction between technical inefficiency and congestion. This is because if, while evaluating, it is recognized that it would be possible to improve some inputs or outputs without other inputs or outputs being deteriorated, there will be technical inefficiency in under evaluation DMU. It should be noted that nothing has been mentioned in terms of decrease in input and increase in output. Various methods have been developed to identify the existence of congestion in a DMU using DEA. In this section the Cooper's method is explained:

A two-stage model to identify congestion

To identify congestion, first DMU_o is evaluated in the following model:

$$\begin{aligned} Max \quad & \varphi+\varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) \qquad (2) \\ s.t \quad & \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = x_{io}, \quad i=1,2,...,m \\ & \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = \varphi y_{ro}, \quad r=1,2,...,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j, s_i^-, s_r^+ \geq 0,\ j=1,\dots,n.; i=1,2,...,m, r=1,2,...,s; \end{aligned}$$

To prove whether congestion exists, suppose that $(\varphi^*, \lambda^*, S^{+^*}, S^{-^*})$ is the optimal solution of (2). We suppose the image of (X_0, Y_o) on the efficiency frontier of $(\hat{X}_0, \hat{Y}_o)$, which is defined as follow: (it can be proved that this image is on the efficiency frontier)

$$\hat{y}_{ro} = \varphi^* y_{ro} + s_r^{+*} \geq y_{ro}, \quad r = 1, \dots, s,$$
$$\hat{x}_{io} = x_{io} - s_i^{-*} \leq x_{io}, \qquad i = 1, \dots, m.$$

So, $\Delta y_{ro} = \hat{y}_{ro} - y_{ro} \geq 0$ is the rth output inefficiency and $\Delta x_{io} = x_{io} - \hat{x}_{io} \geq 0$ is the ith DMU_o input.

Thus, if and only if for some is and rs, $\Delta y_{ro} \neq 0, \Delta x_{io} = 0$, then, by definition, inefficinecy will exist.

Now we solve model (3):

$$\begin{aligned} Max \quad & \sum_{i=1}^{m} \delta_i^- \qquad (3) \\ s.t \quad & \hat{x}_{io} = x_{io} - s_i^{-*} = \sum_{j=1}^{n} x_{ij}\lambda_j - \delta_i^-, \quad i=1,2,...,m \\ & \hat{y}_{ro} = \varphi^* y_{ro} + s_r^{+*} = \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r=1,2,...,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & s_i^{-*} \geq \delta_i^- \\ & \lambda_j \geq 0,\ j=1,\dots,n. \end{aligned}$$

This model allows ith input to increase up to $s_i^{-^*}$ size, so as to produce $\hat{Y}_o$ output.

Finally, to calculate the amount of ith input congestion, the following is applied:

$$s_i^{-c*} = s_i^{-*} - \delta_i^{-*}, i = 1, \dots, m.$$

where, $\delta_i^{-^*}$ is the optimal answer of model (3), and $s_i^{-c^*}$ is the amount of ith input congestion.

A one-stage model to identify congestion

Applying $\delta_i^- = s_i^{-^*} - s_i^{-c}$ in model (3), it could be rewritten. Combining this model and model (4), the following model, known as one-stage model, will be developed:

$$\begin{aligned} Max \quad & \varphi+\varepsilon\left(\sum_{r=1}^{s} s_r^+ - \varepsilon\sum_{i=1}^{m} s_i^{-c}\right) \qquad (4) \\ s.t \quad & \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^{-c} = x_{io}, \quad i=1,2,...,m \\ & \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = \varphi y_{ro}, \quad r=1,2,...,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j, s_r^+, s_i^- \geq 0, r=1,2,...,s, i=1,2,...,m. \end{aligned}$$

Theorem 1: There will be congestion in under evaluation DMU if and only if there is at least $s_i^{-c^*} > 0, (1 \leq i \leq m)$in optimal solution of model (4)$(\varphi^*, \lambda^*, s^{+^*}, s^{-c^*})$.
Proof: See [17].

3. The proposed two-stage congestion model

As explained, DEA is used, as a method, to evaluate the efficiency of congruent DMUs with some inputs and outputs. DMUs can take different forms as hospitals, universities, banks, etc. in some cases, these units act as a two-stage process. The first stage uses some inputs and produces some outputs, which form the inputs of the second stage. The outputs of the first stage are also called intermediate. Using intermediaries, the second stage produces the final outputs of the system. Figure (1) shows an overview of a two-stage DMU. The first stage uses x_{ij}inputs to produce z_{dj} outputs. Then, at the second stage, z_{dj}s act as inputs and produce y_{rj} outputs. As you can see, z_d at first stage acts as an output, while it acts as an input at the second stage. This can be generalized to multi-stage ones.

Applying standard models of DEA for the first stage, the second stage, and the whole process is considered as a usual approach to evaluate the efficiency of DMUs. In this way, the whole two-stage process is considered as a whole unit, in which its inputs are the first-stage inputs and its outputs are the outputs of stage two. This method was proposed by Seiford and Zhou. In this method, however, a whole unit could be efficient, while neither of stages one and two is efficient.

Following that, Chen and Zhou developed a DEA model, in which the rate of efficiency in each stage is identified based on production possibility set of the very stage. Kao and Huwang proposed another method to evaluate the efficiency of two-stage units. In this method, they suggested that two-stage unit efficiency should be the multiplication of the efficiency of stages one and two. In another study, Chen et al defined overall efficiency as convex combination of the efficiency of stages one and two. At constant return to scale, they express their model as follow.

Let n networks are under evaluation so that stage one uses $x_j$and produces z_j and stage two uses z_j to produce y_j. As a result, jth network is shown as $(x_j, z_j, z_j, y_j)^T$.

Property 1) Set of observed activities $(x_j, z_j, z_j, y_j)^T$ j=1,2,….,n belongs to production possibility set.

Property 2) each convex combination of activities related to PPS belongs to production possibility set. In other words, let $(x_1, z_1, z_1, y_1)^T \in T_{network}$ and $(x_2, z_2, z_2, y_2)^T \in T_{network}$, then for each $\lambda \geq 0$, we have

$$\lambda(x_1, z_1, z_1, y_1)^T + (1-\lambda)(x_2, z_2, z_2, y_2)^T \in T_{network}.$$

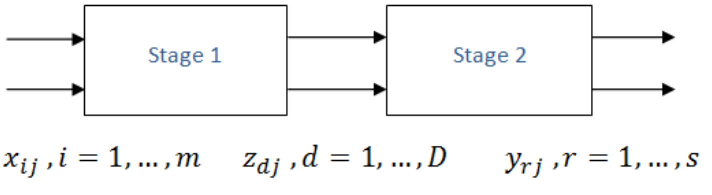

Figure 1: Two-stage decision-making unit.

Property 3) Possibility: let stage one, using vector x, produces output vector z. Then, using each input vector $\bar{x}$ (if $x \leq \bar{x}$), $\bar{z}$ is produced (if $\bar{z} \leq z$).

Also, let stage two, using input vector z, produces output vector y. Then, using each input vector $\bar{z}$ (if $\bar{z} \leq z$)., $\bar{y}$is produced (if $y \leq \bar{y}$). Therfore, if$(x,z,z,y)^T \in T_{network}$ and $(\bar{x},-\bar{z},\bar{z},-\bar{y})^T \geq (x,-z,z,-y)^T \in T_{network}$, then $(\bar{x},\bar{z},\bar{z},\bar{y})^T \in T_{network}$.

Property 4) ray immensity: if activity$(x,z,z,y)^T$ belongs to PPS,then $\lambda(x,z,z,y)^T$belongs to PPS for all positive values of λ.

Accordingly, two-stage PPS network would be as follows:

$$T_{network} = \left\{ \begin{pmatrix} x_j \\ z_j \\ z_j \\ y_j \end{pmatrix} : x \geq \sum_{j=1}^{n} \lambda_j x_j, z \leq \sum_{j=1}^{n} \lambda_j z_j, \; z \geq \sum_{j=1}^{n} \lambda_j z_j, y \leq \sum_{j=1}^{n} \lambda_j y_j, \lambda_j \geq 0 \;\; \forall j \right\}$$

Then, we have:

$$T_{network} = \left\{ \begin{pmatrix} x_j \\ z_j \\ z_j \\ y_j \end{pmatrix} : x \geq \sum_{j=1}^{n} \lambda_j x_j, z = \sum_{j=1}^{n} \lambda_j z_j, \; y \leq \sum_{j=1}^{n} \lambda_j y_j, \lambda_j \geq 0 \;\; \forall j \right\}$$

Here, PPS is introduced for technologies with variable returns to scale.

$$T_V^{\;*} = \left\{ \begin{matrix} \{(x,y): \sum_{j=1}^{n} \lambda_j \left(x_j \theta_j^*\right) \leq x, \\ \sum_{j=1}^{n} \lambda_j \left(y_j \varphi_j^*\right) \geq y, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \end{matrix} \right\}$$

And using BBC, θ_{sj}^* and φ_{mj}^* are obtained for stage one as input and for stage two as output. Geometric interpretation of T_V^* is shown below.

$$\begin{aligned} Min \quad & \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) && (5) \\ s.t \quad & \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta x_{io}, && i = 1,2,...,m \\ & \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{ro}, && r = 1,2,...,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j, s_r^+, s_i^- \geq 0 \;\; r = 1,2,...,s \;, i = 1,2,...,m. \end{aligned}$$

$$\begin{aligned} Max \quad & \varphi + \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) && (6) \\ s.t \quad & \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = x_{io}, && i = 1,2,...,m \\ & \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = \varphi y_{ro}, && r = 1,2,...,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j, s_r^+, s_i^- \geq 0, r = 1,2,...,s \;, i = 1,2,...,m. \end{aligned}$$

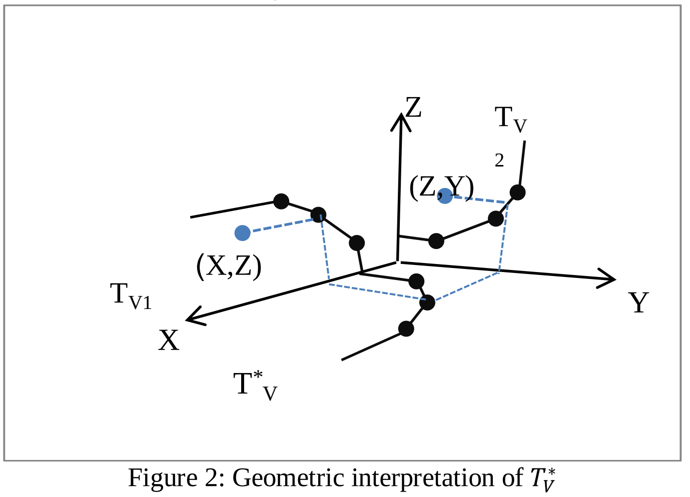

Figure 2: Geometric interpretation of T_V^*

In Figure 2, the graph on the left depicts PPS at stage one (T_V^1***);*** the graph on the right depicts PPS at stage two (T_V^2); and the graph at the bottom depicts network PPS regardless of constant return to scale.
In this study, the modified model to identify congestion, proposed by Cooper et al, is developed as follow:

$$
\begin{aligned}
Max \quad & \tilde{\varphi}^{\circ} + \varepsilon\left(\sum_{r=1}^{s} s_r^{+} - \varepsilon \sum_{i=1}^{m} s_i^{-c}\right) \qquad (7) \\
s.t \quad & \sum_{j=1}^{n} x_{ij}\lambda_j + \sum_{j=n+1}^{2n} (\theta_j^{*} x_{ij})\lambda_j + s_i^{-c} = x_{i\circ}, \ \mathrm{i}=1,2,\ldots,\mathrm{m} \\
& \sum_{j=1}^{n} y_{rj}\lambda_j + \sum_{j=n+1}^{2n} (\varphi_j^{*} y_{rj})\lambda_j - s_r^{+} = \tilde{\varphi}^{\circ} y_{r\circ}, \ \mathrm{r}=1,2,\ldots,\mathrm{s} \\
& \sum_{j=1}^{2n} \lambda_j = 1 \\
& \lambda_j, s_r^{+}, s_i^{-} \geq 0, \ \mathrm{i}=1,2,\ldots,\mathrm{m}, \ \mathrm{r}=1,2,\ldots,\mathrm{s}.
\end{aligned}
$$

Theorem 2: The model (7) is always possible and the amount of objective function will be less than or equal to one.

Proof: Suppose $\left(\tilde{\varphi}^{o}, \lambda_j = e_o, S^{-}=0, S^{+}=0\right)$ is a feasible solution of model (7) and by placing it in model (7), since it applies to the mentioned model and the amount of objective function for this feasible solution will be equal to one and as respects it is maximization so the objective function should be more than and equal to the feasible solution, it means $\tilde{\varphi}^{o} \geq 1$.

Theorem 3. Congestion is present if and only if for an optimal solution $(\tilde{\varphi}^{0*}, \lambda^{*}, s_r^{+*}, s_i^{-c*})$ model (7), there exists at least one $s_r^{+*} > 0$, (r =1, 2... s), and at least one $s_i^{-c*} > 0$, (i=1, 2, …, m).

Proof. It is obvious. Next we improve the theorem by showing that if we have at least one $s_i^{-c*} > 0$, (i=1, 2… m), then it guarantees existing at least one $s_r^{+*} > 0$, (r =1, 2... s), i.e., congestion is present. The following theorem represents the promised improvement.

Theorem 4. Congestion is present if and only if for an optimal solution $(\tilde{\varphi}^{0*}, \lambda^{*}, s_r^{+*}, s_i^{-c*})$ of model (7), there is at least one $s_{io}^{-c*} > 0, \ (1 \leq io \leq m)$.

Proof. (Necessary condition) It is obvious by the congestion definition.
(Sufficient condition) We suppose that $s_{io}^{-c*} > 0, \ (1 \leq io \leq m)$. We must show that there is at least one $s_r^{+*} > 0, \ (1 \leq r \leq s)$ or $\tilde{\varphi}^{0*} > 1$. Suppose to the contrary that $s_r^{+*} = 0$, for all r=1,2… s. Let $\hat{s}_r^{+} = s_r^{+*} = 0$, r=1, 2,…, s; $s_{io}^{-c*} = 0, \ \tilde{\varphi}^{0*} = \hat{\tilde{\varphi}}^{0}$, i=1,2,…,m; $\hat{\lambda}_j = 0$ for $j \neq o$ and $\hat{\lambda}_o = 0$. Then $(\hat{\tilde{\varphi}}^{0}, \hat{\lambda}, \hat{s}_r^{+}, \hat{s}_i^{-c})$ is a feasible solution of (7) and

$$
\begin{aligned}
\hat{\tilde{\varphi}}^{0} + \varepsilon\left(\sum_{r=1}^{s} \hat{s}_r^{+} - \varepsilon \sum_{i=1}^{m} \hat{s}_i^{-c}\right) &= \tilde{\varphi}^{0*} \\
&> \tilde{\varphi}^{0*} - \varepsilon\left(\sum_{i=1}^{m} s_i^{-c*}\right) \\
&= \tilde{\varphi}^{0*} + \varepsilon\left(\sum_{r=1}^{s} s_r^{+*} - \varepsilon \sum_{i=1}^{m} s_i^{-c*}\right)
\end{aligned}
$$

This contradicts the assumption that $(\tilde{\varphi}^{0*}, \lambda^{*}, s_r^{+*}, s_i^{-c*})$ is an optimal solution of model (7). So, there is at least one $s_r^{+*} > 0, (1 \leq r \leq s)$ and in accordance with Theorem 3, congestion is present.

Theorem 5. If DMUo under evaluation strong performance in the first box, then the box is not congestion.

Proof: according to the congestion definition, it is clear.

Illustrative example

Example 1: Table 1 illustrates six two-stage DMUs including A, B, C, D, E, and F, in which X is input, Z is intermediate, and Y is output. The values under

θ_{sj}^{*} *and* φ_{mj}^{*} columns, shown in Table 1, are calculated using input-oriented BBC model in the first stage and output-oriented BBC model in the second stage, respectively.

Table 1: Data of inputs and outputs

DMUs	X	Z	Y	θ_{mj}^{*}	φ_{sj}^{*}
A	1	1	2	1	1
B	3	4	2	1	2.5
C	4	4	1	0.6	5
D	6	3	4	0.5	1.25
E	7	2	5	0.238	1
F	8	1	1	0.125	2

Table 2: The results of solving model (7).

DMU	X	Y	$\tilde{\varphi}^{0*}$	s^{+*}	s^{-c*}
A	1	2	1	0	0
B	3	2	2.5	0	0
C	5	1	5	0	0
D	6	4	1.25	0	0
E	7	5	1	0	0
F	8	1	5	0	1
$\bar{A}$	1	2	1	0	0
$\bar{B}$	3	5	1	0	0
$\bar{C}$	3	5	1	0	0
$\bar{D}$	3	5	1	0	0
$\bar{E}$	1.66	5	1	0	0
$\bar{F}$	1	2	1	0	0

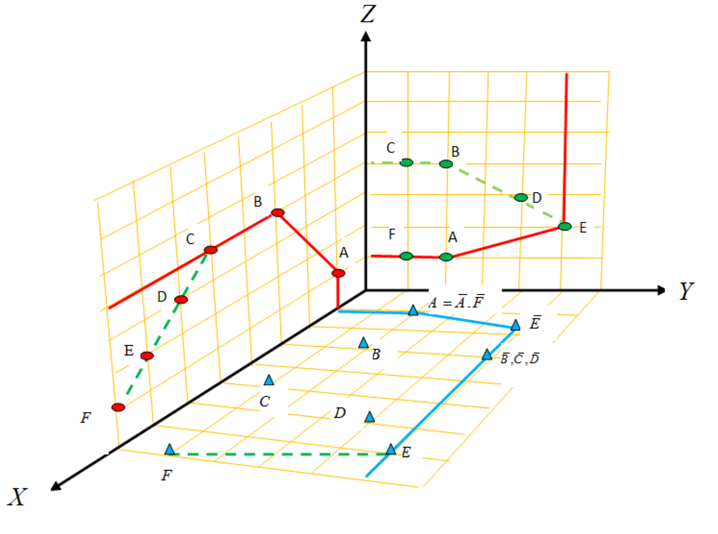

Figure 3: Illustration from DMUF has congestion.

Place: $\overline{DMU}_j = \left(\theta_j^* X_j, \varphi_j^* Y_j\right)$

The second and third columns of Table 2 show inputs and outputs of the basic DMUS and new DMUs of $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$, $\bar{E}$ and $\bar{F}$ that their inputs and outputs obtained by multiplying θ_{mj}^* and φ_{sj}^* in inputs and outputs of the basic DMUs, respectively. The results of model (7) provided in the last three columns of Table 2. The fourth column shows that DMUs B, C, D and F with efficiency score bigger than one are efficient and others are efficient. The last column shows that DMUF has congestion and the amount of congestion is $s^{-c^*} = 1$. Also, Figure 3 confirms that DMUF has congestion.

DMUA in both stages 1and 2 is highly efficient, so it is efficient in the whole process as well. DMUB is highly efficient in stage 1 and has congestion in stage 2, but it is inefficient in the whole process. DMUC is weak sufficient in stage 1, and has congestion in stage2, but it is inefficient in the whole process. DMUD has congestion in both stages 1 and 2. However, it is inefficient in the whole process. DMUE has congestion in stage 1 and is highly efficient in stage 2, but is weak efficient in the whole process. Finally, DMUF has congestion in stage 1 and is weak efficient in stage 2. It. however, has congestion in the whole process.

Example 2: Table 3 presents ten DMUs with two inputs I1, I2, two intermediate measures Z1, Z2 and two outputs O1 and O2. Results of identifying of congestion in the first inputs I1, I2 and intermediate measures Z1 and Z2 as outputs and intermediate measures Z1 and Z2 as inputs and O1 and O2 as the final outputs using model (4) shown in Box1 and Box2 of Table 4, respectively.

In other side, we have solved input-oriented model (5) with two inputs and two intermediate measures as outputs and results have been provided in Box1 of Table 4. Also, the results of solving output-oriented model (6) with two intermediate measures Z1 and Z2 as inputs and two outputs O1 and O2 as the final outputs shown in Box2 of Table 4. By multiplying θ^* in inputs and φ^* in outputs of ten basic DMUs make ten new DMUs as $\overline{\text{DMU}}_j = (\theta_j^* x_{ij}, \phi_j^* y_{rj})$. Since DMUs 1, 3 and 8 are efficient in input-oriented and output-oriented models; therefore, we have seven new DMUs. Data of basic DMUs and new DMUs provided in Table 6. Now we have seventeen DMUs that intend to identify their inputs congestion using model (7). Results have been presented in Table 7. Considering Table 7, DMU2 has congestion in the first input and its value is $s_1^{-c} = 5$. Table 4 shows that DMU2 has congestion in the first input and its value is $s_1^{-c*} = 5$ in Box1, but doesn't have congestion in Box 2. DMU6 is congested in their two inputs in Table 7 and, also, is congested in Box1 and Box2 of Table 4. DMU7 is congested in their two inputs in Table 7 and, also, is congested in Box1 but isn't congested Box2 of Table 4. DMU8 is congested only in the first input and its value is $s_1^{-c*} = 10$ in Table 7, but this DMU is efficient in Box1 and Box2 of Table 4. DMU9 has congestion in Tables 4 and 7. The first input of DMU10 in Table 7 has congestion and this DMU has congestion in Box 1 Table 4.

Table 3: Ten DMUs with two inputs, two intermediate measures and two outputs

DMUS	DMU01	DMU02	DMU03	DMU04	DMU05	DMU06	DMU07	DMU08	DMU09	DMU10
I_1	2	12	3	7	4	10	9	13	8	5
I_2	4	9	4	9	8	10	7	4	8	5
Z_1	3	1	5	6	10	6	1	7	3	1
Z_2	4	2	4	12	11	6	1	1	3	4
O_1	7	1	9	3	4	1	1	2	1	1
O_2	8	1	7	3	2	2	1	1	3	2

Table 4: Identifying of congestion

congestion BOX1	φ^*	$s_1^{-c^*}$	$s_2^{-c^*}$	$s_1^{+^*}$	$s_2^{+^*}$	**congestion** BOX2	φ^*	$s_1^{-c^*}$	$s_2^{-c^*}$	$s_1^{+^*}$	$s_2^{+^*}$
DMU01	1.00	0.00	0.00	0.00	0.00	DMU01	1.00	0.00	0.00	0.00	0.00
DMU02	6.00	5.00	0.00	0.00	0.00	DMU02	1.00	0.00	0.00	0.00	0.33
DMU03	1.00	0.00	0.00	0.00	0.00	DMU03	1.00	0.00	0.00	0.00	0.00
DMU04	1.00	0.00	0.00	0.00	0.00	DMU04	2.56	2.33	8.00	0.00	0.00
DMU05	1.00	0.00	0.00	0.00	0.00	DMU05	2.25	5.00	7.00	0.00	2.50
DMU06	1.67	6.00	2.00	0.00	1.00	DMU06	4.00	3.00	2.00	3.00	0.00
DMU07	8.95	4.25	0.00	0.00	0.00	DMU07	1.00	0.00	0.00	0.00	0.00
DMU08	1.00	0.00	0.00	0.00	0.00	DMU08	1.00	0.00	0.00	0.00	0.00
DMU09	3.33	4.00	0.00	0.00	1.00	DMU09	1.89	0.00	0.00	3.22	0.00
DMU10	1.44	1.75	0.00	4.81	0.00	DMU10	1.00	0.00	0.00	0.00	0.00

Table 5: Results of solving models 5 and 6 with data of Table 3.

Efficiency Box1	θ^*	s_1^{-*}	s_2^{-*}	s_1^{+*}	s_2^{+*}	Efficiency Box2	φ^*	s_1^{-*}	s_2^{-*}	s_1^{+*}	s_2^{+*}
DMU01	1.00	0.00	0.00	0.00	0.00	DMU01	1.00	0.00	0.00	0.00	0.00
DMU02	0.44	2.33	0.00	4.00	2.00	DMU02	1.00	0.00	1.00	0.00	0.00
DMU03	1.00	0.00	0.00	0.00	0.00	DMU03	1.00	0.00	0.00	0.00	0.00
DMU04	1.00	0.00	0.00	0.00	0.00	DMU04	2.56	2.33	8.00	0.00	0.00
DMU05	1.00	0.00	0.00	0.00	0.00	DMU05	2.25	5.00	7.00	0.00	2.50
DMU06	0.51	1.86	0.00	0.43	0.00	DMU06	4.00	3.00	2.00	3.00	0.00
DMU07	0.57	2.14	0.00	4.00	3.00	DMU07	1.00	0.00	0.00	0.00	0.00
DMU08	1.00	0.00	0.00	0.00	0.00	DMU08	1.00	0.00	0.00	0.00	0.00
DMU09	0.50	1.00	0.00	2.00	1.00	DMU09	1.89	0.67	0.00	3.11	0.00
DMU10	0.80	1.00	0.00	4.00	0.00	DMU10	1.00	0.00	0.00	0.00	0.00

Table 6: Data of basic DMUs and new DMUs.

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16	D17
I1	2	12	3	7	4	10	9	13	8	5	5.28	7	4	5.1	5.13	4	4
I2	4	9	4	9	8	10	7	4	8	5	3.96	9	8	5.1	3.99	4	4
O1	7	1	9	3	4	1	1	2	1	1	1	7.68	9	4	1	1.89	1
O2	8	1	7	3	2	2	1	1	3	2	1	7.68	4.5	8	1	5.67	2

Table 7: Identifying congestion using the proposed model, model (7).

DMUs	$\tilde{\varphi}^{\circ*}$	$s_1^{-c^*}$	$s_2^{-c^*}$	$s_1^{+^*}$	$s_2^{+^*}$
DMU01	1.00	0.00	0.00	0.00	0.00
DMU02	7.68	5.00	0.00	0.00	0.00
DMU03	1.00	0.00	0.00	0.00	0.00
DMU04	2.56	0.00	0.00	0.00	0.00
DMU05	2.25	0.00	0.00	0.00	0.00
DMU06	4.00	4.90	4.90	0.00	0.00
DMU07	7.67	3.87	0.00	0.00	0.00
DMU08	4.50	10.00	0.00	0.00	2.50
DMU09	2.67	2.90	2.90	1.33	0.00
DMU10	4.00	0.18	0.00	0.27	0.00
DMU11	1.00	0.00	0.00	0.00	0.00
DMU12	1.00	0.00	0.00	0.00	0.00
DMU13	1.00	0.00	0.00	0.00	0.00
DMU14	1.00	0.00	0.00	0.00	0.00
DMU15	6.00	2.06	0.00	0.00	0.00
DMU16	1.41	2.00	0.00	4.33	0.00
DMU17	4.00	2.00	0.00	3.00	0.00

4. Conclusion

We conclude that congestion may exist at first or second stage without observing any congestion in the whole process. It is also likely not to see congestion in either of stages, while in the whole process it is observed. Therfore, presence or absence of congestion in a two-stage DMU has nothing to do with presence or absence of congestion in each stage. Also, a model was proposed to identify congestion in network.

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