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Evaluation of efficiency using the product method in two-stage systems with undesirable outputs

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Abstract

Data envelopment analysis is a non-parametric method based on mathematics, which is used to evaluate the performance of a set of homogeneous decision-making units in a production technology with multiple inputs and outputs. The idea of performance evaluation in two-stage systems is one of the topics. It is of interest in data coverage analysis. In two-stage systems, all or part of the outputs of the first stage is considered as the input of the tail stage. This input of the second stage can be considered favorable or unfavorable. Arithmetic or geometric models can be used to calculate the efficiency of the decision-making units. Therefore, in this article, the evaluation of efficiency in two-stage systems with undesirable outputs is discussed using the product method. According to the new model presented, the efficiency of 6 regions in China was measured. It should be noted that every region of China has provinces and cities that have the same geographical, natural, etc. conditions.

Keywords: data envelopment analysis, efficiency of two-stage systems, desirable output, undesirable output, product method

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1. Introduction

Data envelopment analysis was described as a method to evaluate the efficiency of homogeneous decision-making units with multiple inputs and multiple outputs. Decision-making units can have different forms such as hospitals, universities, banks, etc. These units in some States act as a two-step process. The first stage uses inputs and produces outputs. All or part of these outputs form the inputs of the second stage. The outputs of the first stage are also called intermediate outputs. Figure 1 shows the general state of a two-stage decision making unit [1].

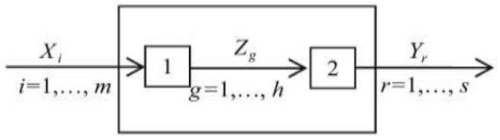

Figure 1: a two-stage decision making unit

The first stage uses inputs x_{ij} to produce outputs z_{dj}. Then this z_{dj} is considered as an input in the second stage and produces the outputs y_{rj}. This form is easily generalizable to multiple states. The models of DEA that we examine to evaluate efficiency are models of two-stage systems in which part of the outputs of the first stage are undesirable. This undesirable output is considered as the input of the second stage to eliminate some of the environmental pollution. Environmental issues such as water pollution, soil pollution, global warming, etc. are among the biggest problems of today's societies. All countries are trying to reduce such pollution in their environment and try to consider these undesirable outputs from production as inputs in the systems to reduce its amount. Figure 2 represents such a system. Is [2].

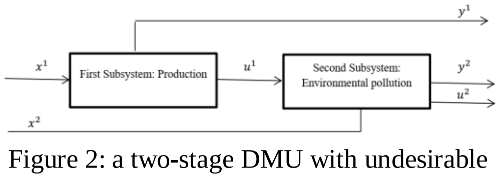

Figure 2: a two-stage DMU with undesirable outputs

In this figure, the first stage of the industrial production system shows that the input X^1 has been used to produce the desired output Y^1 and the undesirable output U^1. The second stage shows the process of environmental pollution, in this stage using the additional input X^2 and the undesirable output U^1 from the first stage as input to produce the desired output Y^2 and a small amount of undesirable output U^2 pay When measuring the efficiency of a DMU, it can be thought of as a closed factory, where the evaluator is located outside the factory and counts the initial quantity and the number of employees who enter the factory to produce products. After a period of time, the evaluator counts the number of products that leave the factory [3]. There are different methods to evaluate the efficiency of two-stage systems with undesirable outputs. Kao has divided related models into three types: independent, connected, and relational. The independent model is the clearest way to check the performance of a network system. We consider each section as an independent DMU and measure its efficiency [4]. In the connected model, distance functions defined in the possibility set are used as system efficiency. In the third type, the efficiency of the system was defined as a function of the efficiency of the subsystems. Cao showed that after the transformation of the network structure, there was always a relationship between the system and the efficiency of the department, which is called a relational model. This relationship is defined as follows:

In relation $E_d = E_d^1 \times E_d^2$, the efficiency of the whole system is defined as the product of the efficiency of two parts.

In relation $E_0 = W^1E_d^1 + W^2E_d^2$, the efficiency of the whole system is defined as the arithmetic mean of the efficiency of two parts.

2. Previous studies

2.1 Environmental efficiency

In recent years, researches in the field of measuring environmental performance and efficiency with the approach of enveloping analysis of data with undesirable outputs have been expanded. Cook investigated the environmental efficiency and energy efficiency for 29 regions of China in the period of 2000-2008 using a modified data envelope analysis model containing adverse output [2]. Far calculated the environmental efficiency of power plants in the United States in order to investigate the impact of the first implementation phase of the acid rain program [5]. Delmas measured the technical, financial and environmental efficiency of livestock units in Belgium. The results of this experimental study show the average technical efficiency, environmental efficiency, specialized efficiency and cost efficiency as 0.843, 0.897, 0.883, 0.985 respectively. Undesirable outputs have been paid. In this study, by simultaneously considering economic activities, carbon dioxide emissions, and energy consumption in the production process, and by using the coverage analysis of adverse output data, energy efficiency has been evaluated in Iran and neighboring countries during the period (2007-2012) [6].

2.2 Production process of two-stage systems

Based on recent studies, the efficiency evaluation method in two-stage systems is divided into three categories: 1. Independent, 2. Dependent, 3. Relational. The independent model is the clearest method to check the performance of a network system, which was investigated by Chambers [7]. In this approach, we consider each section as an independent DMU and measure its efficiency. Serra (2014,2015) have used distance functions defined in the possibility set as system efficiency in the connected model [8,9]. Halkos et al. (2014), In this method, there is a relationship between the overall efficiency and the efficiency of each part, which is defined as multiplicative efficiency and collective efficiency. In the first one, the overall efficiency of the system is considered as the product of the efficiency of each part in the form of equation (1-1) [10]. In the second case, the overall efficiency of the system is defined as relation (1-2) [10]. In all mentioned methods, undesirable intermediate outputs in two-phase systems are not considered. In order to solve this problem, Farr et al. (2004) considered a two-stage system in which in the first stage both the desired output and the undesirable output are produced, and in the second step, the undesirable output is considered as input and the desired output is considered as final output. In addition to the above material, it should be mentioned that Wu (2016) proved that the amount of resources that each decision-making unit considers for production has a significant effect on the efficiency of subsystems and the efficiency of the entire system. Two-stage processes with undesirable outputs will be explained by the multiplication method [11].

3. Statement of the problem and theoretical assumptions

According to the two-stage systems in Figure 1, we make a weighted two-stage DEA model. Suppose there are n decision-making units, which are displayed as $DMU_j, (j = 1, \cdots, n)$. The first subsystem has m_1 external input $X_1 = \left(x_{1j}, x_{2j}, \ldots, x_{m_1 j}\right)^T$ which is used to produce s_1 desired output $X_2 = (x_{1j}, x_{2j}, \ldots, x_{m_2 j})^T$ and o_1 undesirable output $U_1 = \left(u_{1j}, u_{2j}, \ldots, u_{o_1 j}\right)^T$. The second subsystem m_2 external input $X_2 = (x_{1j}, x_{2j}, \dots, x_{m_2 j})^T$ and undesirable output $U_1 = (u_{1j}, u_{2j}, \dots, u_{o_1 j})^T$ consumes from the first stage until s_2 the desired output $y_2 = (y_{1j}, y_{2j}, \dots, y_{s_2 j})^T$ and o_2 the undesirable output $U_2 = (u_{1j}, u_{2j}, \dots, u_{o_2 j})^T$. The output of U_1 from the first stage is considered as the input of the second stage. As you know, all U_j are undesirable outputs, so you can use the transfer vector A suitable w turned this unfavorable negative output into a positive one.

$$u_j^b = -u_j^b + w$$

Now, we write the CCR model, the covering form related to two-phase systems, with the form of model (1).

$$\text{Max} \quad \frac{\sum_{r=1}^{s_2} \mu_{rd}^1 y_{rd}^1 + \sum_{r=1}^{s_2} \mu_{rd}^2 y_{rd}^2 + \sum_{v=1}^{o_2} \tau_{vd}^2 u_{vd}^2}{\sum_{i=1}^{m_1} \omega_{id}^1 x_{id}^1 + \sum_{i=1}^{m_2} \omega_{id}^2 x_{id}^2}$$

s.t.

$$\frac{\sum_{r=1}^{s_1} \mu_{rj}^1 y_{rj}^1 + \sum_{r=1}^{s_2} \mu_{rj}^2 y_{rj}^2 + \sum_{v=1}^{o_2} \tau_{vj}^2 u_{vj}^2}{\sum_{i=1}^{m_1} \omega_{ij}^1 x_{ij}^1 + \sum_{i=1}^{m_2} \omega_{ij}^2 x_{ij}^2} \le 1$$

$j = 1, \dots, n \quad (1.1)$

$$\frac{\sum_{r=1}^{s_1} \mu_{rj}^1 y_{rj}^1 + \sum_{v=1}^{o_1} \tau_{vj}^1 u_{vj}^1}{\sum_{i=1}^{m_1} \omega_{ij}^1 x_{ij}^1} \le 1,$$

$j = 1, \dots, n \quad (1.2)$

$$\frac{\sum_{r=1}^{s_2} \mu_{rj}^2 y_{rj}^2 + \sum_{v=1}^{o_2} \tau_{vj}^2 u_{vj}^2}{\sum_{i=1}^{m_1} \omega_{ij}^2 x_{ij}^2 + \sum_{c=1}^{o_1} \eta_{cj}^1 u_{cj}^1} \le 1$$

$j = 1, \dots, n \quad (1.3)$

$$\frac{\sum_{i=1}^{m_1} \omega_{id}^1 x_{id}^1}{\sum_{i=1}^{m_1} \omega_{id}^2 x_{id}^2 + \sum_{c=1}^{o_1} \eta_{cd}^1 u_{cd}^1} = \frac{\alpha}{1-\alpha} \qquad (1.4)$$

$$\omega_{ij}^1, \omega_{ij}^2, \eta_{cj}^1, \mu_{rj}^1, \mu_{rj}^2, \tau_{vj}^1 \ge 0$$

$j = 1, \dots, n$

Model defaults:

1- The return to scale is constant. 2- The equality of the intermediary size coefficients in all stages, that is, the weights of the undesirable outputs of the first stage are assumed to be equal to the input weights of the second stage.

In the above model, clause (1-1) is related to the efficiency of the whole system for each DMU. Similarly, formulas (1-2), (1-3) show the efficiency of the first subsystem and the efficiency of the second subsystem, respectively. We know that the ratio of input funds from subsystems is equal to the ratio of our priorities for the efficiency of each subsystem. In formula (1-4), if α is greater than 0.5, it indicates that the decision-making unit gives more importance to the efficiency of the first stage, which is production. When α is smaller than 0.5, it indicates that the decision-making unit It gives more importance to the control of environmental pollution (second stage). The model is displayed using variable change as model (2).

$t = \frac{1}{\sum_{i=1}^{m_1} \omega_{ij}^1 x_{ij}^1 + \sum_{i=1}^{m_2} \omega_{ij}^2 x_{ij}^2}$, $\phi_{id}^1 = t\omega_{id}^1$, $\phi_{id}^2 = t\omega_{id}^2$, $\varphi_{rd}^1 = t\mu_{rd}^1$, $\varphi_{rd}^2 = t\mu_{rd}^2$, $\psi_{vd}^1 = t\tau_{vd}^1$, $\psi_{vd}^2 = t\tau_{vd}^2$, $\xi_{cd}^1 = t\eta_{vd}^1$

$\text{Max} \sum_{r=1}^{s_1} \varphi_{rd}^1 y_{rd}^1 + \sum_{r=1}^{s_2} \varphi_{rd}^2 y_{rd}^2 + \sum_{v=1}^{o_2} \psi_{vd}^2 u_{vd}^2$

s.t . (2.1)

$\sum_{r=1}^{s_1} \varphi_{rj}^1 y_{rj}^1 + \sum_{r=1}^{s_2} \varphi_{rj}^2 y_{rj}^2 + \sum_{v=1}^{o_2} \psi_{vj}^2 u_{vj}^2 - \sum_{i=1}^{m_1} \phi_{ij}^1 x_{ij}^1 + \sum_{i=1}^{m_2} \phi_{ij}^2 x_{ij}^2 \le 0 \quad j = 1, .. n$

$\sum_{r=1}^{s_2} \varphi_{rj}^1 y_{rj}^1 + \sum_{v=1}^{o_1} \psi_{vj}^1 u_{vj}^1 - \sum_{i=1}^{m_1} \phi_{ij}^1 x_{ij}^1 \le 0 \; j = 1, \dots, n \quad (2.2)$

$\sum_{r=1}^{s_2} \varphi_{rj}^2 y_{rj}^2 + \sum_{v=1}^{o_2} \psi_{vj}^2 u_{vj}^2 - \sum_{i=1}^{m_1} \phi_{ij}^2 x_{ij}^2 - \sum_{c=1}^{o_1} \psi_{cj}^1 u_{cj}^1 \le 0 \;, j = 1, \dots, n \quad (2.3)$

$\sum_{i=1}^{m_1} \phi_{id}^1 x_{id}^1 + \sum_{i=1}^{m_1} \phi_{id}^2 x_{id}^2 = 1 \quad (2.4)$

$\sum_{i=1}^{m_1} \phi_{id}^1 x_{id}^1 = \alpha\left(1 + \sum_{c=1}^{o_1} \psi_{cj}^1 u_{cj}^1\right)$ (2.5)

$\phi_{ij}^1, \phi_{ij}^2, \varphi_{rj}^1, \varphi_{rj}^2, \psi_{vj}^1 \ge 0 \quad j = 1, .., n$

Now we convert the model (2) in the following simplified form.

$\text{Max} \quad \varphi_1 y_1 + \varphi_2 y_2 + \psi_2 u_2$

s.t . $\varphi_1 y_1 + \varphi_2 y_2 + \psi_2 u_2 - \phi_1 x_1 - \phi_2 x_2 \le 0 \quad (3.1)$

$\varphi_1 y_1 + \psi_1 u_1 - \phi_1 x_1 \le 0 \quad (3.2)$

$\varphi_2 y_2 + \psi_2 u_2 - \phi_2 x_2 - \psi_1 u_1 \le 0 \quad (3.3)$

$\phi_1 x_1 + \phi_2 x_2 = 1 \quad (3.4)$

$\phi_1 x_1 = \alpha(1 + \psi_1 u_1) \quad (3.5)$

$\phi_1, \phi_2, \varphi_1, \varphi_2, \psi_1, \psi_2 \geq 0$

As mentioned in the introduction, there are different methods to evaluate the efficiency of two-stage systems. In this article, the evaluation of the efficiency of the two-stage system with an unfavorable output is discussed using the relational method of the product type.

3.1 Calculate the efficiency of the first stage:

The purpose of calculating the efficiency of the first stage in the figure is to assume that the efficiency of the whole system remains constant and the efficiency E_d is constant. Model (4) is defined as follows:

$$E_d^1 = \text{Max} \frac{\sum_{r=1}^{s_2} \mu_{rd}^1 y_{rd}^1 + \sum_{v=1}^{o_1} \tau_{vd}^1 u_{vd}^1}{\sum_{i=1}^{m_1} \omega_{io}^1 x_{io}^1} \quad (4\text{-}1)$$

$$E_d = \frac{\sum_{r=1}^{s_2} \mu_{rd}^1 y_{rd}^1 + \sum_{r=1}^{s_2} \mu_{rd}^2 y_{rd}^2 + \sum_{v=1}^{o_2} \tau_{vd}^2 u_{vd}^2}{\sum_{i=1}^{m_1} \omega_{id}^1 x_{id}^1 + \sum_{i=1}^{m_2} \omega_{id}^2 x_{id}^2} \leq 1 \quad (4\text{-}2)$$

$$\frac{\sum_{r=1}^{s_2} \mu_{rj}^1 y_{rj}^1 + \sum_{v=1}^{o_1} \tau_{vj}^1 u_{vj}^1}{\sum_{i=1}^{m_1} \omega_{ij}^1 x_{ij}^1} \leq 1 \quad j = 1, \dots, n \quad (4\text{-}3)$$

$$\frac{\sum_{r=1}^{s_2} \mu_{rj}^2 y_{rj}^2 + \sum_{v=1}^{o_2} \tau_{vj}^2 u_{vj}^2}{\sum_{i=1}^{m_1} \omega_{ij}^2 x_{ij}^2 + \sum_{c=1}^{o_1} \eta_{cj}^1 u_{cj}^1} \leq 1 \quad j = 1, \dots, n \quad (4\text{-}4)$$

$$\frac{\sum_{i=1}^{m_1} \omega_{id}^1 x_{id}^1}{\sum_{i=1}^{m_1} \omega_{id}^2 x_{id}^2 + \sum_{c=1}^{o_1} \eta_{cd}^1 u_{cd}^1} = \frac{\alpha}{1-\alpha} \quad (4\text{-}5)$$

$$\omega_{ij}^1, \omega_{ij}^2, \eta_{cj}^1, \mu_{rj}^1, \mu_{rj}^2, \tau_{vj}^1 \geq 0 \quad j = 1, \dots, n \quad (4-6)$$

3.2 Calculate the efficiency of the second stage:

The purpose of calculating the efficiency of the second stage in figure (2-1) is assuming that the efficiency of the whole system remains constant and the efficiency E_d is constant. Model (5) is defined as follows:

$$E_d^2 = \text{Max} \frac{\sum_{r=1}^{s_2} \mu_{rd}^2 y_{rd}^2 + \sum_{v=1}^{o_2} \tau_{vd}^2 u_{vd}^2}{\sum_{i=1}^{m_1} \omega_{id}^2 x_{id}^2 + \sum_{c=1}^{o_1} \eta_{cd}^1 u_{cd}^1} \quad (5\text{-}1)$$

$$E_d = \frac{\sum_{r=1}^{s_2} \mu_{rd}^1 y_{rd}^1 + \sum_{r=1}^{s_2} \mu_{rd}^2 y_{rd}^2 + \sum_{v=1}^{o_2} \tau_{vd}^2 u_{vd}^2}{\sum_{i=1}^{m_1} \omega_{id}^1 x_{id}^1 + \sum_{i=1}^{m_2} \omega_{id}^2 x_{id}^2} \leq 1 \quad (5\text{-}2)$$

$$\frac{\sum_{r=1}^{s_2} \mu_{rj}^1 y_{rj}^1 + \sum_{v=1}^{o_1} \tau_{vj}^1 u_{vj}^1}{\sum_{i=1}^{m_1} \omega_{ij}^1 x_{ij}^1} \leq 1 \quad j = 1, \dots, n \quad (5\text{-}3)$$

$$\frac{\sum_{r=1}^{s_2} \mu_{rj}^2 y_{rj}^2 + \sum_{v=1}^{o_2} \tau_{vj}^2 u_{vj}^2}{\sum_{i=1}^{m_1} \omega_{ij}^2 x_{ij}^2 + \sum_{c=1}^{o_1} \eta_{cj}^1 u_{cj}^1} \leq 1 \quad j = 1, \dots, n \quad (5\text{-}4)$$

$$\frac{\sum_{i=1}^{m_1} \omega_{id}^1 x_{id}^1}{\sum_{i=1}^{m_1} \omega_{id}^2 x_{id}^2 + \sum_{c=1}^{o_1} \eta_{cd}^1 u_{cd}^1} = \frac{\alpha}{1-\alpha} \quad (5\text{-}5)$$

$$\omega_{ij}^1, \omega_{ij}^2, \eta_{cj}^1, \mu_{rj}^1, \mu_{rj}^2, \tau_{vj}^1 \geq 0 \quad j = 1, \dots, n$$

The relationship is defined as follows:

$$E_d = (w^1 E_d^1 + w^2) \times (w^3 E_d^2 + w^4)$$

$$w^1 + w^2 = 1 \ , \ w^3 + w^4 = 1$$

The efficiency of the whole system can be written as a multiplicative combination of the efficiency of each sub-system. Now we can express model (1) in the form of model (6).

$$E_d = \text{Max}(w_d^1 E_d^1 + w_d^2) \times (w_d^3 E_d^2 + w_d^4) \quad (6\text{-}1)$$

$$\text{s.t} \ \frac{\sum_{r=1}^{s_1} \mu_{rj}^1 y_{rj}^1 + \sum_{r=1}^{s_2} \mu_{rj}^2 y_{rj}^2 + \sum_{v=1}^{o_2} \tau_{vj}^2 u_{vj}^2}{\sum_{i=1}^{m_1} \omega_{ij}^1 x_{ij}^1 + \sum_{i=1}^{m_2} \omega_{ij}^2 x_{ij}^2} \leq 1 \quad j = 1, \dots, n \quad (6\text{-}2)$$

$$\frac{\sum_{r=1}^{s_1} \mu_{rj}^1 y_{rj}^1 + \sum_{v=1}^{o_1} \tau_{vj}^1 u_{vj}^1}{\sum_{i=1}^{m_1} \omega_{ij}^1 x_{ij}^1} \leq 1 \quad j = 1, \dots, n \quad (6\text{-}3)$$

$$\frac{\sum_{r=1}^{s_2} \mu_{rj}^2 y_{rj}^2 + \sum_{v=1}^{o_2} \tau_{vj}^2 u_{vj}^2}{\sum_{i=1}^{m_1} \omega_{ij}^2 x_{ij}^2 + \sum_{c=1}^{o_1} \eta_{cj}^1 u_{cj}^1} \leq 1 \quad j = 1, \dots, n \quad (6\text{-}4)$$

$$\frac{\sum_{i=1}^{m_1} \omega_{id}^1 x_{id}^1}{\sum_{i=1}^{m_1} \omega_{id}^2 x_{id}^2 + \sum_{c=1}^{o_1} \eta_{cd}^1 u_{cd}^1} = \frac{\alpha}{1-\alpha} \quad (6\text{-}5)$$

$$E_d^1 = \frac{\sum_{r=1}^{s_1} \mu_{rd}^1 y_{rd}^1 + \sum_{v=1}^{o_1} \tau_{vd}^1 u_{vd}^1}{\sum_{i=1}^{m_1} \omega_{io}^1 x_{io}^1} \quad (6\text{-}6)$$

$$E_d^2 = \frac{\sum_{r=1}^{s_2} \mu_{rd}^2 y_{rd}^2 + \sum_{v=1}^{o_2} \tau_{vd}^2 u_{vd}^2}{\sum_{i=1}^{m_1} \omega_{id}^2 x_{id}^2 + \sum_{c=1}^{o_1} \eta_{cd}^1 u_{cd}^1} \quad (6\text{-}7)$$

$$w_d^1 = \frac{\sum_{i=1}^{m_1} \omega_{io}^1 x_{io}^1}{\sum_{i=1}^{m_1} \omega_{io}^1 x_{io}^1 + \sum_{i=1}^{m_1} \omega_{id}^2 x_{id}^2} \quad (6\text{-}8)$$

$$w_d^2 = \frac{\sum_{i=1}^{m_1} \omega_{io}^2 x_{io}^2}{\sum_{i=1}^{m_1} \omega_{io}^1 x_{io}^1 + \sum_{i=1}^{m_1} \omega_{id}^2 x_{id}^2} \quad (6\text{-}9)$$

$$w_d^3 = \frac{\sum_{i=1}^{m_1} \omega_{io}^2 x_{io}^2 + \sum_{v=1}^{o_1} \tau_{vd}^1 u_{vd}^1}{\sum_{r=1}^{1} \mu_{rd}^1 y_{rd}^1 + \sum_{v=1}^{o_1} \tau_{vd}^1 u_{vd}^1 + \sum_{i=1}^{m_1} \omega_{id}^2 x_{id}^2} \quad (6\text{-}10)$$

$$w_d^4 = \frac{\sum_{r=1}^{1} \mu_{rd}^1 y_{rd}^1}{\sum_{r=1}^{1} \mu_{rd}^1 y_{rd}^1 + \sum_{v=1}^{o_1} \tau_{vd}^1 u_{vd}^1 + \sum_{i=1}^{m_1} \omega_{id}^2 x_{id}^2} \quad (6\text{-}11)$$

$$\omega_{ij}^1, \omega_{ij}^2, \eta_{cj}^1, \mu_{rj}^1, \mu_{rj}^2, \tau_{vj}^1 \geq 0 \quad j = 1, \dots, n$$

In order to evaluate the efficiency of two-stage systems with unfavorable output with variable scale efficiency, we must also perform all these steps for the BCC model of multiple form.

4. Numerical example

Based on what was presented in the previous sections, the evaluation of the efficiency of the two-stage system with unfavorable output is investigated by the product method in 6 regions in China. A series of provinces and cities that have the same geographic, financial, social, natural, etc. conditions are located in one region. These 6 areas are shown in table (1-4). The environmental projects of these 6 areas are representatives of the two-stage systems in Figure 2.

Table (2-4) introduces variables and data related to a unit as an example. x^1،y^1،u^1 in order of inputs (number of workers, capital, cost of each unit of coal), desirable outputs (GPD production, undesirable outputs (water pollution, soil pollution)) are the production stage (first stage). x^2 ,y^2, u^2 respectively inputs (capital, water pollution recycling equipment, soil pollution recycling equipment), desirable outputs (recycled amount), undesirable outputs of the stage of environmental pollution (second stage It should be noted that the undesirable output from the production stage is entered into the system as an input in the second stage.

The average efficiency of the whole system and the efficiency of the production stage (first stage) and environmental pollution stage (second stage) are calculated respectively using models 1, 4 and 5 for 6 regions in China in 2007 when α=0.1. The results are shown in table (3-4). Then, based on the presented model 6, the average efficiency of the entire system was calculated for 6 regions of China in 2007 when α=0.1. The obtained information is shown in table (4-4). In model 6, coefficients W_3, W_4, W_1, W_2 are introduced in the model according to the relations (6-8), (6-9), (6-10) and (6-11). These coefficients can be calculated. After calculating the coefficients and having the efficiency of the production stage and the stage of environmental pollution in each region of China, the total efficiency can be calculated. As mentioned, the efficiency of the whole system was introduced by the product method based on the efficiency of two stages.

Table (1-4) 6 regions in China

Region	
NEC	Northeast China
ECC	Eastern Coastal China
SCC	Southern Coastal China
MYZR	Middle reaches of the Yellow River
SWC	Southwest china
NWC	Northwest china

Table 4.2) Introducing the variables and data of one of the units

	variable	Max	Min	Mean	Standard Deviation
x^1	Labor Capital Coal expenses per GDP	1568 34509.27 4.0899	12 625.81 0.5484	282.92 8170.71 1.43	320.94 7025.8 0.73
y^1	Industrial GDP	92056.84	640.26	16505.86	18891.25
u^1	Industrial waste water Industrial solid waste	287181 31688	5782	79995.29	69490.96
x^2	Pollution treatment investment	844159	3563	156673.95	134703.48
	Industrial waste water recycling and reusing	287181	5782	79995.29	69490.96
	Industrial solid waste recycling and reusing	31688	147	6414.27	5102.86
y^2	The comprehensive values of wastes re-utilization	2863867	10369.2	492400.84	552538.9
u^2	Unclean residuals of including waste water	263760	1	20788.86	43052.47
	Unclean residuals of including solid waste	9631	0.1	1757.36	2188.71

Table 4-3) The average efficiency of subsystems and the efficiency of the whole system for 6 regions in China in 2007 when α=0.1

	Efficiency of stage 1	Efficiency of stage 2	Comprehensive efficiency
NEC	0.327	0.511	0.511
ECC	0.474	0.681	0.681
SCC	0.474	0.610	0.610
MYZR	0.228	0.787	0.787
SWC	0.201	0.764	0.764
NWC	0.584	0.546	0.546

Table 4-4) The average efficiency of the whole system and W_3،W_4،W_1،W_2 based on model (6) for 6 regions in China in 2007 when α=0.1

	Comprehensive efficiency	W_1	W_2	W_3	W_4
NEC	0.478	0.102	0.898	0.999	0.001
ECC	0.642	0.100	0.900	1.000	0
SCC	0.578	0.100	0.900	1.000	0
MYZR	0.726	0.100	0.900	1.000	0
SWC	0.703	0.100	0.900	1.000	0
NWC	0.524	0.100	0.900	1.000	0

5. Conclusion

Based on what was presented in the previous parts, the following results are obtained.

1. In the product-type relationship method to measure the efficiency of a two-stage system with undesirable outputs, the effect of the efficiency of intermediate products is also considered, and the efficiency of the stages is measured simultaneously with the entire system. The efficiency of the system with undesirable output is The product of the efficiency of the first and second steps is obtained. The exact size obtained from the performance evaluation mode is the traditional method. It is clear that the model (6) cannot be linearized by applying Charles Cooper changes. Therefore, with the right values for w^1،w^2،w^3،w^4

$$E_d = (w^1E_d^1+w^2) \times (w^3E_d^2+w^4)$$

We showed a method to linearize the model (6).

A two-stage system is efficient when both stages of the two-stage system are efficient. It is carefully concluded in a two-stage system that any change in the size of the intermediate values and its increase in order to increase the efficiency of the first stage, which is more productive, causes a decrease Efficiency becomes the second step. As a result, environmental pollution has increased and the government should try to reduce its control. Reducing the size of the intermediate values in order to increase the efficiency of the second stage will increase the efficiency of the first stage, which shows that the manufacturer pays more attention to the control of environmental pollution.

2. By comparing the efficiency obtained from model 6 with the efficiency obtained from model 1, it is concluded that there are some minor differences that are caused by the errors obtained in each model.

3. According to the tables (3-4), (4-4), the ranking is as follows:

$$\text{MYZR} \gg \text{SWC} \gg \text{ECC} \gg \text{SCC} \gg \text{NWC} \gg \text{NEC}$$

This relationship results that the ranking is the same using the efficiencies obtained from the two models (1) and (6).

6. Offers

According to the discussed topics, the following suggestions are made:

1. Evaluating the efficiency of two-stage systems with undesirable outputs using the BCC-CCR model
2. Evaluating the efficiency of two-stage systems with undesirable outputs using the CCR-BCC model

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