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Determine of the Malmquist Productivity Index Units in DEA on Fuzzy Inputs and Outputs

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Abstract

Data Envelopment Analysis (DEA) is an extremely flexible and useful methodology, which provides a relative efficiency score for each decision-making unit.
Classic DEA models only evaluate the performance efficiency of units and they cannot provide any information about the progress and regress of units in two periods. Also, they suppose that all inputs and outputs are positive real numbers. But in the real world, due to the existence of uncertainty, this assumption may not always be true. So, the DEA models with fuzzy data (FDEA models) can more realistically represent real-world problems than the conventional DEA models.
In this paper, we assume all inputs, outputs and efficiency measures are triangular fuzzy numbers. So we present a method for obtaining the fuzzy Malmquist productivity index on fuzzy data, and finally, we can determine progress and regression for units with fuzzy data.

Keywords: DEA, Fuzzy data, Fuzzy DEA, Malmquist Productivity Index.

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1. Introduction

Data Envelopment Analysis (DEA) is an extremely flexible and useful methodology, which provides an indicator of the relative efficiency for each different analyzed decision-making unit, where efficiency is a measure of different features related to the environment as well as to the economic or social impacts of the protected area. DEA is a multivariate technique for monitoring productivity and providing insights on the possible directions of improvement of the status quo, when inefficient. It is a nonparametric technique, i.e., it can compare input/output data, making no prior assumptions about the probability distribution under study. The origin of non-parametric programming methodology, with respect to the relative efficiency measurement, lies in the work of Banker et al and Charnes et al [1,2].

DEA has been applied to several benchmarking studies and to the performance analysis of public institutions, such as schools, and hospitals, but also of private ones, such as banks. An exhaustive analysis of its underlying theory and main applications can be found in the literature [3-8].

Also, a comprehensive literature review in Tavaresa and Despotis [9, 10]. The Malmquist productivity index was introduced by Caves, Christensen and Diewert. They extended an idea of Malmquist, who in a consumer context used Ratios of input distance functions to construct an input quantity index [11, 12]. Fare et al. followed and extended the work of Caves et al [11, 13]. They calculated an adjacent Malmquist productivity index consisting of the geometric mean of two Malmquist indexes as defined by Caves et al [11]. Later, Berg et al., introduced a base period Malmquist Productivity index, which except for the fixed base period technology is the same as one of the indexes defined by Caves et al [11, 14]. The adjacent and the base period versions of the Malmquist index measure the efficiency Change between two time periods equivalently. The technical change, however, is measured differently. The adjacent version is primarily a two-period notion and measures the shift in the technology frontier as the shift in the frontier at time t and t + 1. It only uses adjacent time periods to measure the technology changes. Finally, the two shifts are averaged geometrically.

In real-world situations, however, this assumption may not always be true. Due to the existence of uncertainty, DEA sometimes faces the situation of imprecise data, especially when a set of Decision-Making Units (DMUs) contains missing data, judgment data, forecasting data or ordinal preference information. Generally speaking, uncertain information or imprecise data can be expressed in interval or fuzzy numbers. The DEA models with fuzzy data (FDEA models) can more realistically represent real-world problems than the conventional DEA models, see [15-21]. The fuzzy set theory also allows linguistic data to be used directly within the DEA models. The fuzzy DEA models take the form of fuzzy linear programming models. A typical approach to fuzzy linear programming requires a method to rank fuzzy sets and different fuzzy ranking methods may lead to different results. The problem of ranking fuzzy sets has been addressed by many researchers. In this paper, we assume all inputs, outputs and efficiency measures are triangular fuzzy numbers. We use the concept of the symmetric triangular fuzzy number and introduce an approach to defuzzy a general fuzzy quantity. The basic idea of the new method is to obtain the "nearest” symmetric triangular approximation of fuzzy numbers. By use of a ranking function, we introduce the approach to solving the fuzzy CCR model (FCCR) with fuzzy data.

2. DEA background

Consider n decision making units DMUj, j={1,...,n}, which each DMU consumes inputs levels x_{ij}, i={1,...,m}, to produce outputs levels y_{rj}, r={1,...,s}. Let is j={1,...,n} and suppose that $X_j = (x_{1j}, \dots, x_{mj})^T$ and $Y_j = (y_{1j}, \dots, y_{sj})^T$ are the vectors of inputs and outputs values respectively, for DMUj , in which it has been assumed that Xj ≥ 0, Xj $\neq 0$ and Yj $\geq$ 0, Yj $\neq 0$. The relative efficiency score of the DMUo, o$\in\{1,...,n\}$, is obtained from the following model which is called the input-oriented CCR envelopment model:

$$\theta^k(X_o^p, Y_o^p) = Min\ \theta \qquad (1)$$

$$s.t \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \quad i = 1, \dots m,$$

$$\sum \lambda_j y_{rj} \geq y_{ro,}, \quad r = 1, \dots s,$$

$$\lambda_j \geq 0, \ j = 1, \dots n.$$

The well-known Fare et al [7] adjacent Malmquist input-based productivity index is defined as the geometric mean of two Malmquist input-based Productivity indexes which is introduced by Caves et al, i.e., as [5]:

$$MPI = \sqrt{\frac{\theta^t(X_p^{t+1}, Y_p^{t+1})\ \theta^{t+1}(X_p^{t+1}, Y_p^{t+1})}{\theta^t(X_p^t, Y_p^t)\ \theta^{t+1}(X_p^t, Y_p^t)}} \qquad (2)$$

The decomposition of (2) in an efficiency change component and a technical change component is given as:

$$MPI = \underbrace{\frac{\theta_o^{t+1}(X_0^{t+1}, Y_0^{t+1})}{\theta_o^t(X_0^t, Y_0^t)}}_{\text{Efficiency Change}} \underbrace{\left[\frac{\theta_o^t(X_o^{t+1}, Y_o^{t+1})}{\theta_o^{t+1}(X_0^{t+1}, Y_0^{t+1})} \cdot \frac{\theta_o^t(X_0^t, Y_0^t)}{\theta_o^{t+1}(X_o^t, Y_o^t)}\right]^{\frac{1}{2}}}_{\text{Technical Change}} \qquad (3)$$

The efficiency change share in (3) is equal to the ratio of the Farrell technical efficiency measure at time t + 1, divided by the Farrell technical efficiency measure at time t. The technical change part is captured by the geometric average of the two ratios reflecting the shifts in the frontier at time t, and t + 1.M >1 indicate an improvement in total productivity, M <1 a decline, and M = 1 an unchanged productivity growth [5, 6].

3. An introduction to fuzzy sets

Considering what Maleki et al. have presented in [22] we will review a background in fuzzy logic. We represent an arbitrary fuzzy number by an ordered pair of functions $\tilde{u} =: (\underline{u}(r), \overline{u}(r))$, $0 < r < 1$, which satisfy the following requirements:

- $\underline{u}(r)$ is a bounded left continuous non-decreasing function over [0,1].
- $\overline{u}(r)$ is a bounded right continuous non-increasing function over [0,1].
- $\underline{u}(r)$ and $\overline{u}(r)$ are right continuous in 0
- $\underline{u}(r) \leq \overline{u}(r)$, $0 < r < 1$.

A crisp number α is simply represented by $\overline{u}(r) = \underline{u}(r) = \alpha$, $0 < r < 1$.

Definition1. The fuzzy number $\tilde{t} =: (C_{\tilde{t}} - W_{\tilde{t}}^L + W_{\tilde{t}}^L r, C_{\tilde{t}} + W_{\tilde{t}}^R - W_{\tilde{t}}^R r)$ $=: (C_{\tilde{t}}, W_{\tilde{t}}^L, W_{\tilde{t}}^R)$, $0 < r < 1$ is a non-symmetric triangular fuzzy number N.S.T. As a matter of

Fact $C_{\tilde{t}} - W_{\tilde{t}}^L + W_{\tilde{t}}^L r = \underline{t}(r)$ and $C_{\tilde{t}} + W_{\tilde{t}}^R - W_{\tilde{t}}^R r = \overline{t}(r)$.

A popular fuzzy number is the symmetric triangular fuzzy number (S.T) $S[x_o, \sigma]$ where $W_{\tilde{t}}^L = W_{\tilde{t}}^R = \sigma$ centered at x_o with basis 2σ.

Definition2. Let $\tilde{t} =: (C_{\tilde{t}}, W_{\tilde{t}}^L, W_{\tilde{t}}^R)$ and $\tilde{u} =: (C_{\tilde{u}}, W_{\tilde{u}}^L, W_{\tilde{u}}^R)$ are N.S.T and $K \in \Re$

by using extension principal we can define:

1. $\tilde{t} = \tilde{u}$ if and only if $C_{\tilde{t}} = C_{\tilde{u}}$ and $W_{\tilde{t}}^{L} = W_{\tilde{u}}^{L}$ and $W_{\tilde{t}}^{R} = W_{\tilde{u}}^{R}$

2. $\tilde{t} + \tilde{u} = (C_{\tilde{t}} + C_{\tilde{u}}, W_{\tilde{t}}^{L} + W_{\tilde{u}}^{L}, W_{\tilde{t}}^{R} + W_{\tilde{u}}^{R})$

3.

$$k\tilde{t} = \begin{cases} (kC_{\tilde{t}}, kW_{\tilde{t}}^{L}, kW_{\tilde{t}}^{R}) & k \geq 0 \\ (kC_{\tilde{t}}, -kW_{\tilde{t}}^{R}, -kW_{\tilde{t}}^{L}) & k < 0 \end{cases} \quad (4)$$

Definition3. (Ordering on S.T) Let $\tilde{t} = (x_{o_1}, \sigma_1)$ *and* $\tilde{u} = (x_{o2}, \sigma_2)$

We say $\tilde{t} \prec \tilde{u}$ if and only if:

1. $x_{o_1} < x_{o2}$

or

2. $x_{o_1} = x_{o2}$ and $\sigma_2 < \sigma_1$

In the case equality we have $\tilde{t} \cong \tilde{u}$ if and only if (($x_{o_1} = x_{o2}$) $\wedge$ ($\sigma_2 = \sigma_1$)).

And $\tilde{t} \lesssim \tilde{u}$ if and only if ($\tilde{t} \prec \tilde{u} \vee \tilde{t} \cong \tilde{u}$).

We extend this Ordering on S.T to an efficient approach for ordering the elements of $F(\Re)$, that is to define a ranking function $\psi : F(\Re) \rightarrow \Re$ which maps each fuzzy number into the line, where a natural order exists. We define orders on $F(\Re)$ by

1. $\tilde{t} \prec \tilde{u}$ if and only if $\psi(\tilde{t}) < \psi(\tilde{u})$.
2. $\tilde{t} \lesssim \tilde{u}$ if and only if $\psi(\tilde{t}) \leq \psi(\tilde{u})$.
3. $\tilde{t} \cong \tilde{u}$ if and only if $\psi(\tilde{t}) = \psi(\tilde{u})$.

Where $\tilde{t}$ and $\tilde{u}$ are in $F(\Re)$.

We restrict our attention to linear ranking function, that is, a ranking function ψ such that:

$$\psi(k\tilde{t} + \tilde{u}) = k\psi(\tilde{t}) + \psi(\tilde{u}) \quad (5)$$

for any $\tilde{t}$ and $\tilde{u}$ are in $F(\Re)$ and any $k \in \Re$.

Here, we introduce a linear ranking function that is similar to the ranking function adopted by Maleki (FJMS) [9]. For a fuzzy number $\tilde{u} =: (\underline{u}(r), \overline{u}(r))$, we define ranking function as follows:

$$\psi(\tilde{u}) = 1/2\int_0^1 (\underline{u}(r) + \overline{u}(r))dr - \varepsilon 3/2\int_0^1 (\overline{u}(r) - \underline{u}(r))(1-r)dr. \quad (6)$$

Where

$$\sigma(\tilde{u}) = 3/2\int_0^1 (\overline{u}(r) - \underline{u}(r))(1-r)dr, \quad (7)$$

$$x_o(\tilde{u}) = 1/2\int_0^1 (\underline{u}(r) + \overline{u}(r))dr, \quad (8)$$

i.e., the nearest symmetric triangular defuzzification of $\tilde{u}$ is given by the Center $x_o(\tilde{u})$ and fuzziness $\sigma(\tilde{u})$ [9]. It should be noted that is a non- Archimedean infinitesimal number.

For the non-symmetric triangular fuzzy number $\tilde{u} =: (C_{\tilde{u}}, W_{\tilde{u}}^{L}, W_{\tilde{u}}^{R})$ we have:

$$\psi(\tilde{u}) = C_{\tilde{u}} - 1/4W_{\tilde{u}}^{L} + 1/4W_{\tilde{u}}^{R} - \varepsilon 1/2(W_{\tilde{u}}^{L} + W_{\tilde{u}}^{R}). \quad (9)$$

Definition4. For two fuzzy numbers in parametric $\tilde{t} =: (C_{\tilde{t}}, W_{\tilde{t}}^{L}, W_{\tilde{t}}^{R})$ and $\tilde{u} =: (C_{\tilde{u}}, W_{\tilde{u}}^{L}, W_{\tilde{u}}^{R})$ we have:

$\tilde{t}\,\tilde{u} = \tilde{l} = (\underline{l}(r), \bar{l}(r))$,

Where

$$\underline{l}(r) = \min\left\{\overline{t}(r)\overline{u}(r), \underline{t}(r)\underline{u}(r), \overline{t}(r)\underline{u}(r), \underline{t}(r)\overline{u}(r)\right\}$$

$$\underline{l}(r) = \max\left\{\overline{t}(r)\overline{u}(r), \underline{t}(r)\underline{u}(r), \overline{t}(r)\underline{u}(r), \underline{t}(r)\overline{u}(r)\right\}$$

for example for two positive N.S.T s, $\tilde{t} =: (C_{\tilde{t}}, W_{\tilde{t}}^{L}, W_{\tilde{t}}^{R})$ and $\tilde{u} =: (C_{\tilde{u}}, W_{\tilde{u}}^{L}, W_{\tilde{u}}^{R})$ where $(C_{\tilde{t}} - W_{\tilde{t}}^{L} \geq 0)$ and $(C_{\tilde{u}} - W_{\tilde{u}}^{L} \geq 0)$ we have:

$$\begin{aligned} \underline{l}(r) &= C_{\tilde{t}}C_{\tilde{u}} + C_{\tilde{t}}W_{\tilde{u}}^{L}(r-1) \\ &+ W_{\tilde{t}}^{L}C_{\tilde{u}}(r-1) + W_{\tilde{t}}^{L}W_{\tilde{u}}^{L}(r-1)^2, \\ \bar{l}(r) &= C_{\tilde{t}}C_{\tilde{u}} + C_{\tilde{t}}W_{\tilde{u}}^{R}(1-r) \\ &+ W_{\tilde{t}}^{R}C_{\tilde{u}}(1-r) + W_{\tilde{t}}^{R}W_{\tilde{u}}^{R}(1-r)^2. \end{aligned} \quad (10)$$

Definition5. Extended Division. Division is also neither an increasing nor a decreasing operation if $\tilde{t}$ and $\tilde{u}$ are strictly positive fuzzy numbers in parametric $\tilde{t} =: (C_{\tilde{t}}, W_{\tilde{t}}^L, W_{\tilde{t}}^R)$ and $\tilde{u} =: (C_{\tilde{u}}, W_{\tilde{u}}^L, W_{\tilde{u}}^R)$ we have:

$$\frac{\tilde{t}}{\tilde{u}} \cong \left(\frac{C_{\tilde{t}}}{C_{\tilde{u}}}, \frac{C_{\tilde{t}} W_{\tilde{u}}^R + C_{\tilde{u}} W_{\tilde{t}}^L}{(C_{\tilde{u}})^2}, \frac{C_{\tilde{t}} W_{\tilde{u}}^L + C_{\tilde{u}} W_{\tilde{t}}^R}{(C_{\tilde{u}})^2}\right) \quad (11)$$

4. Results and discussion DEA with fuzzy data

In recent years, the fuzzy set theory has been proposed as a way to quantify imprecise and vague data in DEA models. Fuzzy DEA models take the form of fuzzy linear programming model. The CCR model with fuzzy coefficients and its dual is formulated as the following linear programming models:

$$\tilde{\theta}^p(\tilde{X}_o^k, \tilde{Y}_o^k) = Min\ \tilde{\theta} \quad (12)$$

$$\begin{aligned} s.t \quad & \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij}^p \tilde{\le} \tilde{\theta} \tilde{x}_{io}^k, \quad && i = 1, \dots m, \\ & \sum \lambda_j \tilde{y}_{rj}^p \tilde{\ge} \tilde{y}_{ro}^k, && r = 1, \dots s, \\ & \lambda_j \ge 0, && j = 1, \dots n. \end{aligned}$$

Where $\tilde{X}_o$ is the column vector of fuzzy inputs consumed by the target DMU (DMUo), $\tilde{X}$ is the matrix of fuzzy inputs of all DMUs, $\tilde{Y}_o$ is the column vector of fuzzy outputs consumed by the target DMU (DMUo), $\tilde{Y}$ is the matrix of fuzzy outputs of all DMUs. In the above model indices k, an p denotes the time period and technology respectively.

The fuzzy CCR models cannot be solved by a standard LP solver like a crisp CCR model because coefficients in the fuzzy CCR model are fuzzy sets. With the fuzzy inputs and fuzzy outputs, the optimality conditions for the crisp DEA model need to be clarified and generalized. This approach by using the concept FFLP can be used to solve the above FCCR then. According to what Maleki has provided in the following model will be resulted [22].

Theorem 1. The following linear programming problem (CCR) and the FCCR are equivalent:

$$\tilde{\theta}^p(\tilde{X}_o^k, \tilde{Y}_o^k) = Min\ \tilde{\theta} \quad (13)$$

$$\begin{aligned} s.t & \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij}^p \tilde{\le} \tilde{\theta} \tilde{x}_{io}^k, \quad && i = 1, \dots m, \\ & \sum \lambda_j \tilde{y}_{rj}^p \tilde{\ge} \tilde{y}_{ro}^k, && r = 1, \dots s, \\ & \lambda_j \ge 0, && j = 1, \dots n. \end{aligned}$$

And

$$\psi(\tilde{\theta}^p(\tilde{X}_o^k, \tilde{Y}_o^k)) = Min\ \psi(\tilde{\theta}) \quad (14)$$

$$\begin{aligned} s.t \ & \sum_{j=1}^{n} \lambda_j \psi(\tilde{x}_{ij}^p) \le \psi(\tilde{\theta} \tilde{x}_{io}^k), i = 1, \dots m, \\ & \sum \lambda_j \psi(\tilde{y}_{rj}^p) \ge \psi(\tilde{y}_{ro}^k), r = 1, \dots s, \\ & \lambda_j \ge 0, \quad j = 1, \dots n. \end{aligned}$$

It should be mentioned that in model (10) multiplication of two fuzzy numbers are considered according to definition 4. In recent years, the fuzzy set theory has been proposed as a way to quantify imprecise and vague data in DEA models. Fuzzy DEA models take the form of fuzzy linear programming model. The fuzzy CCR models cannot be solved by a standard LP solver like a crisp CCR model because coefficients in the fuzzy CCR model are fuzzy sets. With the fuzzy inputs and fuzzy outputs, the optimality conditions for the crisp DEA model need to be clarified and generalized. In this section, we are in purpose to evaluate the Malmquist productivity index for DMUs with fuzzy data. Therefore, assume that fuzzy numbers, $\tilde{x}_{ij}^t$ and $\tilde{y}_{rj}^t$ are the i-th input and the r-th output for DMUj in time t. The two single period measures can be obtained by using the FCCR DEA model

(12). The efficiency $\tilde{\theta}_o^t(t)$ determines the amount by which observed inputs can be proportionally reduced, while still producing the given output level. Using (t + 1) instead of (t) for the above method, we get $\tilde{\theta}_o^{t+1}(t+1)$ the technical efficiency score for DMUo in time period (t + 1). The first of the mixed period measures, which is defined as $\tilde{\theta}_o^t(t+1)$ for each DMUo, is computed as the optimal value to the (12) fuzzy linear programming problem, where p=t and k = t +1. Similarly, the other mixed period measure, $\tilde{\theta}_o^{t+1}(t)$, which is needed in the computation of the Malmquist productivity index, is the optimal value to the (12) fuzzy linear problem, where p=t+1 and k=t. The Fuzzy Malmquist productivity index, which measures the productivity change of a particular DMUo, $o \in J = \{1, ..., n\}$, in time t + 1 and t is given as:

$$(FMPI_o)^2 = \frac{\psi(\tilde{\theta}_o^t(t+1))\psi(\tilde{\theta}_o^{t+1}(t+1))}{\psi(\tilde{\theta}_o^t(t))\psi(\tilde{\theta}_o^{t+1}(t))} \quad (15)$$

For obtaining $(FMPI_o)^2$, we apply relation (10) and then relation (11) for division of fuzzy numbers.

It can be seen that the above measure actually is the geometric mean of two Caves, et al.'s [11] Malmquist productivity indexes. Thus, following Caves et al.'s [11] suit, Fare, et al. [13] define that $(FMPI_o)^2 > 1$, indicates productivity gain; $(FMPI_o)^2 < 1$ indicates productivity loss, and $(FMPI_o)^2 = 1)$ means no change in productivity from time t to t + 1.

5. Conclusion

This paper has presented a method for obtaining Fuzzy Malmquist Productivity Index on fuzzy data. In most practical applications of Data Envelopment Analysis (DEA), some or all inputs-outputs indexes of decision-making units are not exactly defined. Therefore, it is need that the presented models are revised in order to obtain measure of progress and regress. Fortunately, we can determine progress and regress for some units, but there are units which we cannot evaluate their progress or regress, and it is a drawback. In order to remove above difficult, we suggested an index for determining progress and regress for any evaluated unit.

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