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Evaluating Efficiency by Multiplier Method in Two-Stage Unit with Undesirable Outputs

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Abstract

Data Envelopment Analysis (DEA) is a linear programming based method to measure performance a group of homogeneous decision making units (DMUs) units such as firms with multiple inputs and outputs. In two-stage unit all or part of the outputs of the first stage are assumed to be the second stage's inputs and measuring performance of these DMUs is one of the important issues in DEA. There are two ways, multiplier or additive model, to calculate efficiency of DMU in DEA which in this paper, a new model is proposed by using multiplier method to evaluate efficiency of such unit then according to the presented model, the efficiency of a hose and plastic product manufacturer in 10 years is examined.

Keywords: Data Envelopment Analysis (DEA); Efficiency; Two-Stage Unit; Undesirable and Desirable Outputs; Multiplier Model.

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1. Introduction

Data envelopment analysis (DEA) is a mathematical non-parametric method used to evaluate the performance of a set of homogeneous decision making units in a production technology with multiple inputs and outputs. The variety of DEA applications along the development of methodology and its concept is the reason for the importance of using this technique in recent decades. In fact, among many evaluation methods, DEA has been widely used to measure the relative efficiency of DMUs because these models need not to recourse to the exact production function regarding multiple inputs and outputs. Decision making units (DMUs) can have a variety types such as banks, factories, universities, etc… and in practice, these units operate as a two-stage process so that all or a part of outputs of the first stage are used as inputs of the second stage. This intermediate product can be considered desirable or undesirable. For instance, a color manufacturing factory uses labor, equipment, raw material, etc. to produce color. Along the color producing, undesirable outputs are produced such as air pollution, color wastage, and so on. Many of these factories bring the color wastage to the second stage and recycle it. Noting the importance of evaluating the performance of network decision making units, several network methods have been proposed to calculate the efficiency of these complex structure units with undesirable middle product. Due to the fact that the Classical DEA models make no assumptions concerning the internal operations of a DMU. Rather, the DEA model treats each DMU as a ‘‘black box’’ by considering only the initial inputs and final outputs produced by it and neglects internal linking.

In order to better understand these types of systems, the process of two- stage systems is generally shown in figure 1.

In this figure, the first stage of the industrial production system has been shown that is used input X^1 for producing desirable output Y^1 and undesirable output U^1. The second stage shows the environmental pollution process, which in this stage, desirable output Y^2 and a little amount of undesirable output U^2 are produced by using an extra input X^2 and undesirable output U^1 from the first stage as input. The undesirable output U^1 from the first stage that is used as input in the second stage is called middle product. In the two-stage systems, producing subsystem in the first phase provides the basic needs of individuals in the society such as food, energy, clothing, health, etc. in the future.

On the other hand, in the coming years, environmental pollution threatens the society. So, the result is that reducing and controlling this pollution is considered as a long-term goal of that unit. From the above, it can be concluded that two- stage systems present short-term and long-term targets, respectively.

The organization of this paper is as follows: In section 2 the literature of DEA is reviewed. The background and innovation are given in section 3 and 4, respectively. In addition, application analysis by testing with a numerical example on Petropuyan company is presented in section 5. In the next section, conclusion is given. Finally, suggestions are given in the last section.

2. Literature review.

The principles of DEA date back to Furrel who proposed evaluation of DMUs in a non-parametric method in 1957 [1]. After that DEA has developed by Charnes et al. [2]. They proposed a non-parametric linear programing to evaluate efficiency of DMUs, taking into account the principle of constant return to scale, and considered the production possibility set (PPS) to estimate the efficiency frontier.

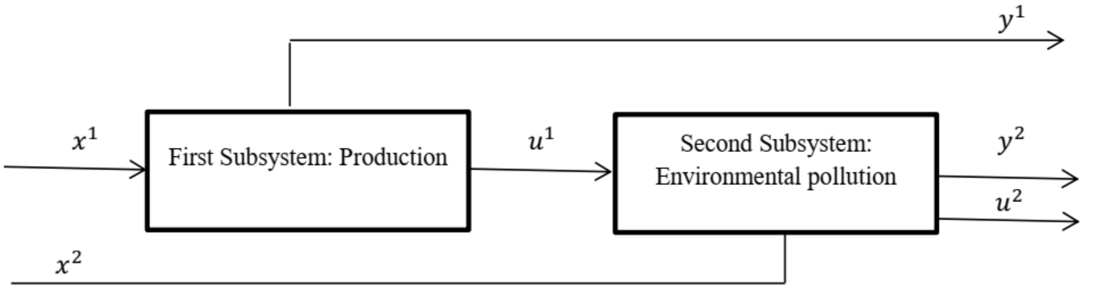

Figure (1): Two-Stage Environmental Network

In recent years, research on measuring on environmental analysis performance and efficiency has expanded with an approach to data envelopment analysis with undesirable outputs. Fare et. al. (2005) calculated the environmental performance of U.S. generation stations to consider the effect of the first phase of the acid rain conflict scheme. Cooley (2006) measured the technical, economic and environmental efficiency of animal husbandry units in Belgium. Sajjadifar et. al. has calculated the DEA with undesirable outputs. In this study, considering economic activities, carbon dioxide energy consumption in the production process, simultaneously, and using data envelopment analysis of undesirable outputs, the energy efficiency of Iran and neighboring countries in period (2007-2012) has been evaluated. By using modified DEA model with undesirable output, Wang et. al. (2013) checked out environmental and energy efficiency for twenty-nine China regions in the period 2000-2008. The result of this experimental study shows the average of technical efficiency, environmental efficiency, specialized efficiency and cost efficiency with 0.897, 0.843, 0.985 and 0.883, respectively [3].

DEA in multi-stage decision making units was first used by Farell and Primont (1984) [4] to evaluate the performance of producing units with multi- stage generation process and then, Whittaker (1995) [5] developed it. In 1991, by using introductory inputs, Farel [6] examined the efficiency of a producing unit. Farell and Primont (1993) [7] presented a method for analyzing performance and measuring efficiency of a bank. Fare and Grosskopf (1996) [1] have evaluated multi- stage units which named Network Data Envelopment Analysis and Dynamic Networks in a way, how network activities are effective in the next period. A two-stage network was proposed as the simplest network structure by Wang et. al. (1997) [8] that overall efficiency of the DMU is calculated regardless of the intermediate products. Then, Seiford (1999) [9] developed the approach and used it to evaluate the function of fifty-five banks. After that, Chen and Zhu (2004) [10] introduced a model fr the efficiency of a two- stage network in which they assigned a weight to efficiency of every stage based on its importance. Liang et. al. (2006) [11] presented models for assessing the supply chain efficiency (overall efficiency) and its members (individual efficiency) in the following way:

- Calculating the efficiency of the general process (overall efficiency) and then computing the efficiency of each stage according to the efficiency of general process
- Calculating the efficiency of the general process as the average efficiency of each stage

Liang et. al. (2008) [12] calculated the efficiency of the whole two- stage system as a product of the efficiency of the stages.

Assuming constant return to scale, Kao and Hwang (2008) [13] computed the efficiency of the two-stage network as a product of the efficiency of stages and by using the multiplier model. Chen et. al. (2009) [14] have modified the proposed methods by presenting a model which the total system efficiency is calculated with a total weighted efficiency of two stages. Wang and Chen (2010) [15] developed Kao's Model in the variable return to scale and generalized Chen et. al. (2009) [16] model by computing overall efficiency and by using the weighted average efficiency of the stages. Using common input in the process of constructing a two- stage unit, Zha and Liang (2010) [17] evaluated it.

3. Background

Before presenting the new model to evaluate the efficiency of two-stage DMUs with undesirable outputs by multiplier models, we would like to remind some preliminary as follow.

Assume that there are *n* DMUs to be evaluated. Each DMU has *m* inputs and *s* outputs. The inputs and outputs of each DMU*j* *(j=1,2,…,n)* are denoted as *xij* (*i* = 1, 2,...,*m*) and y_{rj} ,(r=1, 2,...,s)

$$T=\left\{\binom{X}{Y} \mid X \geq \prod_{i=1}^{k} x_i^{\lambda_i} \ \&Y \leq \prod_{j=1}^{n} y_j^{\lambda_j}\right\}$$

respectively, all data are non-negative and there are at least one positive component of each input and output. Production possibility set (PPS) is achieved by four principles observed, free disposability, geometric convexity and minimum extrapolation.

The multiplier model to measure performance of the under evaluated DMU (DMU_o), by considering constant return to scale, is as follow:

$$\begin{aligned}
&\text{Min } \theta && (1)\\
&\prod_{j=1}^{n} x_{ij}^{\lambda_j} \leq x_{io}^{\theta} , && i=1,\dots,m\\
&\prod_{i=1}^{n} y_{rj}^{\lambda_j} \leq y_{io} , && r=1,\dots,s\\
&\lambda_j \geq 0 , && j=1,\dots,n
\end{aligned}$$

4. Innovation.

As the Figure.1, Assume that there are *n* DMUs to be evaluated as the external input vector $X_1=(x_{1j},x_{2j},\dots,x_{m1j})$ is used to product the desirable output $Y_1=(y_{1j},y_{2j},\dots,y_{s1j})^T$ and undesirable output $U_1=(u_{1j},u_{2j},\dots,u_{o_1j})^T$ in the first stage. The second sub-unit (stage two) utilizes the external input $X_2=(x_{1j},x_{2j},\dots,x_{m2j})$ and undesirable output $U_1=(u_{1j},u_{2j},\dots,u_{o_1j})^T$, from stage 1, to product the desirable output $Y_2=(y_{1j},y_{2j},\dots,y_{s2j})^T$ and undesirable output $U_2=(u_{1j},u_{2j},\dots,u_{o_2j})^T$. Now, we would like to introduce model (2) in order to measure the overall efficiency of the two-stage DMU by using multiplier model in input-oriented with constant return to scale.

$$\begin{aligned}
&\text{Min } \left(\theta_1{}^{\rho} * \theta_2{}^{(1-\rho)}\right)\\
&\text{s.t.} && (2)\\
&\prod_{j=1}^{n}(x_{i_1j}^1)^{\lambda_j} \leq (x_{i_1d}^1)^{\theta_1}, && i_1=1,\dots,m_1\\
&\prod_{j=1}^{n}(y_{r_1j}^1)^{\lambda_j} \geq y_{r_1d}^1 , && r_1=1,\dots,s_1\\
&\prod_{j=1}^{n}(u_{v_1j}^1)^{\lambda_j} = \prod_{j=1}^{n}(u_{v_1j}^1)^{\mu_j}, && v_1=1,\dots,o_1\\
&\prod_{j=1}^{n}(x_{i_2j}^2)^{\mu_j} \leq (x_{i_2d}^2)^{\theta_2} , && i_2=1,\dots,m_2\\
&\prod_{j=1}^{n}(y_{r_2j}^2)^{\mu_j} \geq y_{r_2d}^2 , && r_2=1,\dots,s_2\\
&\prod_{j=1}^{n}(u_{v_2j}^2)^{\mu_j} \leq u_{v_2d}^2 , && v_2=1,\dots,o_2\\
&\lambda_j,\mu_j \geq 0 , && j=1,\dots,n , \ \rho \geq 0
\end{aligned}$$

It should be noted that the value of the coefficient ρ is determined by the decision maker in the objective function.

Now, the model (2) is transformed using exponential logarithm definition, as model (3).

$$\begin{aligned}
&\text{Min } (\rho\ln(\theta_1)+(1-\rho)\ln(\theta_2))\\
&\text{s.t.} && (3)\\
&\sum_{j=1}^{n} \lambda_j\left(\ln x_{i_1j}^1\right) \leq \theta_1\left(\ln x_{i_1d}^1\right)\\
&i_1=1,\dots,m_1\\
&\sum_{j=1}^{n} \lambda_j(\ln y_{r_1j}^1) \geq (\ln y_{r_1d}^1)\\
&r_1=1,\dots,s_1
\end{aligned}$$

$\sum_{v_1=1}^{o_1} \lambda_j (\ln u_{v_1 j}^1) \leq (\ln u_{v_1 j}^1)$,
$v_1=1,\ldots,o_1$
$\sum_{j=1}^{n} \mu_j (\ln x_{i_2 j}^2) \leq \theta_2 (\ln x_{i_2 d}^2) i_2=1,\ldots,m_2$
$\sum_{j=1}^{n} \mu_j (\ln u_{v_1 j}^1) \geq (\ln u_{v_1 j}^1)$, $v_1=1,\ldots,o_1$
$\sum_{j=1}^{n} \mu_j (\ln y_{r_2 j}^2) \geq (\ln y_{r_2 d}^2)$, $r_2=1,\ldots,s_2$
$\sum_{j=1}^{n} \mu_j (\ln u_{v_2 j}^2) \leq (\ln u_{v_2 d}^2)$ $v_2=1,\ldots,o_2$
$\lambda_j, \mu_j \geq 0$, $j=1,\ldots,n$, $\rho \geq 0$

Finally, a simplified form of model (4) is obtained by changing the variables as follow:
$(\ln x_{i_1 j}^1) = x_{i_1 j}^{*1}, (\ln y_{r_1 j}^1) = y_{r_1 j}^{*1}$,
$(\ln u_{v_1 j}^1) = u_{v_1 j}^{*1}$, $(\ln x_{i_2 j}^2) = x_{i_2 j}^{*2}$
$(\ln x_{i_2 j}^2) = x_{i_2 j}^{*2}$, $(\ln y_{r_2 j}^2) = y_{r_2 j}^{*2}$
, $(\ln u_{v_2 j}^2) = u_{v_2 j}^{*2}$

Min $(\rho \ln(\theta_1) + (1-\rho)\ln(\theta_2))$
s.t. (4)
$\sum_{j=1}^{n} \lambda_j (x_{i_1 j}^{*1}) \leq \theta_1 (x_{i_1 d}^{*1})$, $i_1=1,\ldots,m_1$
$\sum_{j=1}^{n} \lambda_j (y_{r_1 j}^{*1}) \geq (y_{r_1 d}^{*1})$, $r_1=1,\ldots,s_1$
$\sum_{v_1=1}^{o_1} \lambda_j (u_{v_1 j}^{*1}) \leq (u_{v_1 d}^{*1})$, $v_1=1,\ldots,o_1$
$\sum_{j=1}^{n} \mu_j (x_{i_2 j}^{*2}) \leq \theta_2 (x_{i_2 d}^{*2})$, $i_2=1,\ldots,m_2$
$\sum_{j=1}^{n} \mu_j (u_{v_1 j}^{*1}) \geq (u_{v_1 d}^{*1})$, $v_1=1,\ldots,o_1$
$\sum_{j=1}^{n} \mu_j (y_{r_2 j}^{*2}) \geq (y_{r_2 d}^{*2})$, $r_2=1,\ldots,s_2$
$\sum_{j=1}^{n} \mu_j (u_{v_2 j}^{*2}) \leq (u_{v_2 d}^{*2})$, $v_2=1,\ldots,o_2$
$\lambda_j, \mu_j \geq 0$, $j=1,\ldots,n$, $\rho \geq 0$

It should be considered that in case of existence the weak disposal, the two noted constraints by (**) are equal, then the previous model is rewired as a model (5).
Min $(\rho \ln(\theta_1) + (1-\rho)\ln(\theta_2))$
s.t. (5)
$\sum_{j=1}^{n} \lambda_j (x_{i_1 j}^{*1}) \leq \theta_1 (x_{i_1 d}^{*1})$, $i_1=1,\ldots,m_1$
$\sum_{j=1}^{n} \lambda_j (y_{r_1 j}^{*1}) \geq (y_{r_1 d}^{*1})$, $r_1=1,\ldots,s_1$
$\sum_{v_1=1}^{o_1} \lambda_j (u_{v_1 j}^{*1}) = \sum_{j=1}^{n} \mu_j (u_{v_1 j}^{*1})$
$v_1=1,\ldots,o_1$
$\sum_{j=1}^{n} \mu_j (x_{i_2 j}^{*2}) \leq \theta_2 (x_{i_2 d}^{*2})$, $i_2=1,\ldots,m_2$
$\sum_{j=1}^{n} \mu_j (y_{r_2 j}^{*2}) \geq (y_{r_2 d}^{*2})$, $r_2=1,\ldots,s_2$
$\sum_{j=1}^{n} \mu_j (u_{v_2 j}^{*2}) \leq (u_{v_2 d}^{*2})$, $v_2=1,\ldots,o_2$
$\lambda_j, \mu_j \geq 0$, $j=1,\ldots,n$, $\rho \geq 0$

Due to the existence of a logarithmic in the objective function model (4) and (5) are non-linear but by regarding the logarithmic function is an ascending function, we use the equation $(p\theta_1 + (1-p)\theta_2)$ in the objective function. Hence, model (6) is obtained.
Min $(\rho\theta_1 + (1-\rho)\theta_2)$
s.t. (6)
$\sum_{j=1}^{n} \lambda_j (x_{i_1 j}^{*1}) \leq \theta_1 (x_{i_1 d}^{*1})$, $i_1=1,\ldots,m_1$
$\sum_{j=1}^{n} \lambda_j (y_{r_1 j}^{*1}) \geq (y_{r_1 d}^{*1})$, $r_1=1,\ldots,s_1$
$\sum_{v_1=1}^{o_1} \lambda_j (u_{v_1 j}^{*1}) = \sum_{j=1}^{n} \mu_j (u_{v_1 j}^{*1})$
$v_1=1,\ldots,o_1$
$\sum_{j=1}^{n} \mu_j (x_{i_2 j}^{*2}) \leq \theta_2 (x_{i_2 d}^{*2})$, $i_2=1,\ldots,m_2$
$\sum_{j=1}^{n} \mu_j (y_{r_2 j}^{*2}) \geq (y_{r_2 d}^{*2})$, $r_2=1,\ldots,s_2$
$\sum_{j=1}^{n} \mu_j (u_{v_2 j}^{*2}) \leq (u_{v_2 d}^{*2})$, $v_2=1,\ldots,o_2$
$\lambda_j, \mu_j \geq 0$, $j=1,\ldots,n$, $\rho \geq 0$

5. Application Analysis.

In this section, the concepts which are expressed in section (4) is described in the form of an application analysis to better understand the issue. So, we collected PetroPoyan company 10-year data. The company is a manufacturer of hosepipe and plastic goods. Furthermore, we consider each year as a decision making unit. This unit uses three inputs annually to produce four desirable outputs and an undesirable output, which is wastage of production. Since, this unit has the ability to utilize wastage and return it to the production cycle, by combining two desirable external outputs and wastage in special mixers, desirable and undesirable outputs are generated. After collecting data, the relevant information is shown in table (1).

Table (1): Annual Report on Input and Outputs of PetroPouyan Company from 1384 to 1394

		1385	1386	1387	1388	1389	1390	1391	1392	1393	1394
x	Raw Material Salary Surplus Producti on	30,904, 536,605 273,697 ,638 844,865 ,077	33,591,8 87,615 304,108, 486 971,109, 284	36,512, 921,320 337,898 ,318 1,116,2 17,567	39,687 ,957,9 57 375,44 2,576 1,283, 008,68 9	43,139 ,084,7 36 417,15 8,418 1,474, 722,64 2	46,890 ,309,4 95 436,50 9,353 1,695, 083,49 6	50,967, 727,71 2 515,01 0,392 1,948, 371,83 5	55,399, 704,035 572,233 ,769 2,265,5 48,645	60,412, 323,249 637,862,1 25 268,465,2 15	60,640, 395,248 705,402, 325 2,971, 361,554
y	Hosepipe Type (1) Hosepipe Type (2) P.V.C. Compou nd P.V.C. Granule	12,396, 277,476 10,689, 325,992 331,249 ,581 9,036,1 62,807	13,498,0 86,524 11,637,4 39,680 360,469, 236 9,837,94 2,041	14,699, 108,313 12,671, 126,144 392,651 ,384 10,711, 727,043	16,008 ,418,8 30 13,797 ,792,0 20 427,55 7,263 11,664 ,077,3 26	17,436 ,895,7 07 15,026 ,130,5 06 465,64 4,439 12,702 ,334,8 96	18,994 ,258,7 97 16,365 ,274,8 80 507,13 0,145 13,834 ,113,8 70	20,693, 118,32 0 17,825, 306,68 8 55,239, 525 15,068, 710,75 5	22,556, 701,120 19,426, 690,100 602,000 ,000 16,422, 173,190	24,674,86 8,160 21,179,49 6,500 338,246,0 00 18,025,10 3,250	24,719,6 56,750 21,132,2 26,700 391,088, 960 18,162,4 02,300
u	Wastage	429,537 ,059	466,927, 238	507,529 ,606	551,66 2,616	599,63 3,278	651,77 5,302	708,45 1,415	770,055 ,886	839,731,2 93	836,837, 454
x	Salary Surplus Producti on	49,796, 904 18,153, 493	54,127,0 70 19,732,0 58	58,833, 772 21,447, 889	63,949 ,752 23,312 ,922	69,510 ,600 25,340 ,133	75,555 ,000 27,543 ,623	82,125, 000 29,838, 721	91,250, 000 33,265, 245	101,250,0 00 36,215,46 5	112,500, 000 41,236,5 42
y	Grade 2 Hosepipe	282,739 ,464	307,331, 010	334,049 ,940	363,09 8,862	40,091 ,058	428,99 7,114	466,27 6,986	509,533 ,794	557,326,5 75	572,148, 268
u	Scrap Wastage	214,786 ,529	233,463, 619	253,764 ,803	275,83 1,308	299,81 6,639	325,88 7,651	354,22 5,708	385,027 ,943	419,865,6 47	418,418, 727

Table (2): Petropouyan's Efficiency over a Decade Based on Wastage Recycling

Year	**Efficiency**
1385	1
1386	0.89
1387	0.99
1388	0.84
1389	0.99
1390	0.90
1391	0.92
1392	1
1393	1
1394	0.99

As shown in table (1), x^1 is used as input to the first stage. Also, y^1 and u^1 are the desirable and undesirable outputs of the first stage, respectively. In the second stage, the wastage (u^1) is used with the external input x^2 to produce the desirable output y^2 and undesirable output u^2 that is wastage. Now, by using the collected data, we consider model (4).

After reviewing the data in Table (1) and implementing them using model (4), the efficiency of each year is shown in Table (2).

According to the review of table (2), we conclude that PetroPoyan was efficient in the years of 1385, 1392 and 1394 in the field of wastage recycling and production. In other years, it must change its policy to be effective. Also, it should be noted that this efficiency in the years of 85, 92 and 94 does not present the 100% efficiency of this company. Because, there are many factors affecting the production of a unit and we only consider sum of them.

6. Conclusion

The following results are achieved after this research.

1- Due to the fact that in the economy, we usually dealing with units with multiple inputs and outputs, using the multiplier model are the best one to evaluate the efficiency. Therefore, this problem has solved in two-stage systems by providing the model (4) and (5) based on strong and weak disposal.
2- After investigating the results of PetroPoyan, we found that the company is efficient in the process of recycling wastage, in 1385, 1392 and 1394. While in many countries, one hundred percent of these wastage products become usable products, due to the lack of facilities and professional equipment, the company is not able to turn the entire wastage of the first stage into useful and usable materials.

6. Suggestions.

1- Not only does the amount of efficiency in multi-stage environmental systems is very important but also the long-term decisions have been made to reduce undesirable outputs in the environmental. Accordingly, it is possible to calculate the efficiency of two-stage systems with undesired output in a multiplier model with variable return to scale. This is because that can be determined what influence is on the environmental performance by changing the scale of a DMU.
2- Determining the ideal point, in order to get better for unit, could be one of the studies to be considered in the future.

References

[1] Farell R.,Grosskopf S.,1996. Intertemporal production Frontiers, With Dynamic DEA. Kluwer Academic Poblishers.

[2] Charnes, A., Cooper, W. W., Rhodes, E., 1978. Measuring the efficiency of decision making units, European Journal of Operational Research. 2 (6), 429-444.

[3] Wang, K., Yu, S., & Zhang, W. (2013). China's regional energy and environmental efficiency: a DEA window analysis based dynamic evaluation. Mathematical and Computer Modelling, 58(5), 1117-1127.

[4] Farell R., Primont D.,1984. efficiency measures for multi-plant firms,Operations Reaserche Letters 3, 257-260.

[5] Farell R.,Wittaker G.,1995. An intermediate input model of dairy production using complex survey data, Journal of Agricultural Economic .46,201-213.

[6] Farell R.,1991. Measuring Farrel efficiency for a firm with interediate inputs,Academia Economic Papers 19(2):329-340.

[7] Farell R., Primont D.,1993. Measuring the efficiency of Multi-Unit Banking, Journal of Banking and Finance.3,539-544.

[8] Wang Y.M., Chin K.S., 2010, Some Alternative DEA models for Tow-stage process, Expert System with Applications. 37(12), 8799-8802.

[9] Siford L.M., Zhu J., 1999. Profitability and marketability of the top 55 US commercial banks, Management Science. 45, 1270-1288.

[10] Chen Y.,Zhu J., 2004, Measuring information technology' s indirect impact on firm performance, Information Technology and Management Journal. 5,9-22.

[11] Liang L., Yang F., Cook W.D., Zhu J.,2006. DEA models for supply chain efficiency evaluation, Annals of Operations Research. 145, 35-49.

[12] Liang L., Cook W.D., Zhu J., 2008. DEA models for Two-stage processes, Game approach and efficiency decomposition. Naval Research Logistics 55, 643.

[13] Kao C., Hwang S.N., 2008, Efficiency decomposition in Two-stage data envelopment analysis, An application to non-life insurance companies in TaiWan. European Journal of Operational Research. 185, 418-429.

[14] Chen Y., Liang L.,Zhu J., 2009b, Equivalence in Two-stage DEA approaches, European Journal of Operational Research. 193(2), 600-604.

[15] Tone, K, Tsutsui, M., (2009). “Network DEA: A slack-based measure approach.” European Journal of Operational Research. 197, 243-252.

[16] Chen, Y., Cook, W.D., Li, N., Zhu, J., (2009). “Additive efficiency decomposition in two-stage DEA”. European Journal of Operational Research 196, 1170–1176.

[17]Zhu J., Liang L., 2010, Two-stage cooperation model with input freely distributed among the stage, European Journal of Operational Research. 205, 332-338.